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A Study on the New Developments in Longevity Risks Transfer Market

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Abstract

In this paper, we introduce the challenges confronting the insurance industry with regards to population ageing. The primary aim of this study is to analyze a new evolution in the longevity risk transfer marketplace. Constructing the longevity swap transaction table based on the capital markets. Estimating the longevity risks associated with it, hedging the aggregate longevity risks using index based longevity-linked and mortality linked securities. This study reveals the major issues in the transfer market and stops with an interrogation of how new longevity derivatives can be used as a risk management instrument for hedging longevity risks.

Keywords

Age Pension, Longevity Risks, Hedging

Introduction

Longevity risks are a risk to which a pension fund (or) life insurance company could be exposed as a result of higher than expected payout ratios. With life spans lengthening due to improved medical care, breakthrough in disease control and prevention, healthier lifestyles, etc., some pension funds, insurers and reinsurers find their exposure to longevity risk growing, therefore the risk of paying out on pensions and annuities for longer than anticipated is increasing.

Longevity risk and related capital market solution have grown increasingly important in recent years, both in academic research and in the market. We refer to as the new life market, i.e., the capital market that trades longevity-linked assets and liabilities. Longevity risk growth around the world are putting more and more pressure to on government, pension funds, life insurance companies, as well as individuals, to deal with the longevity risk they face. At the same time, capital market can, in principle, provides vehicles to hedge longevity risk effectively and transfer the risk.

For those unwilling or unable to manage it to those willing to invest in this risk in exchange for appropriate risk adjusted returns (or) to those who have a counterpoising risk that longevity risk can hedge. Many new investment products have been created both by the insurance/reinsurance industry and by the capital market Longevity swaps are an example of a successful insurance-linked security. Some new innovative capital market solution for transferring longevity risk include longevity (or survival bonds), longevity swap and mortality (q-forward) contract. In this study we are going to see about the how longevity risks can be tackled through life market and what are the new instrument available in the market.

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Tackling Longevity Risk

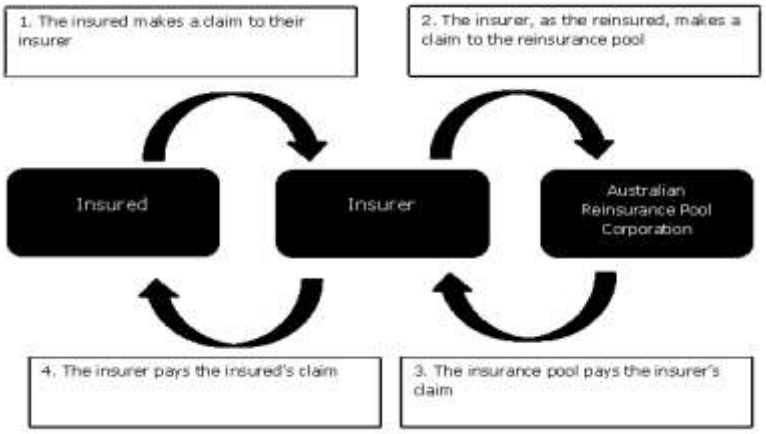
Currently, there are three ways for financial institution to tackle longevity risk.

- ✓ Traditional insurance and reinsurance
- ✓ Natural Hedging
- ✓ Capital Market Solution

Traditional Insurance and Reinsurance

An organization can transfer longevity risk in a number of ways. The simplest way is through a single premium immediate annuity (SPIA), whereby a risk holder pays a premium to an insurer and passes both asset and liability risk. This strategy would involve a large transfer of assets to a third party, with the possibility of material credit risk exposure. Alternatively, it is possible to eliminate only longevity risk while retaining the underlying assets via reinsurance of the liability. In this model, instead of paying a single premium, the premium is spread over the likely duration of 50 or 60 years (expected term of liability), aligning premiums and claims and moving uncertain cash flows to certain ones. Traditional insurance and reinsurance, in which the risk is passed on to an insurer or reinsurer after paying a premium. For Example: an insurer can purchase a reinsurance contract to hedge away the risk, or a pension plan can buy annuities from an insurer to cover the risk for its members.

Figure 1: Structure of traditional insurance and reinsurance



Natural Hedging

Natural hedging, which exploits the opposite movement between the values of annuities and life insurance. This diverse approach may be viable for large institutions with the financial resources and structure to sell both kinds of products.

It may also be implemented by using an astonished mortality swap to build an external hedge between two separate parties who have life insurance and annuities respectively on their books.

Capital Market Solution

Capital market solution, including insurance securitization, mortality – or longevity – linked securities, derivatives. This method has seen much development in the UK in recent years. Insurance securitization means securitizing a class of business as a complex bundle into highly structured securities for sale to market investor. Some

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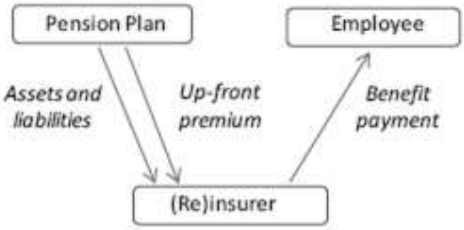
popular de-risking solution such as buy-ins, buy-outs, and longevity swaps are bespoke transactions for hedging specific portfolios.

Buy – Out

In a buy-out transaction all of the pension fund's assets and liabilities are transferred to an insurer in return for an up-front premium. The pension liabilities and their offsetting assets are removed from the pension fund sponsor's balance sheet and the insurer takes over full responsibility for making payments to pensioners.

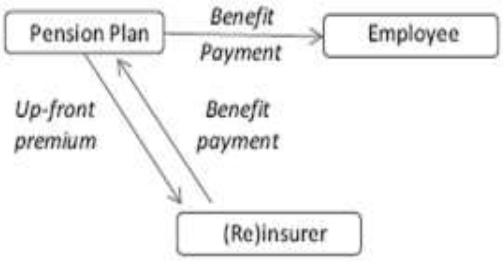
Figure 2: Structure of buy-out transaction



Buy – In's

In a buy-in, the sponsor pays an up-front premium to the insurer who then makes periodic payments to the pension fund sponsor equal to those made by the sponsor to its members. This "insurance policy" is held as an asset by the pension plan for which the premium is the cost of the insurance policy that guarantees payments even if retirees live longer than expected.

Figure 3: Structure of buy-out transaction



The apparent high cost of buy-outs and buy-ins is a result of insurance companies being typically subject to more stringent regulation than pension funds, such as the necessity to hold resilience test reserves in case of extreme scenarios—while pension funds can temporarily run funding gaps (where the discounted present value of their liabilities exceeds the value of their assets. Additionally, buy-outs can appear expensive, in part, because any initial underfunding requires a lump-sum payment by the sponsor to reach full funding before the plan can be sold to a third party. This option nevertheless remains the most used in the UK pension market because many corporate sponsors prefer not to have a DB plan to weigh on their balance sheets. Buy-outs appear to be particularly attractive to smaller pension schemes for whom the pricing gap is not as significant, and who do not have the capability to hedge individual risks on their own. In fact, almost all of the large UK LRT transactions (i.e., larger than £500 million) since 2007 have been either buy-ins or longevity swaps. Furthermore, all of the recent large LRT transactions in the United States have been buy-outs. Compared to the other types of longevity risk

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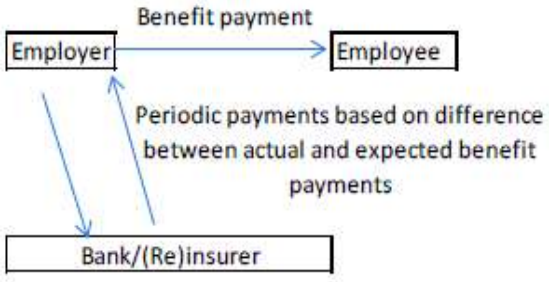
transfer, buy-outs have the potential disadvantage for employees that the government-backed guarantee of their pension entitlements is lost , that is should such a guarantee exist.

Longevity Swap

In a longevity swap, the pension fund obtains a similar protection from higher-than-expected pension payouts. The plan sponsor makes periodic fixed "premium" payments to the swap counterparty, which in turn makes periodic payments that are based on the difference between the actual and expected benefit payments. The sponsor maintains full responsibility for making benefit payments to its employees. An advantage of buy-ins and swaps is that they can be used to hedge the longevity risk associated with specific subsets of the underlying population. An advantage of swaps is that longevity risk can be isolated, whereas buy-in and buy-out transactions typically also transfer the investment risk of the assets. Longevity swaps can also be combined with other types of derivative contracts, such as inflation, interest rate and total return swaps, to create so called "synthetic" buy-ins that do transfer all of the risks.

Figure 4: Structure of a Longevity Swap Transaction



Furthermore, swaps are more likely to activate broader capital market interest. For example, the €12 billion longevity swaps between Dutch insurer AEGON and Deutsche Bank used standard International Swaps and Derivatives Association (ISDA) documentation. Also, it had a 20-year maturity with a closed mechanism that determined the final payment, as opposed to the open-ended maturities of more traditional transactions. In addition, the longevity-indexed floating payments are floored and capped so that investors are not exposed to open-ended risk if longevity is either under- or overestimated. Finally, it used a longevity index based on publicly-available data to drive cash flows, as opposed to the actual longevity experience of AEGON's annuity book.

Longevity swaps require the posting of high-quality liquid securities as collateral, which can potentially involve significant costs. However, the collateral requirements are based on only the net payments - the difference between what each swap participant owes the other. Biffis et. al., (2011) show that the cost of collateral to secure longevity swaps can be relatively low, especially when counterparty default risk and collateral rules are symmetric. However, transactions with (re) insurers typically take the form of insurance contracts that may not involve collateral posting.

While there are similarities between insurance and swap transactions, there is an important difference regarding counterparty risk. In longevity insurance, the risk ceding party is exposed to an insurance company and to an insurance company only. In swaps, the risk may be distributed more broadly, yet may return to the swap intermediary (which could be an investment bank) in case of a tail event (e.g. Cure for cancer). Whether this matters in terms of the economics depends on how the committed risk-capital of the investors and the investment bank on the one hand compare to the

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committed risk-capital of the insurer on the other. There is reason to suspect that, because of more stringent regulation, insurance-based transactions lead to more complete risk transfer, as a result of lower counterparty risk.

Table 1: Recent Longevity Swaps Transaction

Pension Fund / Sponsor	Provider(s)	Solution(s)	Amount	Date
Delta Lloyd	RGA Re	Index-based longevity swap	€12 billion	Jun 2015
Aegon	Canada Life Re	Longevity swap and reinsurance	€6 billion	Jul 2015
Manweb (ScottishPower)	Abbey Life	Longevity swap	£1 billion	Aug 2016
AXA France	RGA Re	Longevity swap and reinsurance	€1.3 billion	Nov 2016
Pension Insurance Corporation	SCOR	Longevity swap and reinsurance	£1 billion	Jul 2017
MMC UK Pension Fund	Canada Life Reinsurance, The Prudential Insurance Company of America (PICA)	Longevity swap and reinsurance	£3.4 billion	Sep 2017

Figure 5: UK Longevity Risk Transfer Transactions (GBP Millions of Total Pension Liabilities)

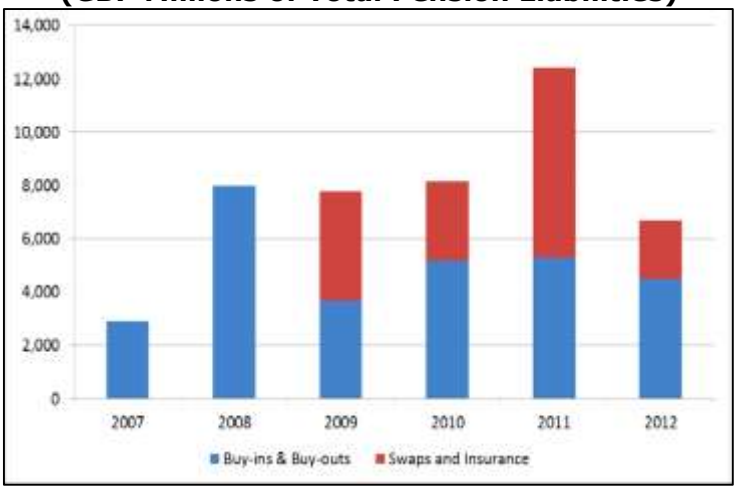

Figure 6: Removing Risk using a Longevity Swap

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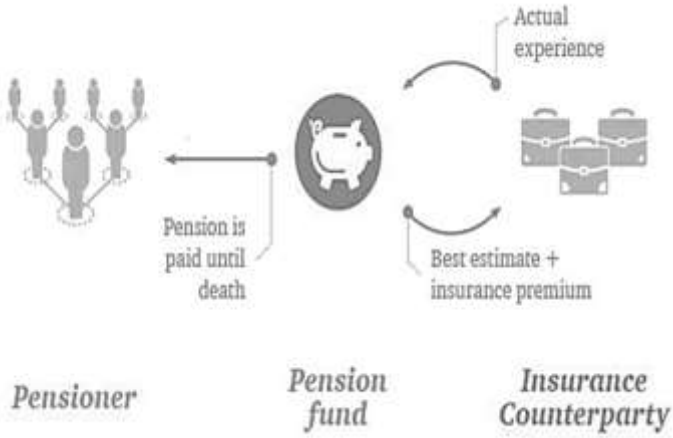

On the other hand, standardized mortality or longevity-linked securities and derivatives have their cash flows linked to a selected reference or index population, instead of the population underlying the portfolio to be hedged. There would then be a potential mismatch between the hedging tool and the portfolio, in terms of demographic difference. Moreover, a small portfolio would have high sampling variability, which makes it more likely to derive from the index population. Also, the payoff structures would often be different between the hedging instrument and the portfolio being hedged. All these discrepancies are referred to as longevity basis risk, and are under intense research. The major types of standardized, index-based securities proposed in the longevity literature are listed in Table 1.

Index – Based Securities

The simplest type of longevity and mortality derivatives is a mortality forward rate contract, which we call a q-forward, S-forward, K-forward and mortality option. We will now describe some new index linked securities. Broadly speaking, these can be classified into various types of: (1) longevity bond, (2) longevity swap, (3) mortality option, and (4) survivor options. These securities have the usual features which we would expect of bonds, swaps, forward, and options. In particular, there is the distinction between those which are traded over-the-counter (e.g. Swaps) and those which are traded in organized exchanges (e.g. Forward). The former have the attraction that they can be tailor made to the requirements of a user (which keeps down basis risk), the latter has the attraction of greater market liquidity (which facilitates unwinding)

Longevity Bonds

The payout on longevity bonds depends on the longevity experience of a given population, so that the payment is related to the number of survivors in the population. Basically, it would pay out more, the higher the proportion of survivors in the reference population. One disadvantage is that, unlike a swap, the bond buyer makes a large up-front payment to the issuer, resulting in counterparty risk exposure to the issuer. However, counterparty risk would be minimized if the bonds are issued by a high-quality sovereign or supranational, or by a special purpose vehicle that invests the proceeds in low risk highly liquid fixed income securities, the income from which covers the bond payouts. The issuer might also transfer some or all of the longevity risk to a reinsurer probably via a longevity swap contract.

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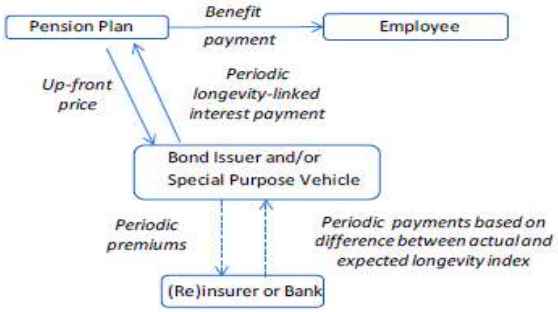
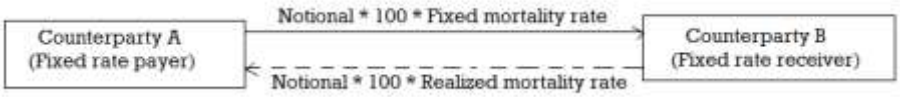
Figure 7: Structure of Longevity Bond Transaction



q-FORWARD

A q-forward is an agreement between two parties to exchange at a future date (the maturity of the contract) an amount proportional to the realized mortality rate of a given population (or subpopulation), in return for an amount proportional to a fixed mortality rate that has been mutually agreed at inception. In other words, a q-forward is a zero-coupon swap that exchanges fixed mortality for realizing mortality at maturity. The reference rate for settling the contract is the realized mortality rate as determined by the appropriate index, such as the Life Metrics Index.

Figure 8: Structure of q-forward



In a fair market, the fixed mortality rate at which the transaction takes place defines the "forward mortality rate" for the population (or subpopulation) in question. If the q-forward is fairly priced, no payment changes hands at the inception of the trade. At maturity, however, a net payment will be made by one counterpart or the other.

S-FORWARD

The pricing of the s-forward, which is "an agreement between two counterparties to exchange at a future date(the maturity of the contract) an amount equal to the realized survival rate agreed at the inception of the contract". A related contract is the s-forward (or) survivor forward contract. Which is based on the survivor index, S(t,x) which itself is derived from the more fundamental mortality rates. A "s-forward" is the basic building block of a longevity (survivor) swap. A longevity swap is composed of a stream of s-forward with different maturity dates. On another word the payoffs are similar to those of the q-forward, with the mortality rate being replaced by the percentage of the index population who are alive at maturity.

K-FORWARD

By the previously developed q-forward and s-forwards, we propose as a concept a standardized longevity security called k-forward, which can be regarded as a zero coupon swap that exchanges on maturity date a fixed amount for a random amount that is proportional to a CBD mortality index at some future time. Mathematically, it can be expressed as k-forward written on the i^{th} CBD (castarophe bonds) mortality index, for $i = 1, 2$. Suppose that the k-forward is issued at time t_0 and matures at time T (where

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$T > t_0$). From the perspective of the fixed rate receiver, the payoffs of this k-forward at maturity is $F^{(i)}(T,K) = K - k_T^{(i)}$. Where K represents the fixed leg that is predetermined when the k-forward is issued. In other word to a fixed rate receiver, a payoff of $\left(K_{T,i}^{forward} - K_{T,i}\right)$ is made after $T+1$ years (maturity), in which $K_{T,i}^{forward}$ is the forward CBD mortality index set at the starts , and $K_{T,i}$ is the CBD mortality index calculated from the actual observation of the index population in year T; for a floating rate receiver, the payoff is $\left(K_{T,i} - K_{T,i}^{forward}\right)$ instead.

Mortality Option

Mortality options give payoffs which are nonlinear functions of underlying variables, and a natural first question with options is why market participants would prefer the non-linear payoffs which they generate over the (broadly) linear payoffs of, say, annuity futures. A key part of the answer must be that options might be useful too:

- Hedgers, who might wish to protect their downside exposure, but leave any upside potential; and
- Speculators who want to trade views on volatility rather than views on the level of mortality (or related, e.g. annuity) rates.

For both of these purposes, options are (usually) the best type of instrument. The valuation of options as well as their risk management requires the use of a good mortality model. To the fixed rate receiver, the payoff of the max mortality option is $q_{X,T} - q_{X,T}^{Strike}$. The main feature of mortality option is to a call holder, a payoff of max $\left(q_{X,T} - q_{X,T}^{Strike}, 0\right)$ is made after $T+1$ years (maturity), in which $q_{X,T}^{Strike}$ is a fixed rate (strike price) set at the start, and $q_{X,T}$, is the actual mortality rate of the index population observed in year T; for a put holde, the payoff is max $\left(q_{X,T}^{Strike} - q_{X,T}, 0\right)$ instead.

Survivor Option

The main features of survivor option are the payoff is similar to that of mortality option, with the mortality rate being replaced by the percentage of the index population who alive on maturity date.

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Table 2: Index-Based Longevity-Linked and Mortality Linked Securities

Security	Main Features
Longevity bond	Coupons are linked to the percentage of the index population who are alive on the coupon payment dates
Longevity swap	Two series of future cash flows are exchanged, one of which is linked to the percentage of the index population who are alive on the payment dates, and the other series is fixed at the start
q-forward	To a fixed rate receiver, a payoff of $\left(q_{x,T}^{\text{forward}} - q_{x,T}\right)$ is made after $T+1$ years (maturity), in which $q_{x,T}^{\text{forward}}$ is the forward mortality rate set at the start, and $q_{x,T}$ is the actual mortality rate of the index population observed in year T; for a floating rate receiver, the payoff is $\left(q_{x,T} - q_{x,T}^{\text{forward}}\right)$ instead
S-forward	The payoffs are similar to those of the q-forward, with the mortality rate being replaced by the percentage of the index population who are alive on maturity
K-forward	To a fixed rate receiver, a payoff of $\left(\kappa_{T,t}^{\text{forward}} - \kappa_{T,t}\right)$ is made after $T+1$ years (maturity), in which $\kappa_{T,t}^{\text{forward}}$ is the forward CBD mortality index set at the start, and $\kappa_{T,t}$ is the CBD mortality index calculated from the actual observations of the index population in year T; for a floating rate receiver, the payoff is $\left(\kappa_{T,t} - \kappa_{T,t}^{\text{forward}}\right)$ instead
Mortality option	To a call holder, a payoff of $\max\left(q_{x,T} - q_{x,T}^{\text{strike}}, 0\right)$ is made after $T+1$ years (maturity), in which $q_{x,T}^{\text{strike}}$ is a fixed rate (strike price) set at the start, and $q_{x,T}$ is the actual mortality rate of the index population observed in year T; for a put holder, the payoff is $\max\left(q_{x,T}^{\text{strike}} - q_{x,T}, 0\right)$ instead
Survivor option	The payoffs are similar to those of the mortality option, with the mortality rate being replaced by the percentage of the index population who are alive on maturity

Table 3: Transaction Table

Fund/Sponsor	Provider(S)	Solution
Legal & General	Unnamed UK Pension	Longevity Swap & Reinsurance
AVIVA	Prudential insurance Company of America	Longevity Reinsurance
NN Group	Hannover Re	Index-Based longevity Hedge
SSE PLC Pension	Pension insurance Corporation PLC	Buy-Ins, Longevity Insurance & Reinsurance
Philips UK Pension Fund	Pension insurance Corp. & Hannover Re	Buy-Out & Longevity Reinsurance
Delta Lloyd	RGA Re	Index-Based longevity Derivatives(Swap)

While most longevity transactions so far have been customized in nature, index based solutions and standardized products could draw more interest from financial entities both inside and outside the insurance industry.

They have much potential in providing effective risk management at lower costs and significant capital saving. Some of the products in Table 3 have already been issued and tested in practice, with different levels of success.

There is a huge potential for the life market to continue to grow. When global economic conditions improve, hedging instrument become more widely affordable, financial institutions offer more innovative products, and pension plans sponsors and market

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investors have a better understanding of longevity risk transfer, life market will have a great chance to flourish. Actuaries working in life insurance and superannuation are encouraged to keep up-to-date with the latest development in the area.

The Life and Longevity Markets Association (LLAS) was established in the UK by several global insurers, reinsurers, and investment banks in 2010 to develop a liquid 'life market', which serves as a platform for insurers, reinsurers, and market investors to trade various longevity and mortality-linked assets and liabilities. Market investors may accept longevity risks from insurers and pension plans in exchange for appropriate risks adjusted returns. The risks are significant and increasing in many countries internationally. In many countries "life market" can be used as a risk management instrument for hedging longevity risk.

Conclusion

Longevity risk has been around a long time, but its importance has only recently been fully recognized. It has major implications for pensions providers and life insurers, whose ability to make good on their promises depends, to a considerable extent, on how well they manage this risk. This is of major importance to their policyholders, whose retirements could be ruined if their pension providers fail to deliver. We believe that new index based securities are available in the life market to give better value and more certainly to policyholders. While a market for longevity derivatives is beginning to develop the experience with longevity swaps and bonds. Several financial institutions are currently considering the launch of new index-based and mortality-linked financial instruments. Hence, we conclude that among the life market new longevity derivatives are the best solution for market investors.

References

1. New Development in Longevity risk transfer market -By Jackie Li, and Leonie Tickle (link-http://www.actuaries.digital)
2. Mortality-linked investment products (IAA Mortality Working Group)
3. Joint Forum-Longevity risk transfer market: Market structure, growth drivers and impediments, and potential risks.
4. Discussion Paper PI-1704 Longevity risk and capital market (link: http://www.pension-institute.org/)
5. Wai-Sum Chan, Johnny Siu-Hang Li* and Jackie Li – The CBD Mortality Indexes: Modeling and Application
6. By D.Blake*,A . J. G Cairns, K. DowD, And A. R Kessler – Still Living With Mortality: The Longevity Risk Transfer Market After One Decade. (http://www.artemis.bm/deal_directory/vita-capital-Itd/)
7. City, University of London Institutional Repository http://openaccess.city.ac.uk/21039/
8. Dowd, K., Cairns, A. J. G & Blake, D. (2006b). Mortality-dependent financial risk measure. Insurance : Mathematics and Economics,38, 427-440
9. Ederington, L.H. (1979). The hedging performance of the new futures market. Journal of finance,39,157-170

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Emerging Need of Long Term Care in India

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ABSTRACT

Older persons are the fastest growing population segment in India and many of them require or will require long-term care in the future. Although long-term care receives far less policy attention than health care does, long-term care is going to assume much significance in future. The emergence of nuclear families and growth of old age homes is paving the way to roll out Long Term Care in a big way. Even though there is a steady growth of old age homes, they are unable to fulfill actual needs of long term care due to financial constraints of old age homes as well as beneficiaries, hence this triggers the need for innovative LTC products to meet long term care of aged population which is going to grow in a faster pace. The paper discusses the trend of population ageing in India, government policies such as the National Policy on Older Persons and the National Programme for Health Care for the Elderly, the prevailing provision of long-term care for older persons and its potentials & finally concludes with recommendations of how actuaries can play a vital role in developing LTC products to suit the growing needs of LTC in India.

1. CURRENT AND PROJECTED DEMOGRAPHIC OF ELDERLY POPULATION IN INDIA

The census of India 2011 reported that 8% of the Indian population is above age 60. The elderly population in India is the fastest growing segment in the population. . The elderly population is heterogeneous with variations in morbidity across variables like gender, location, culture, status, etc.

The proportion of elderly population is projected to rise significantly as shown in below table due to economic wellbeing, better medical facilities and reduction in Fertility rates, this poses social and financial challenges and puts a strain on the health care system. The elderly in India 2016 report by ministry of statistics and programme implementation stated that only 29% of the elderly reside in urban areas while 71% reside in rural areas, these 2/3rd of the population have poor socio-economic status,

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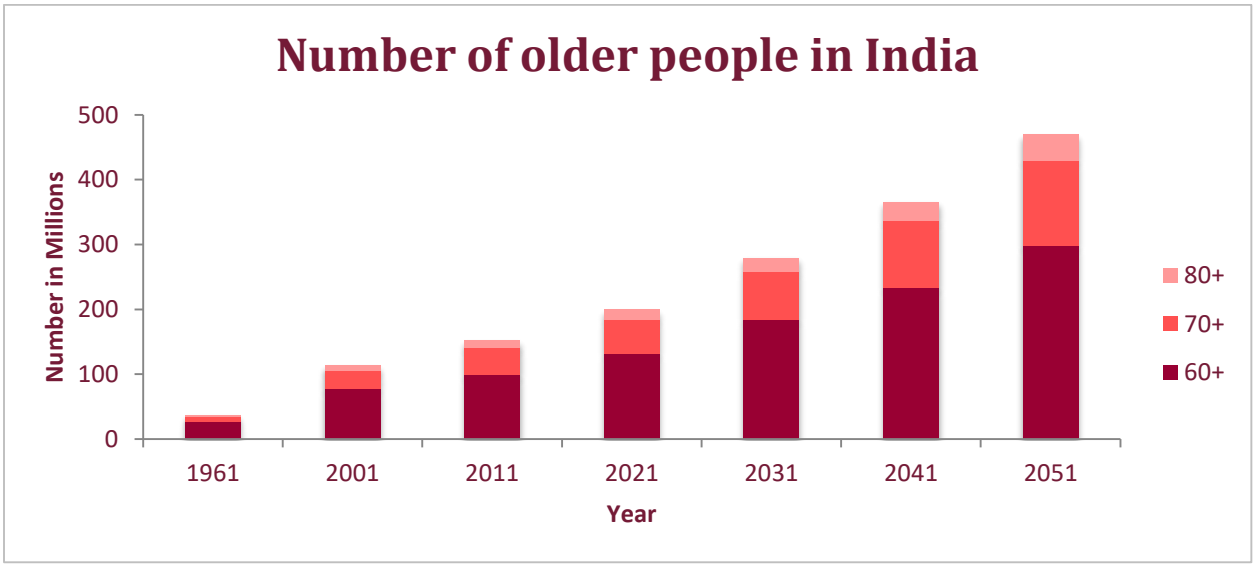
are illiterate and economically dependent. The elderly population is increasing at a rapid speed and 30 years down the lane, it would almost double in its number.

Number and Percentage of Elderly population in India							
Age	Number (in Millions)						
	1961	**2001**	**2011**	**2021**	**2031**	**2041**	**2051**
60+	25	77	98	132	184	233	298
70+	9	29	43	52	75	104	131
80+	2	8	11	16	20	29	41
Age	Percentage of Elderly Population to the total population						
	1961	**2001**	**2011**	**2021**	**2031**	**2041**	**2051**
60+	5.6	7.5	8.1	9.7	12.1	14.0	16.6
70+	2.0	2.9	3.6	3.8	4.9	6.3	7.3
80+	0.6	0.8	0.9	1.2	1.3	1.7	2.3



The median age of the Indian population has been increasing in the past few years and if the same trend continues, then median age would reach 44 by 2051 which is the current situation in Japan, where

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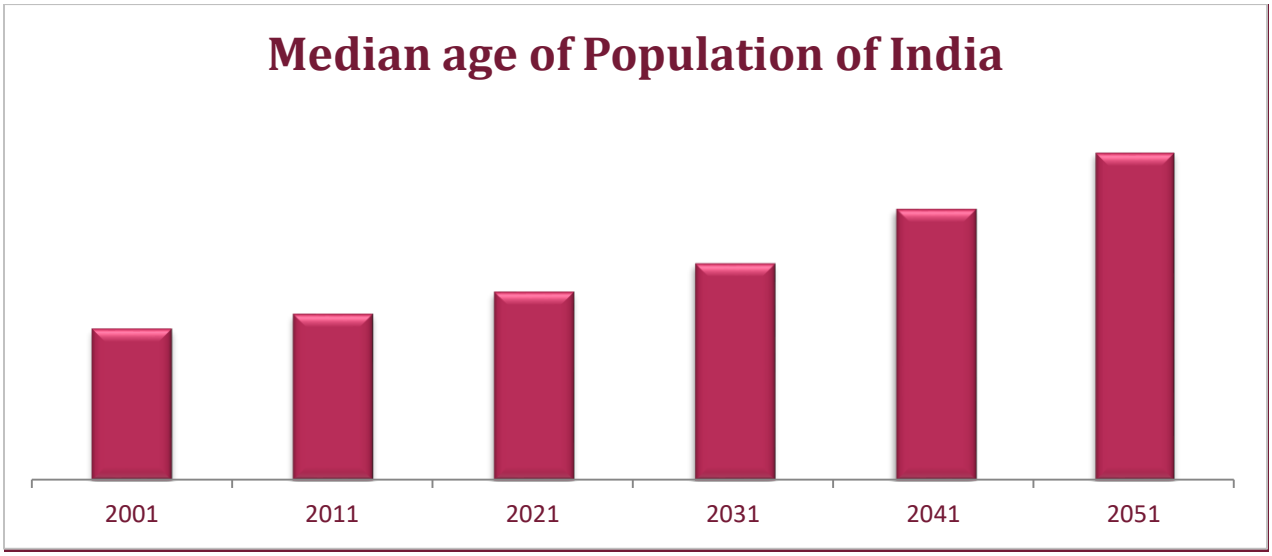
it's difficult to spot young children on the streets as the median age of the population is currently around 48-50.

Year	2001	2011	2021	2031	2041	2051
Median age	**20.5**	**22.5**	**25.5**	**29.3**	**36.7**	**44.3**



Data Source: Health and long term care thesis by Dr. A B Dey. AIIMS

2. ISSUES FACED BY THE ELDERLY POPULATION

The increase in employment opportunities for women who used to traditionally take care of the elders, urbanization, rise of nuclear families, abuse in family and institutional settings have led to loneliness, marginalization and isolation of the elderly. With the dynamic environment and ever changing needs of youth, the gap between generations has increased. Due to the poor geographical access, high cost of treatment (A 24-hr home care in India can cost about Rupees 3,000/- to 10,000/-, depending on the type of services) and poor utilization of health among the elders, they suffer from diseases, impairment of sensory functions, disability and restricted mobility.

Statistics shows that 50% of the chronic diseases like diabetes, depression, vascular and lung diseases will be borne by those aged above 50 in the upcoming years with almost all bearing at least one of the disease and around 30% bearing two or more diseases, 40% of the elderly are disabled, 8% of those

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above 60 and 25% of those above 80 are bed bound. They also suffer from various issues like visual, locomotive, cardiovascular, neurological, psychiatric, etc. With increase in diseases, the complexity and expenditure increase, this causes stress and economic burden to the members of the family who have to look after the elders.

3. PROGRAMS FOR ELDERLY

To overcome the issues faced by the elderly population of India, the government of India had initiated several programs.

Maintenance and Welfare of Parents and Senior Citizens Act passed in 2007 by Indian parliament makes it a legal obligation for the heirs to provide maintenance to senior citizens and parents through monthly allowance.

National Program for Health Care of Elderly (NPCHE) introduced in 2011 to address the health related problems of elderly, funding for homecare, diagnosis, etc and provide accessible, affordable and quality health care and to build capacity of medical and paramedical staff.

The National Policy on Senior Citizens mentions that state government will take actions to provide facility, improve quality of life and ensure availability of user friendly public services to older persons. It also introduced National **Initiative for Care of the Elderly (NICE).**

Even though the government introduced these policies, they are not implemented properly and regulated by defining roles and responsibilities. Due to the increasing cost of health care and insufficient availability of professionals in the field of health care of elders, the government programs, savings, and pension plans are not sufficient to meet the long term care of elders by themselves. The government and NGO's face the same issue since the elderly population is increasing in large numbers. Hence Long Term Care product is an ideal solution to address the issues faced by elderly population.

4. LONG TERM CARE

Long Term Care (LTC) is a combination of medical and non-medical services that support the needs of elderly persons with chronic diseases and ageing issues that affect their ability to perform everyday activities. An individual who is fully not capable of health care can opt to have LTC if he can afford to hire such services. It is a paid service, provided mostly by the private sector at home or at institutions which offers medical, social and personal aspects of care with preventive, curative, restorative and rehabilitative measures.

5. WHY LONG TERM CARE?

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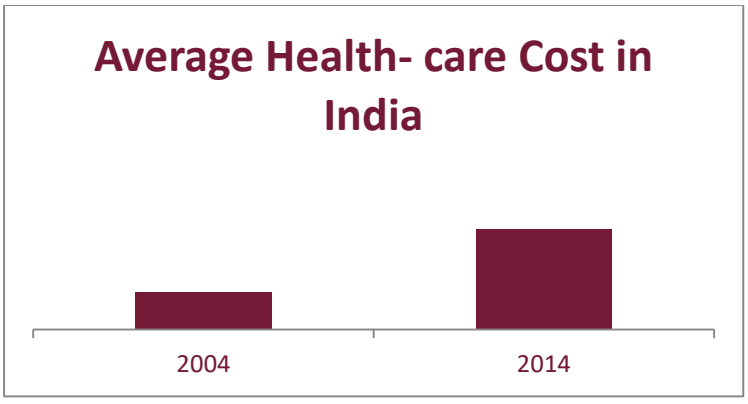
LTC is an organized and structured system which improves the quality of the life of elderly persons by offering a range of medical and non-medical needs. They assist in Activities of Daily Living (ADL), such as dressing, bathing, toileting, etc and Instrumental Activities of Daily Living (IADL) like medication management, housework, etc. ADLs are essential for basic functioning and IADLs impact a person's continued ability to manage independently. LTC helps in maximizing the independence and maintaining the dignity of older persons. It also promotes healthy ageing through application of proper technology, incorporating social-economic, cultural and political elements of the country.

The need for LTC around the world has increased in the past few years due to the increase in expectancy of life by those above the age of 65 or those with disabling condition that needs constant supervision. In our country, traditionally it was the children who took care of their parents in their old age, but due to globalization and change in the mind set of older people, they do not want to rely on children or family members for support and in such a situation, the family care giver system is under stress. Pension and social security is also restricted to those who have worked in the government or public sector or the organized sector of the industry.

Health care has become significantly expensive and with the rise in inflation, it is expected to double the inflation rate, making it a costlier affair. Long term care insurance offers financial independence and eliminates the need to borrow to cover LTC costs by saving during the most productive years of an Individual.



Even though health insurance sounds like the best solution to eliminate the financial disaster, senior citizens are considered as high risk by policy makers as they think in terms of investment and returns. Increase in life expectancy, globalization, expanding middle class, growing workforce of women, increase in population has given an opportunity for the private sector to provide long term care for the elderly.

In India, a country with a large population, and growing elderly population, LTC insurance is the perfect solution, yet the LTC insurance has not gained much fame.

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6. CHALLENGES

Insurance cover that is elderly-sensitive is virtually non- existent in India and, when provided, it's very costly. The concept of geriatric care has remained a neglected area of medicine in the country. Healthcare system for elderly in India is extremely bad and very costly which many can't afford as compared to developed countries where government schemes and healthcare system look after their senior citizens. Long term care is considered expensive by almost a major part of the population and most LTC insurance policies will cover only a specific amount for each day you spend in a nursing facility or for each home-care visit and hence people hesitate to purchase the policy. Most people, especially those in rural areas are not aware of such insurance and the government too hasn't taken any serious step in this matter. The introduction of alternatives for LTC which are comparatively cheaper is also a major challenge.

7. OVERCOMING THE SHORT COMINGS OF LTC IN INDIA

India already has a strong public health infrastructure, but focus should be given on building human resource capacity and competent professional care takers through specialized education, training and research activities. High priority must be given to include every aspect and dimension of LTC like availability, affordability, accessibility and maximum quality. A common reserve funding structure should be created by creating an autonomous body that can pool in reserves from various sources to utilize for the LTC.

The general public should gain awareness about ageing. Positive ageing should be promoted through addressing the public on various age related issues through the use of media and technology. Since India is a country with high family values, the younger family members should be encouraged to pursue the elderly of the family to take up a LTC. Health literacy should be encouraged among elders such that it helps them to posses' skills that help them to gain access, understand and use information to maintain good health, which in turn makes them to understand the importance of LTC insurance.

The youth of the nation should be competent and well equipped with suitable knowledge about ageing and the scope in offering such professional work and service to the elderly. They should be encouraged to initiate innovations so that a skilled and diverse workforce consisting of physicians, dieticians, dentists and non-professional care takers can be specifically built for LTC of the older people.

The government should raise capacity of health professionals in the field of LTC through specialized courses and training such that organized caretakers and service providers can develop a system to meet the needs and challenges of elderly population. The government should also ensure proper availability of healthcare equipment such as wheelchairs, relief materials, etc and improve the transparency of such systems so that the elderly can access the benefits easily. The NGO's can help launch a comprehensive program of action for elderly and take steps and actions to sort out emerging LTC needs.

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Finally much awareness to be created to start saving for LTC by purchasing suitable LTC product at very early ages and should be treated at par / should be given equal importance similar to Insurance & Health Care.

8. ROLE OF ACTUARIES IN DEVELOPING SUITABLE LTC PRODUCTS

Data regarding the elderly population are not yet properly available in India, so Actuaries can play a constructive role by coordinating with government and other agencies:

- To build strong and reliable data that is collected from every nook and corner of the country
- Formalize the process for periodical updating of data
- To work closely with geriatric professionals for research and development purpose as R&D on LTC in India mostly does not exist
- Design innovative LTC products based on gathered data by closely working with Insurance Companies & help them in the new product roll out process
- Design products keeping in mind affordability to mass population.

9. CONCLUSION

Long Term Care is definitely going to be a niche segment for Actuaries, Insurance Companies and Old age homes in the years to come, so Actuaries to play a vital role to achieve in creating awareness about LTC in all available forums so that;

- Government must consider the importance and formulate suitable regulations to enable insurance companies to settle the LTC benefit directly to Old Age home and LTC providers;
- Old age home providers gear up with required infrastructure to meet future demand and licensing requirement that make them qualify to operate as LTC provider;
- General public is made aware of the benefits of LTC & there by start saving at young age.

REFERENCES

https://www.livemint.com/Money/3yZMVlaJLgRzseWhnC2D1O/Rising-cost-of-healthcare-The-challenge-of-managing-health.html

International Journal of Management, IT & Engineering
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https://www.thehindubusinessline.com/economy/rising-healthcare-costs-push-55-cr-indians-below-poverty-line/article24116816.ece

https://www.nerdwallet.com/blog/insurance/long-term-care-insurance

https://www.pwc.com/us/en/industries/health-industries/library/behind-the-numbers.html

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4264287

https://www.inia.org.mt/wp-content/uploads/2017/09/2.1-8-India-59-to-71-1-rev-RFB.pdf

Health and Long-term Care of Older Persons in India by Dr A B Dey

SDD-SPPS Project working Papers Series: Long- Term Care for Older Persons in Asia and the Pacific

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Economic Burden of Cancer in India: Mediclaim Vs Cancer Cover

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Abstract

With the ongoing demographic and epidemiological transition, cancer is emerging as a major public health concern in India .This paper uses WHO data to examine the overall prevalence and economic burden of cancer in India .A comparative analysis of Mediclaim and Cancer Cover has been conducted by use of literature review. This paper provides different factors to be taken into consideration while purchasing a policy and assists in selecting the best insurance policy. To check the awareness among general public this paper uses questionnaire method to collect samples and the results are interpreted. Finally this paper also provides recommendations for Cancer Cover to make it more effective.

Keywords

Cancer, Cancer cover, Mediclaim.

Introduction

The term "Cancer" is derived from the Greek word "Karkinos" (for crab) which refers to a generic non-communicable disease (NCD) characterized by growth of malignant (Cancerous or Neo-Plasms) abnormal cells (tumor/lump) in any part of the human body. Although several forms of cancer have been detected, the most common sites of these tumors in human bodies are breasts, cervical and oral cavity. These four cancers account for more than 50% of all cancer deaths in India. Incidence of cancer is increasing in India. Around 2.5 million people live with cancer in India. Over seven lakhs new patients register every year and cancer mortality is around 5,56,400 in a year.

Globally, the cancer etiology as well as epidemiology has received significant attention of researchers and policymakers. In fact, cancer is the second leading cause of deaths worldwide and accounts for a share of 13 percent in total global deaths (or 8.7 million deaths). The prevalence of cancer was conventionally much evident in developed nations, but in recent years, it has increased substantially in developing countries as well. The estimates from Global Burden of Disease (GBD) suggest that about 70 percent of all cancer deaths are now concentrated among low- and middle-income countries. However, cancer research and treatment are one of the most challenging fields in biomedical sciences and oncologists have been struggling to ensure greater survival chances among cancer patients. In general, there is a consensus that about 60 percent of cancer deaths can be prevented with improved preventive (removing the causes of disease so theta exposure to risk is minimal) and screening (test or procedure used to detect disease) facilities. Given the fact that much of the cancer survival is associated with early diagnosis, access to state-of-the-art medical technology is a prominent policy concern for low-and middle-income countries. The problem increases manifold for developing nations such as India that has poor geographical coverage of medical services and negligible financial protection in health.

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According to WHO, India has a cancer mortality rate of 79 per 100,000 deaths and accounts for over 6 percent of total deaths. Further, the cancer mortality in India is projected to increase to over 900,000 deaths by the end of this decade.

Against this backdrop, this paper makes a comparative analysis of Mediclaim and cancer cover .Reasons to purchase a cancer cover and parameters to purchase a cancer cover. This paper also provides recommendations for cancer cover to make it more effective.

Background of Cancer in India

Number of new cases in 2018, both the sex at all age

Type of Cancer	No. of new cases in 2018	Percentage
Breast	1,62,468	14%
Lip, Oral Cavity	1,19,992	10.48%
Cervix Uteri	96,992	8.4%
Lung	67,795	5.9%
Stomach	57,394	5%
Other Cancer	6,52,723	56.4%
Total	**11,57,294**	

Cancer incidence and mortality in India is estimated to be 17.3 lakhs (new cases) and 8.8 lakhs respectively in 2020. The Indian Council of Medical Research recently projected that India might register over 17 lakh new cases of cancer and over 8 lakh deaths because of the disease by 2020.

More than 50 percent of the cases in India are diagnosed in stage 3 or 4, which decreases the patients' chances of survival. Reports say India has highest mortality-to-incidence ratio in the whole world.

What is Cancer Insurance?

Cancer insurance is special insurance which is designed to provide financial aid to the insured if he/she is diagnosed with cancer. Cancer insurance plans are comprehensive insurance policies that save you from paying an arm and a leg by providing coverage for cancer-related treatment, i.e. radiation, hospitalization, surgery, chemotherapy, blood transfusion, nursing care and the cost of prescribed medicine.

Comparative Analysis of Mediclaim and Cancer Cover

A comparative analysis of Mediclaim and cancer cover is done based on

What is Mediclaim /cancer cover?

Mediclaim policy is health insurance policy that offers health cover up to the sum insured in the case of an illness or accident leading to hospitalization. Mediclaim policy in India is issued for a specific time period.

Cancer cover is a special type of a policy designed to offer financial assistance to those diagnosed with the disease after free lookup period mentioned in the purchased cover insurance policy's terms and condition.

What does it cover?

Mediclaim covers actual medical and surgical expenses incurred by the insured. Coverage is restricted to hospital expenses only, with strict sub-limits for room rent, etc.

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A cancer cover policy offers cover for various costs associated with cancer diagnosis and treatment, including hospitalization, chemotherapy, radiation, surgery, etc. At all stages of cancer (pre-stage, early stage and major stage)

Reason's to purchase

A Mediclaim is a basic indemnity- based plan, which means that the insurer will reimburse hospital expenses or provide cashless treatment. Ideal for covering the rising cost of hospitalization, treatments, diagnosis, medical aid, etc.

Cancer cover covers all expenses of cancer. This is a benefit- based policy, Can use it to pay for your medical expenses or as a replacement of your monthly income

Who is it for?

Mediclaim is a basic policy that is a must for everyone with rising overall health care cost, a health insurance policy should be given precedence before cancer cover and critical illness plan .However in case Mediclaim might prove to be inadequate .Hence the need to add cancer cover.

Cancer cover is ideal for those individuals who identify the risks and understand the costs associated with cancer .A cancer insurance policy should be purchased in addition to a basic health insurance policy.

Premiums

In Mediclaim policy Premiums are based on age, gender, personal history, medical examination, sum assured, etc.

In cancer cover premium calculation is done on the basis of sum assured, policyholder's age, gender, coverage term, health and family history.

Duration

Mediclaim needs to be renewed after every 1 or 2 years.

Cancer cover is being available for long terms like 20 years.

Sum assured

In Mediclaim policy the minimum sum assured is 1 lakh and the maximum sum assured is 1 crore.

In cancer cover the minimum sum assured is 5 lakh and the maximum sum assured is 50 lakh.

Eligibility

For a Mediclaim policy the minimum entry age is 1 day (new born) and there is no maximum age limit.

Eligibility for a cancer cover is 18 to 65 years of age.

Pre-existing disease

Pre- existing disease is not covered in both the disease.

Benefits

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Benefits of Mediclaim are:

- ✓ It is a cost-effective solution to afford adequate healthcare.
- ✓ It reduces the financial burden faced by an individual in the event of a sudden illness or accident that leads to hospitalization.
- ✓ Mediclaim policy offers cashless hospitalization Cashless Mediclaim policy ensures you don't have out-of-pocket medical expense.
- ✓ You can get tax deductions on premiums paid.

Benefits of cancer cover policy are:

- ✓ Covers cancer of multiple stages
- ✓ Lump sum is paid out on cancer diagnosis
- ✓ Premium waiver is available under certain conditions like early stage cancer diagnosis
- ✓ If there are no claims during the year ,the sum assured increases by a pre-specified percentage
- ✓ Premium discounts are available
- ✓ Insurance cover is not ceased after the first diagnosis
- ✓ Tax benefits under section 80d of the Income tax act

Exclusions

Although most health insurance policies available in the market today cover almost all major critical illnesses, including cancer, but these policies generally pay only for inpatient hospitalization and for treatment at hospitals in India. They do not cover the entire cost of treatment.

Exclusions for cancer cover are

- o Skin cancer.
- o Any kind of cancer directly or indirectly caused by or contributed to by sexually transmitted diseases, HIV or AIDS.
- o Cancer resulting from any congenital condition, pre-existing condition; biological, nuclear, or chemical contamination; contact with radiation or radioactivity from any non-diagnostic or therapeutic sources.

Why you should consider a cancer policy?

Although most health insurance policies available in the market today cover almost all major critical illnesses, including cancer, but these policies generally pay only for inpatient hospitalization and for treatment at hospitals in India. They do not cover the entire cost of treatment. Also, the policy amount may not be enough as the common man generally does not go for a health policy of more than Rs. 5 lakh. Such limitations may be overcome by opting for critical illness (CI) insurance which is designed to help cover medical expenses that your normal health insurance will not cover.

"Comprehensive medical insurance (usually floater policies which cover all your family members) provides insurance to take care of a wide range of medical problems. However, the insurance cover provided by such policies comes with sub-limits on the amount payable in different situations. Critical illness insurance addresses this problem. The purpose of a critical illness cover is to help pay for expensive treatments required for specific diseases. Additionally, CI insurance is much cheaper than comprehensive medical insurance plans as only specific illnesses are covered," informs Gaurav Roy, co-founder & head-products, BigDecisions.com.

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However, despite many additional features and plus points, such as the option of bigger coverage of, say, Rs. 15 to Rs. 50 lakh, critical illness insurance plans also have some limitations. "The biggest drawback is that such plans cover cancer only at an advanced stage. While the plan may cover your first heart attack and entry stage kidney disorders, and even pay for dialysis, it will pay for cancer only if the malignant tumor shows uncontrolled growth, with invasion and destruction of normal tissues-an advanced stage," according to an ET Wealth report.

Also, a regular critical ailment plan provides only a lump-sum benefit and does not waive future premiums payable by the insured. Such things can be taken care of by a dedicated cancer care product.

Things to consider before Purchasing a Cancer Cover

While buying a cancer specific insurance plan it is important to take the sum insured in sync with the treatment expense. The cancer insurance plan will act just like a superficial form of financial back up if, it would not be able to meet the overall treatment cost.

Here are some of the points that should be kept in mind before buying a cancer insurance plan.

Go for High Sum Assured

Medical expenses are mushrooming rapidly over the years, and if we talk about cancer treatment, then it can easily leave a dent in your savings. Also, the duration of cancer treatment is longer. So, it is wiser to opt for cancer insurance plan that offers a high sum assured amount. This will act as a financial cushion at the time of a crisis.

The Coverage should be of Longer Duration

Apart from focusing on the coverage, it is also important that the plan you choose provides coverage for a longer duration. The time period of cancer-related treatment is usually very long; so, the longer the tenure of your policy, the more coverage you will get for cancer treatment.

Your Plan should cover all Stages of Cancer

Even though the rates of cancer insurance policy are high, go for a cancer insurance plan that covers you through different stages of cancer. Hence, plans which provide stage-based claim payouts should be your first choice. Generally, a percentage of sum assured is provided by the cancer insurance policy for different stages of cancer.

The Plan Should Provide Premium Waiver and Income Benefit

Cancer can severely affect the health of an individual. Moreover, it also burns a hole in the pocket. The high cost of cancer treatment can inevitably take your income for a toss. Hence, go for a plan that acts as a financial backup during these situations. A plan with premium waiver benefit will not only keep the policy active but will also provide financial assistance to the family of the insured. The same could be said for regular income benefit in a cancer policy.

Go through the Terms and Conditions of Survival and Waiting Period of the Policy

Check and re-check the waiting period of the policy i.e. the time period you will have to wait before the policy starts providing coverage. Moreover, check the survival period of the policy is also very important as it is what makes the insured eligible to claim benefits

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of the policy. The survival period is the minimum number of days the insured has to survive after a cancer diagnosis.

Check Family Health History and weigh risk Factor

Buying a cancer insurance policy makes more sense if you have a family history of cancer. You can check your current medical insurance plan to analyze how a cancer insurance plan can complement your current health insurance policy. Keep in mind that cancer insurance can only provide coverage for cancer and does not cover any other critical illnesses.

Double Policy Does Not Mean Double Coverage

Having a separate cancer insurance policy along with a comprehensive health insurance plan does not mean that you can avail the benefits of both the plans at the same time. Maximum insurance policies include the clause of 'coordination of benefits' which means that one plan will not cover the expenses that the other one does. Thus, it is important to carefully analyze and compare various plans before purchasing a particular cancer insurance plan.

Awareness on Cancer Cover

To measure the awareness about cancer cover among general public a questionnaire was given to a sample of say289 members and following results were obtained using SPSS:

Percentage of Male and Females in the Samples:

Male - 54.7 %

Female - 45.3%

Age of the Respondent

Age	frequency
15-30	86
30-45	105
45-60	78
60-75	17
Above 75	3
Total	289

Percentage of People Having Health Insurance Policy in the Sample

31.8% of people are having health insurance while 68.2% of the population don't have.

Percentage of People who have Purchased Cancer cover

Only one person out of 289 samples have purchased cancer cover.

Percentage of People having family history of Cancer

18 % of the sample population has family history of cancer.

Percentage of People who are Qualified for Government Health Insurance Scheme

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28.4% of the sample population are qualified for government health insurance scheme.

Family History of Cancer and Knowledge about Cancer Cover Cross tabulation

52 member out of a sample of 289 have family history of cancer and among them only 18 persons are aware of cancer cover.

237 members don't have family history of cancer and among them only 32 persons are aware of cancer cover.

ANOVA analysis

As per ANOVA test there is a significant difference in knowledge about cancer cover w.r.t occupation.

And there is a significant difference between cancer cover policy holders and occupation.

Occupation and Knowledge about Cancer Cover

		Knowledge about Cancer insurance		Total
		Yes	No	
Occupation	Doctor	6	5	11
	Teaching Profession	2	17	19
	Business men	10	45	55
	Physical worker	0	31	31
	Employee	5	41	46
	Housewife	17	63	80
	Student	6	18	24
	Banking & IT	2	10	12
	Others	2	9	11
Total		**50**	**239**	**289**

So as per all cross tabulations and test it is being highly proved that the awareness among the general public is very less and necessary steps should be taken to create awareness.

Recommendations for Cancer Cover

Screening tests can help find cancer at an early stage, before symptoms appear. When abnormal tissue or cancer is found early, it may be easier to treat or cure. By the time symptoms appear, the cancer may have grown and spread. This can make the cancer harder to treat or cure.

Some cancer screening tests have been found to lower the death rate (mortality rate) from certain cancers. Examples of some common cancer screening tests that are known to lower cancer death rates include colonoscopy for colon cancer, mammography for breast cancer, and Pap smear for cervical cancer .Some cancer screening tests are recommended only for people at high risk of developing cancer, such as MRI of the breast for women at increased risk of breast cancer.

Screening Tests

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Many cancer screening tests are in use. Some tests have been shown both to find cancer early and to lower the chance of dying from the disease. Others have been shown to find cancer early but have not been shown to reduce the risk of dying from cancer; however, they may still be offered to people, especially those who are known to be at increased risk of cancer.

Cancer cover should include the cost of

Since these test have proved to reduce the cancer mortality the insurance should cover the cost of screening for breast cancer ,cervical cancer, mouth and oral cavity and lung cancer because the incidence of these cancer are very high in India. And the frequency for screening respective of their ages for above mentioned cancer are as follows:

Brest Cancer Screening

- ✓ CBE for women between ages 25-40 (recommended every 1 to 3 yrs)
- ✓ Annual CBE and screening mammography for women aged > 40
- ✓ Mammography costs nearly 1250-1500

Lip And Oral Cavity Screening

- ✓ Clinical examination done by dentist and doctors

Cervical Cancer Screening

- ✓ Pap test (alone) for women between ages 21 to 29 (every 3 years).
- ✓ Pap test and HPV (co -testing) for women > 30 (every 5 years preferred)
- ✓ Pap test alone every 3 years (for women > 30)
- ✓ Pap test costs – Rs.1000
- ✓ HPV costs around -2000

Conclusion

Data from population-based registries under the 'National Cancer Registry Program' indicates that the commonest forms of cancer in India are those affecting the oral cavity, female breast and the uterine cervix, colon and rectum, the lung and stomach. People who already have a health insurance plan or a critical illness plan should also opt for cancer insurance as a top-up. Check that your cancer cover goes on with your parameter and checklist. Cancer cover can be more effective if the recommendations are considered. Awareness among population regarding cancer cover is a must.

References

1. (n.d.). Retrieved from https://www.indiainfoline.com/article/news-personal-finance/5-things-tofocus-beforebuying-
2. (n.d.). Retrieved from http://pib.nic.in/newsite/PrintRelease.aspx?relid=177200
3. DeNoon, D. J. (n.d.). Routine Cancer Screening: Weighing Risks and Benefits. Retrieved from https://www.webmd.com/cancer/features/cancer-screening-risks-and-benefits#1

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4. Figure 2f from: Irimia R, Gottschling M (2016) Taxonomic revision of Rochefortia Sw. (Ehretiaceae, Boraginales). Biodiversity Data Journal 4: E7720.
5. https://doi.org/10.3897/BDJ.4.e7720. (n.d.). doi:10.3897/bdj.4.e7720.figure2f
6. Figure 2f from: Poorani J (2015) Two new species of Scymnini (Coleoptera: Coccinellidae) from Karnataka, India. Biodiversity Data Journal 3: E5296.
7. https://doi.org/10.3897/BDJ.3.e5296. (n.d.). doi:10.3897/bdj.3.e5296.figure2f Judge decides Title IX cant cure harassment. (2017). *Student Affairs Today*, *20*(9), 11-11.
8. doi:10.1002/say.30417 Kamal, Y., Cheng, C., Frost, H. R., & Amos, C. I. (2018). Predictors of disease aggressiveness influence outcome from immunotherapy treatment in renal clear cell carcinoma.
9. *OncoImmunology*, *8*(1). doi:10.1080/2162402x.2018.1500106
10. Kenigsberg, M. I. (1998). Why you should consider board certification. *PsycEXTRA Dataset*. doi:10.1037/e533092009-006
11. Screening for various cancers. (2016, January 21). Retrieved from http://www.who.int/cancer/detection/variouscancer/en/
12. Screening for various cancers. (2016, January 21). Retrieved from http://www.who.int/cancer/detection/variouscancer/en/
13. Screening Tests for Cancer: Facts on Early Cancer Detection. (n.d.). Retrieved from https://www.medicinenet.com/cancer_detection/article.htm#cancer_screening _test_fact s
14. What is cancer? (n.d.). Retrieved from https://www.allaboutcancer.fi/facts-about-cancer/what-iscancer/

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Third-Party Administrator (TPA)-the potential Game Changer

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Abstract

Third-party administrator (TPA) is an IRDA approved organization functioning as an intermediary between the insurer and the insured. Various services are rendered by TPAs and this study is an attempt to understand TPAs elaborately. They play a vital role in health insurance market ensuring better services to policy holders. Since it is difficult for the insurer to connect with large networks, the need for TPAs is increasing by leaps and bounds. At the same time, there are challenges in effectively institutionalizing the services of TPAs. A clear indication is made in this study that the regulatory body needs to focus on developing mechanisms, which would help TPAs to strengthen their human capital and ensure smooth delivery of their services.

Keywords

Health insurance, IRDA, employee retirement plans, flexible spending accounts, commercial liability insurance.

Introduction

Who is a TPA?

A TPA is neither the insurer nor the insured, but handles the administration of the plan which includes: underwriting, processing, adjudication and negotiation of claims; record-keeping, maintenance, customer services. TPAs are licensed by Insurance Regulatory and Development Authority (IRDA). Many employee benefit plans have highly technical aspects and difficult administration that can make using a specialized entity such as a TPA more cost effective than doing the same process in house. For all the services, TPAs charge a percentage of premiums from insurance company as regulated by IRDA, called SLA or service level agreement. TPA can serve multiple insurance companies too. In short, they are like a Business Process Outsourcing in insurance.

Figure 1: Image depicting TPA acts as an intermediary between insurer and insured.

Review of Literature

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(Krishnekumaar S.T., Meenakshisundaram K.S., 2016), This study is an attempt to know the relationship in the role of Third Party Administrators in claim settlement through the public sector insurance, private sector insurance, National Health Scheme, group insurance, individual health insurance and self-insurance. This study concluded that the TPAs have to maintain the data to assist the insurers for their claim settlement and out of pocket expenses, copayment decision has to really depend on the claims experience and not by age band alone.

(Keerti Singh, Shivali Chopra and Sanjay Gupta, 2015), In this study, the researcher discussed about the evolution of TPA, role of TPA in health insurance, key services rendered by TPA**,** benefits of TPA to the insurer. This study also explained the risks associated with TPA, shrinking role of TPA and future role of TPA.

(Ramesh Bhat, Sunil Maheshwari and Somen Saha, 2005), The major findings from this study are TPAs insist on standardization of fees structure of medical services/procedures across providers, healthcare providers do experience substantial delays in settling of their claims by the TPAs, hospital administrators perceive significant burden in terms of effort and expenditure after introduction of TPA and no substantial increase in patient turnover after empanelling with TPAs.

TPAs play a prominent role in:

Commercial General Liability

Commercial General Liability (CGL) is the foundation for any corporate insurance program. It provides the company with coverage (including legal defense, expenses and settlement costs) for claims of property damage or bodily injury to a third party caused by company's people, products or services. In simple terms, many refer to this as "trip and fall" coverage. In CGL, liabilities are written with a large SIR (A self-insured retention is an amount specified in a liability insurance policy that must be paid by the insured and after that amount, the insurer would make any additional payments for defense and indemnity that were covered by the policy).

The TPA acts like a claims adjuster for the insurance company and sometimes works in conjunction with the inside insurance company claims adjuster or an outside claims adjuster as well as the defense counsel. The defense counsel in some situations is selected by the TPA. The point is that the larger the SIR, the more responsibility the TPA has.

401(k) and employee retirement plans

In the United States, a 401(k) plan is a qualified employer-sponsored retirement plan. It lets workers save and invest a piece of their paycheck before taxes are taken out. Taxes aren't paid until the money is withdrawn from the account. 401(k) and other retirement benefit plans can be highly technical field. Retirement plans are often partly managed by an investment company. Instead of handling all the plan contributions by employees, distributions to employees and other aspects of plan processing, the investment company may contract with a third-party administrator to handle much of the administrative work and only handle the remaining investment work. Keeping up with changes in the retirement plan industry, best practices, compliance, and other administration tasks makes hiring TPAs essential in employee retirement plans.

Flexible Spending Account

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A flexible spending account (FSA) is a type of savings account that provides the account holder with specific tax advantages. Set up by an employer for an employee, the account allows employees to contribute a portion of their regular earnings to pay for qualified expenses, such as medical expenses or dependent care expenses.

Third-party administrators handle many aspects of FSA and are the "go-to source" for everything related to Flexible Spending Account. TPAs have access to FSA including FSA balance, plan information and guidelines, and plan deadlines. TPAs also answer questions about FSA eligible products or FSA eligible services which qualify for one's individual plan. TPA is also responsible for claims reimbursement.

Health insurance

TPAs are the backbone of health insurance since they have undertaken major responsibilities on behalf of health insurance companies. Health insurance policies can be either self-insured or fully insured. The risk falls on the insurance company in a fully insured plan, whereas in a self-insured plan the employer or company assumes most of the risk.

Though many health insurance policies promote cashless claim settlement, there are a few plans where the claim is settled through reimbursement. Cashless hospitalization means the TPA may authorize upon a policyholder's request for direct settlement of eligible services and the according charges to a network hospital. In such case, the TPA will directly settle all eligible amounts to the network hospital to the extent of these services covered under the policy. In case, clients pay the hospitalization expenses by themselves, the claim needs to be submitted the TPA within a specified period of time from the date of discharge. Documents related to hospitalization like discharge summary, IP bill with detailed breakup, cash receipts, lab reports, images & films etc., are to be submitted in original.

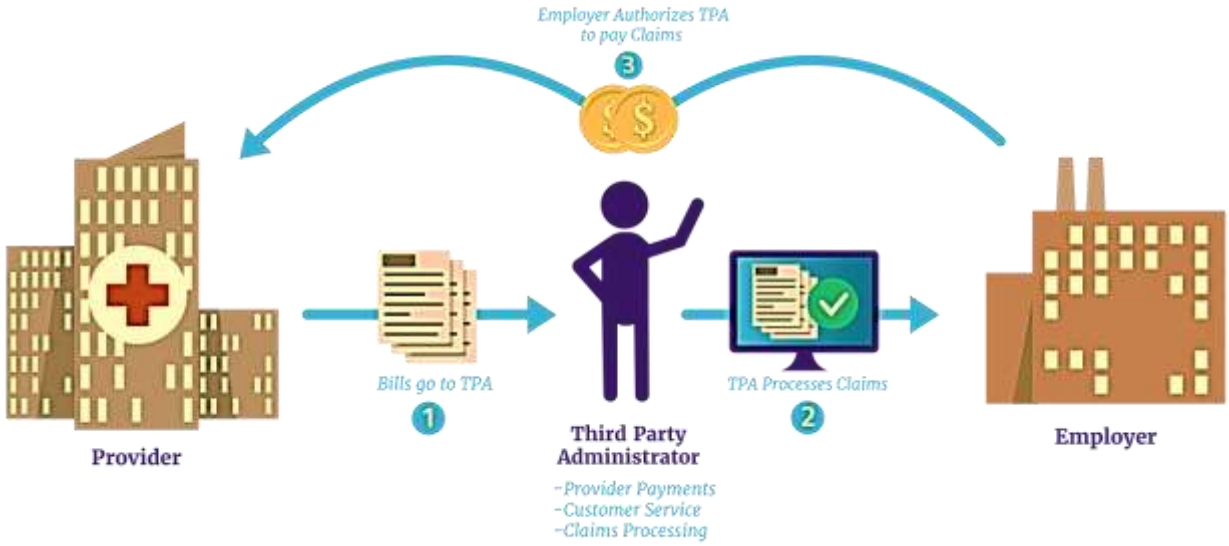

Figure 2: Work of TPA in cashless claim settlement of self-insured health plan.

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Findings-Awareness and Perception about TPAs

Through convenient sampling method, a sample of 100 policyholders is selected from Trichy for the purpose of survey. Out of the 100 respondents, only 14 have the knowledge about the existence of TPAs. It is quite evident that policyholders have little information about their insurance policy. Many individuals are skeptical to make a claim in their insurance policies because they don't know the procedure.

In health insurance, only 5% of the policyholders have adequate knowledge on illnesses covered in their policies, exclusion of illnesses in the policy and only 12% of the policyholders know about cashless reimbursement and the list of empanelled hospitals.

How can TPAs further improve?

i. Need for TPAs in Agriculture: Agriculture is dependent on monsoon which is always unpredictable. It leads to operating risk in cultivation of different crops. To cover this risk, crop insurance is the only mechanism available to safeguard. Awareness about crop insurance is very minimal among farmers. They select the option of suicide over crop insurance. Hence, there is need for TPAs at local level for creating awareness about crop insurance among farmers and helping the farmers in claims assessment and receiving the indemnity payment.

ii. TPAs should conduct education programs frequently and create awareness to policyholders about the policies they have undertaken.

iii. Currently, there are no mechanisms in place to appraise the performance of the TPAs. The IRDA's present role of TPA appraisal is more based on their financial performance rather than consumer satisfaction. There is a need to link incentive of TPAs with their performance.

Conclusion

TPAs need to focus on developing their competencies and capacities and take care of various operational issues in provision of services. This will need significant amount of investment on training their human capital, systems and automation. This study explained the work of TPAs and the various roles performed by them. From the survey, it is observed that general awareness about the TPAs and the services they provide is very low. Suggestions are given for the better functioning of TPAs.

In a nutshell, Third-Party Administrators are the potential game changers in insurance.

References

1. Amol H. Bobade , Shrikrishna S. Mahajan (2012). "Growth of NAIS: A study of crop insurance in India", Volume-3, No.-1, January-April, 2012.
2. https://en.wikipedia.org/wiki/Third-party_administrator
3. https://fsastore.com/learn/how-can-contact-third-party-fsa-administrator
4. https://linfordco.com/blog/third-party-administrator-tpa/
5. https://tra401k.com/what-is-a-third-party-administrator-tpa/
6. https://www.varipro.com/self-funded-health-plans/

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7. http://www.vidalhealthtpa.com

8. Keerti Singh, Shivali Chopra, Sanjay Gupta (2015). "Role of TPA in Health Insurance and associated risk in having a TPA", Institute of Actuaries of India, 24th Indian Fellowship Seminar, 10-11th Dec 2015, Mumbai.

9. Krishnekumaar S.T., Meenakshisundaram K.S. (2016), A Study on the role of Third Party Administrators in Health Insurance in India", Fifth International Conference on Global Business, Economics, Finance and Social Sciences (GB16Chennai Conference), Chennai-India. 1-3 April, 2016.

10. Ramesh Bhat, Sunil Maheshwari, Somen Saha (2005). "Third Party Administrators and Health Insurance in India: Perception of Providers and Policyholders", Indian Institute of Management Ahmedabad, .January 2005.

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Exponential Fractional age assumption for the survival function and mortality rate

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Abstract

In actuarial estimation, there are three main assumptions prevail, namely, Uniform Distribution of Death (UDD), which leads to increasing force of mortality, Balducci assumption, which leads to the decreasing force of mortality and Constant Force of Mortality (CFM) assumption. These are the assumptions that are being used in practice to evaluate the survival function l_{x+t}, for all ages $x \geq 0$ in the interval $0 \leq t \leq 1$. All the above three assumptions are based on the interpolated values of the survival function between two consecutive ages of the survival function. This leads to a continuous survival function l_{x+t}, for all $x \geq 0$ in the interval $0 \leq t \leq 1$. In this paper, we introduce an exponential fractional age assumption to value the survival function at different age and compare the same with the existing assumptions. The numerical illustration demonstrates that exponential fractional age assumption and the existing assumptions have no significant difference in the survival function, whereas, in the case of mortality rate, the overall mean mortality rate based on the above three assumptions is simply the average of the mortality rate calculated based on fractional age assumption.

Key Words: Mortality rate, survival function, exponential fractional age, uniform distribution of death, constant force of mortality

1. Introduction

In a life insurance contract, an insured person has to pay the premium to the insurer and the insurer has to provide the benefit to the insured on the occurrence of the contingent events. Therefore the insurer needs to evaluate the correct premium that has to be charged so that it will be able to provide the benefit. The calculation of premium depends on the probability of survival or death and therefore the estimation of the probabilities play a very crucial role in the actuarial estimation of life insurance and pension contract. Since q_x is a population parameter, its true value cannot be known with complete certainty. So in actuarial estimation, Binomial, Poisson or Maximum likelihood method (Broffitt 1984) is being used to estimate the population parameter. These estimated values listed in the life table which provides probabilities of survival and death for integer ages and durations.

An individual can take an insurance contract at any point of time during the year, not just at integer age. Similarly, the insurance company needs to provide the benefit when the contingent event occurs not just at integer age. Therefore there is a need to know the probabilities of survival and death at fractional age.
In the actuarial estimations, typically the three assumptions that are being used are:

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1. Uniform distribution of death, which assumes the death are linear between the integer ages and expressed the probability of death as ${}_t q_x = t\, q_x$.
2. The constant force of mortality, which assumes that force of mortality is constant between integer ages and expresses the probability of survival as ${}_t p_x = (p_x)^t$.
3. Balducci assumption, which assumed that the reciprocal of the survival function is linear between integer ages and expresses the probability of death as ${}_{(1-t)} q_x = (1-t)\, q_x$

The probability of survival and death and its approximation to fractional age can be found in textbooks like Bowers et al. (1986), Jordan (1975) and Neill (1977). Mereu (1961) in his paper discussed the relevance of the different assumption on actuarial approximation. Willmot (1997) has discussed a family of Fractional age assumption by considering the "fractional independence" property. This family, of which uniform distribution of death is a member, is convenient in actuarial present value calculations. Jones and Mereu (2000) introduce and analyze a family of FAAs that generalizes the three well-known assumptions and allowed to vary across ages so as to produce a more reasonable force of mortality function and more accurate actuarial present values. Jones and Mereu (2002) further extended Jones and Mereu (2000) paper and present an optimality criterion based on the length of the probability density function over the range of the mortality table choosing specific fractional age assumptions. Frostig (2003) derived some properties of the power family of fractional age approximations introduced by Jones and Mereu (2000, 2002) and compared the different approximations with respect to stochastic ordering. Hossain (2010) introduced a quadratic fractional age assumption which makes the force of mortality and survival function continuous at all ages but was restricted to certain assumptions.

This paper considers the exponential fractional age assumption for survival function and compares the same with the existing assumptions. The rest of the paper is organised as follows. Section 2 describes the general model and explores the different results which are necessary to be a valid survival function. Our new result is being compared with the existing assumption and presented as the numerical illustration in Section 3. At the end, Section 4 deals with the concluding remarks and suggestion for future work of research.

2. Model development

In this paper x is assumed to be the integral age. Let the function $f_x(t)$ be the rate of death in the interval x to $x+1$. Therefore the survival function l_{x+t} can be expressed as

$$l_{x+t} = l_x - f_x(t)\, d_x \qquad (1)$$

Where l_x is the number of lives aged x and d_x be the observed number of deaths between the age x and $x+1$. The Equation (1) must satisfy the following boundary conditions in order to have a valid survivorship function at all integral ages.

(i) $\lim_{t \to 0} l_{x+t} = l_x$

(ii) $\lim_{t \to 1} l_{x+t} = l_{x+1}$

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Now applying the above conditions on Equation (1), $f_x(t)$ has the following boundary conditions

(a) $f_x(0) = 0$

(b) $f_x(1) = 1$

Assuming the rate of death function to be an exponential we have

$$f_x(t) = e^{a_x t} - 1 \qquad 0 \le t \le 1 \tag{2}$$

where a_x is a constant to be determined.

Now applying the boundary conditions on Equation (2) and solving for a_x we get $a_x = \log 2$

Substituting the value of a_x in Equation (2) we will obtain

$$f_x(t) = 2^t - 1, \ 0 \le t \le 1 \tag{3}$$

Hence the new survival function assuming exponential fractional age assumption becomes

$$l_{x+t} = l_x - (2^t - 1) d_x, \ 0 \le t \le 1 \tag{4}$$

Result 1: The survival function l_{x+t} is non-increasing and non-negative integer for $x \ge 0$ on the interval $0 \le t \le 1$.

Proof: From Equation (4) we have the new survival function as

$$l_{x+t} = l_x - (2^t - 1) d_x$$

Differentiating the new survival function with respect to t yields

$$l'_{x+t} = -2^t (\log 2) d_x$$

Since $d_x \ge 0$ for all integer values of x we must have $l'_{x+t} \le 0$ for all $x \ge 0$ and $0 \le t \le 1$.

Hence l_{x+t} is a decreasing function of t, $0 \le t \le 1$.

We know that

$1 \le 2^t \le 2$ for $0 \le t \le 1$

Hence $0 \le 2^t - 1 \le 1$

Also $l_x \ge d_x$ for all x

Hence $l_x - (2^t - 1) d_x \ge 0$ for all x and $0 \le t \le 1$

Therefore l_{x+t} is non-increasing and non negative for all integers $x \ge 0$ and $0 \le t \le 1$

Now based on the exponential fractional age assumption and the new survival function, the important life table parameters can be derived and expressed as follows.

$$_t p_x = 1 - (2^t - 1) q_x \tag{5}$$

$$_t q_x = (2^t - 1) q_x \tag{6}$$

$$\mu_{x+t} = \frac{2^t (\log 2) q_x}{1 - (2^t - 1) q_x} \tag{7}$$

$$_t p_x \mu_{x+t} = 2^t (\log 2) q_x \tag{8}$$

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$$ {}_{1-t}q_{x+t} = \frac{(2-2^t)q_x}{1-(2^t-1)q_x} \tag{9} $$

For $0 \le s < t \le 1$ we have

$$ {}_{t-s}q_{x+s} = \frac{(2^t-2^s)q_x}{1-(2^s-1)q_x} \tag{10} $$

From Equation (8) it is clear that the probability density function of the survival function is positive

Result 2: The survival function ${}_tp_x$ is non-negative decreasing function in t for all integer $x \ge 0$ and $0 \le t \le 1$

Proof: From Equation (5) we have the survival probability as

$$ {}_tp_x = 1-(2^t-1)q_x $$

Differentiating this with respect to t we obtain

$$ {}_tp_x^{'} = -2^t(\log 2)q_x $$

Clearly ${}_tp_x^{'} < 0$ for all integer $x \ge 0$ and $0 \le t \le 1$

Also $1-(2^t-1)q_x \ge 0$ for all integer $x \ge 0$, since $(2^t-1) \ge 0$ for $0 \le t \le 1$

Therefore ${}_tp_x$ is a non-negative decreasing function in t for all integer $x \ge 0$ and $0 \le t \le 1$.

Result 3: The probability of death ${}_tq_x$ is a non-negative increasing function in t for all integer $x \ge 0$ and $0 \le t \le 1$

Proof: From Equation (6) we have

$$ {}_tq_x = (2^t-1)q_x $$

Differentiating with respect to t we obtain

$$ {}_tq_x^{'} = 2^t(\log 2)q_x $$

Clearly ${}_tq_x^{'} > 0$ for all integer $x \ge 0$ and $0 \le t \le 1$

Also $(2^t-1)q_x \ge 0$ for all integer $x \ge 0$ and for $0 \le t \le 1$

Therefore ${}_tq_x$ is a non-negative increasing function in t for all integer $x \ge 0$ and $0 \le t \le 1$

Result 4: The force of mortality μ_{x+t} is a non-negative increasing function in t for all integer $x \ge 0$ and $0 \le t \le 1$

Proof: From Equation (7) we have the force of mortality as

$$ \mu_{x+t} = \frac{2^t(\log 2)q_x}{1-(2^t-1)q_x} $$

Differentiating this with respect to t we obtain

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$$\mu'_{x+t} = \frac{2^t (\log 2)^2 q_x (1+q_x)}{[1-(2^t-1)q_x]}$$

We have $1-(2^t-1)q_x \geq 0$ for all integer $x \geq 0$ and $0 \leq t \leq 1$

Hence $\mu'_{x+t} \geq 0$ for all integer $x \geq 0$ and for $0 \leq t \leq 1$

Also $\mu_{x+t} \geq 0$ for all integer $x \geq 0$ and for $0 \leq t \leq 1$

Therefore μ_{x+t} is a non-negative increasing function in t for all integer $x \geq 0$ and $0 \leq t \leq 1$

3. Numerical Illustration

Since the proposed model is not restricted to any condition, an arbitrary data set can be chosen for the validation of this proposed model. For simplicity and comparison purpose we shall take the range of age from age 50 to 60 of the Indian Assured life mortality (2012-14) Ult. (IALM 2012-14).

Table 1: Survivor at different ages

x	l_x	d_x
50	100,000	444
51	99,556	492
52	99,064	550
53	98,514	608
54	97,906	669
55	97,237	731
56	96,506	793
57	95,713	854
58	94,859	915
59	93,944	976
60	92,968	1038

3.1 Survival function based on different assumptions:

- In the case of exponential fractional age assumption the survival function l_{x+t} can be expressed as $l^e_{x+t} = l_x - (2^t - 1)d_x$
- In the case of Uniform distribution of death assumption the survival function l_{x+t} can be expressed as $l^u_{x+t} = l_x - td_x$
- In the case of Balducci assumption the survival function l_{x+t} can be expressed as $l^b_{x+t} = \frac{l_{x+1}}{1-(1-t)q_x}$

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- In the case of Constant force of mortality assumption the survival function l_{x+t} can be expressed as $l^{c}_{x+t} = l_x\left(\frac{l_{x+1}}{l_x}\right)^t$

Table 2: The survival function under different assumption

x	l^{m}_{x+t}	l^{u}_{x+t}	l^{b}_{x+t}	l^{c}_{x+t}
50	$100{,}00-(2^t-1)444$	$100{,}000-t444$	$\frac{99{,}556}{1-(1-t)0.00444}$	$100{,}000\left(\frac{99{,}556}{100{,}000}\right)^t$
51	$99{,}556-(2^t-1)492$	$99{,}556-t\,492$	$\frac{99{,}064}{1-(1-t)0.00495}$	$99{,}556\left(\frac{99{,}064}{99{,}556}\right)^t$
52	$99{,}064-(2^t-1)550$	$99{,}064-t\,550$	$\frac{98{,}514}{1-(1-t)0.00555}$	$99{,}064\left(\frac{98{,}514}{99{,}064}\right)^t$
53	$98{,}514-(2^t-1)608$	$98{,}514-t\,608$	$\frac{97{,}906}{1-(1-t)0.00617}$	$98{,}514\left(\frac{97{,}906}{98{,}514}\right)^t$
54	$97{,}906-(2^t-1)669$	$97{,}906-t\,669$	$\frac{97{,}232}{1-(1-t)0.00683}$	$97{,}906\left(\frac{97{,}237}{97{,}906}\right)^t$
55	$97{,}237-(2^t-1)731$	$97{,}237-t\,731$	$\frac{96{,}506}{1-(1-t)0.00751}$	$97{,}237\left(\frac{96{,}506}{97{,}237}\right)^t$
56	$96{,}506-(2^t-1)793$	$96{,}506-t\,793$	$\frac{95{,}713}{1-(1-t)0.00821}$	$96{,}506\left(\frac{95{,}713}{96{,}506}\right)^t$
57	$95{,}713-(2^t-1)854$	$95{,}713-t\,854$	$\frac{94{,}859}{1-(1-t)0.00892}$	$95{,}713\left(\frac{94{,}859}{95{,}713}\right)^t$
58	$94{,}859-(2^t-1)915$	$94{,}859-t\,915$	$\frac{93{,}944}{1-(1-t)0.00965}$	$94{,}859\left(\frac{93{,}944}{94{,}859}\right)^t$
59	$93{,}944-(2^t-1)976$	$93{,}944-t\,976$	$\frac{92{,}968}{1-(1-t)0.01039}$	$93{,}944\left(\frac{92{,}968}{93{,}944}\right)^t$
60	$92{,}968-(2^t-1)1{,}038$	$92{,}968-t\,1{,}038$	--	--

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Figure 1: Survival Function

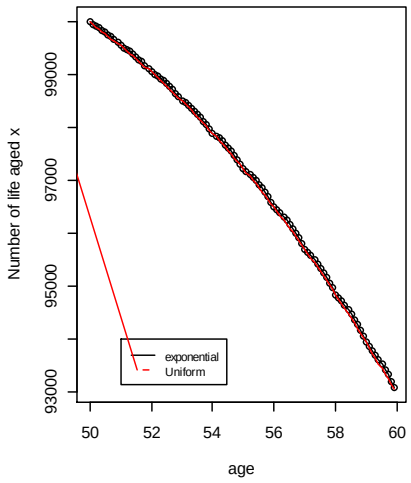

Figure 2: Survival Function

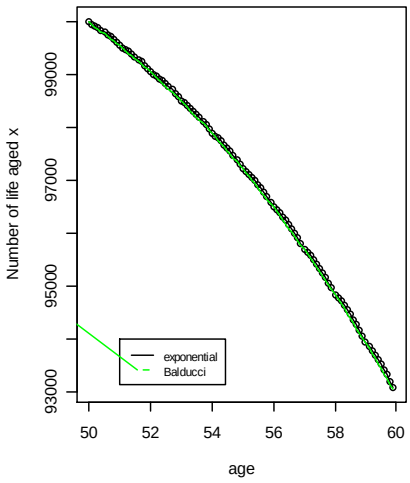

Figure 3: Survival Function

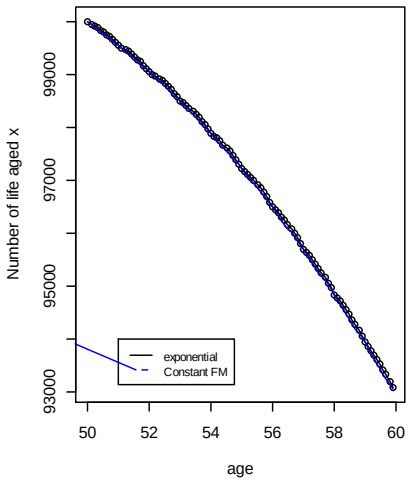

Figure 1, 2 and 3 shows the comparison of the survival function under exponential fraction age assumption with that of the Uniform distribution of death, Balducci and constant force of mortality assumptions respectively. From the figure it can be observed that there is no significance difference in the survival function between exponential fraction age assumptions and the earlier assumptions.

3.2 Force of Mortality based on different assumptions:

- In the case of exponential fractional age assumption the force of mortality μ_{x+t} can be expressed as $\mu^{e}_{x+t} = \dfrac{2^{t}(\log 2)\,d_x}{l_x - (2^{t}-1)\,d_x}$
- In the case of Uniform distribution of death assumption the force of mortality μ_{x+t} can be expressed as $\mu^{u}_{x+t} = \dfrac{d_x}{l_x - t d_x}$
- In the case of Balducci assumption the force of mortality μ_{x+t} can be expressed as $\mu^{b}_{x+t} = \dfrac{d_x}{l_x - (1-t)\,d_x}$
- In the case of Constant force of mortality assumption the force of mortality μ_{x+t} can be expressed as $\mu^{c}_{x+t} = -\dfrac{1}{t}\log\left(\dfrac{l_{x+t}}{l_x}\right)$

Table 3: The force of mortality under different assumption

x	μ^{m}_{x+t}	μ^{u}_{x+t}	μ^{b}_{x+t}	μ^{c}_{x+t}
50	$\dfrac{2^{t}(\log 2)444}{100{,}000-(2^{t}-1)444}$	$\dfrac{444}{100{,}000-t444}$	$\dfrac{444}{100{,}000-(1-t)444}$	$-\log\left(\dfrac{99{,}556}{100{,}000}\right)$
51	$\dfrac{2^{t}(\log 2)492}{99{,}556-(2^{t}-1)492}$	$\dfrac{492}{99{,}556-t492}$	$\dfrac{492}{99{,}556-(1-t)492}$	$-\log\left(\dfrac{99{,}064}{99{,}556}\right)$

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52	$\frac{2^t(\log 2)550}{99{,}064-(2^t-1)550}$	$\frac{550}{99{,}064-t550}$	$\frac{550}{99{,}064-(1-t)550}$	$-\log\left(\frac{98{,}514}{99{,}064}\right)$
53	$\frac{2^t(\log 2)608}{98{,}514-(2^t-1)608}$	$\frac{608}{98{,}514-t608}$	$\frac{608}{98{,}514-(1-t)608}$	$-\log\left(\frac{97{,}906}{98{,}514}\right)$
54	$\frac{2^t(\log 2)669}{97{,}906-(2^t-1)669}$	$\frac{669}{97{,}906-t669}$	$\frac{669}{97{,}906-(1-t)669}$	$-\log\left(\frac{97{,}232}{97{,}906}\right)$
55	$\frac{2^t(\log 2)731}{97{,}237-(2^t-1)731}$	$\frac{731}{97{,}237-t731}$	$\frac{731}{97{,}237-(1-t)731}$	$-\log\left(\frac{96{,}506}{97{,}232}\right)$
56	$\frac{2^t(\log 2)793}{96{,}506-(2^t-1)793}$	$\frac{793}{96{,}506-t793}$	$\frac{793}{96{,}506-(1-t)793}$	$-\log\left(\frac{95{,}713}{96{,}506}\right)$
57	$\frac{2^t(\log 2)854}{95{,}713-(2^t-1)854}$	$\frac{854}{95{,}713-t854}$	$\frac{854}{95{,}713-(1-t)854}$	$-\log\left(\frac{94{,}859}{95{,}713}\right)$
58	$\frac{2^t(\log 2)915}{94{,}859-(2^t-1)915}$	$\frac{915}{94{,}859-t915}$	$\frac{915}{94{,}859-(1-t)915}$	$-\log\left(\frac{93{,}944}{94{,}859}\right)$
59	$\frac{2^t(\log 2)976}{93{,}944-(2^t-1)976}$	$\frac{976}{93{,}944-t976}$	$\frac{976}{93{,}944-(1-t)976}$	$-\log\left(\frac{92{,}968}{93{,}944}\right)$
60	$\frac{2^t(\log 2)1038}{92{,}968-(2^t-1)1038}$	$\frac{1038}{92{,}968-t1038}$	$\frac{1038}{92{,}968-(1-t)1038}$	—

Figure 4: Mortality rate

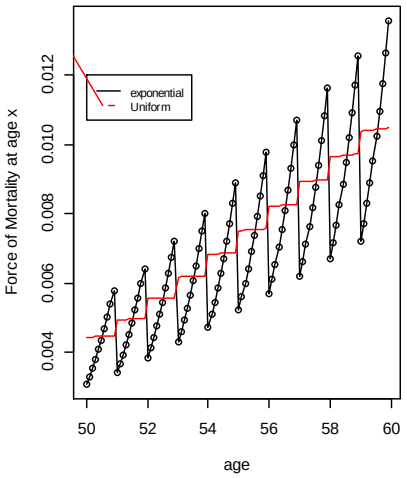

Figure 5: Mortality rate

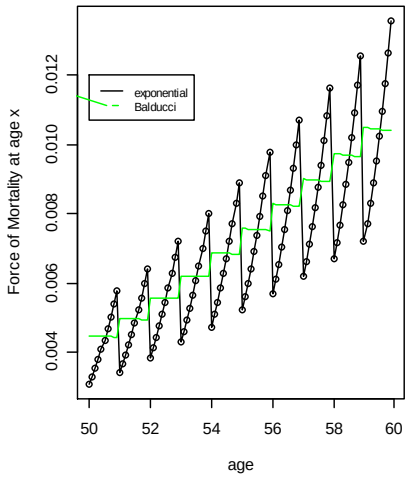

Figure 6: Mortality rate

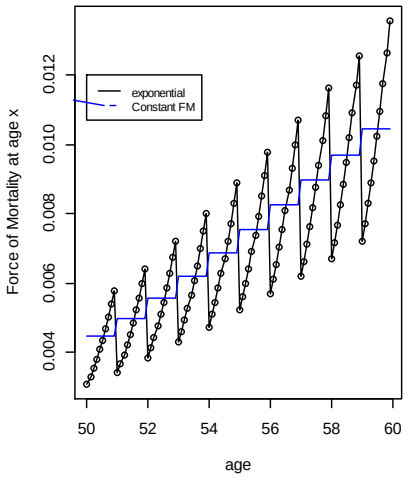

Figure 4, 5 and 6 shows the comparison of the mortality rate under exponential fraction age assumption with that of the Uniform distribution of death, Balducci and constant force of mortality assumptions respectively. From the figure it can be observed that for exponential fraction age assumptions, there is a lower mortality rate at the beginning of the year, while it increases exponentially during the year and it is higher at the end and the year. The mean of the year mortality rate based on exponential fractional age assumption leads to the mortality rate evaluated based on the other three assumptions.

4. Conclusion

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This paper proposes a new model for the fractional age assumption for the distribution of deaths over integer ages. Based on the exponential fractional age assumption the different life table parameter was derived and checked for the necessary and sufficient condition for the existence of such a model. From the actuarial literature, it is known that the survival function is a decreasing function of time and in the case of new exponential fractional age model the survival function is also non-increasing and in fact it decreases exponentially between the integer ages. The force of mortality under UDD is increasing and concave up, for the Balducci, it is decreasing and concave up, and for the constant force of mortality, it is constant between adjacent integer age intervals. In contrast, the new exponential fractional age the force of mortality is increasing and concave up during the integer age. This makes the model practical and more meaningful in its applications. For future research, a different fractional age assumption can be considered which may also make the force of mortality continuous between integer ages.

Reference

1. Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A., Nesbit, C.J., (1986). Actuarial Mathematics. The Society of Actuaries, Itasca, IL.
2. Broffitt, J.D., (1984). Maximum likelihood alternatives to actuarial estimators of mortality rates. Transactions of the Society of Actuaries XXXVI, 77–142.
3. Frostig E, (2003). Properties of the power family of fractional age approximations. Insur Math Econ 33:163–171
4. Hossain S.A, (2010). Quadratic fractional age assumption revisited. DOI 10.1007/s10985-010-9153-1
5. Jones BL, Mereu JA (2000). A family of fractional age assumptions. Insur Math Econ 27:261–276
6. Jones BL, Mereu JA (2002). A critique of fractional age assumptions. Insur Math Econ 30:363–370
7. Jordan, C.W., (1975). Life Contingencies, 2nd Edition. Society of Actuaries, Chicago, IL.
8. Mereu, J.A., (1961). Some observations on actuarial approximations (with discussion). Transactions of the Society of Actuaries Part I XIII, 87–115.
9. Neill, A., (1977). Life Contingencies. Heinmann, London.
10. Willmot GE, (1997). Statistical independence and fractional age assumptions. N Am Actuar J 1(1):84–99

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A SURVEY ON DIFFERENT AUTHENTICATION SCHEMES IN CLOUD COMPUTING ENVIRONMENT

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Abstract: Cloud computing technology has been a new buzzword in the IT industry and expecting a new horizon for the coming world and it is also considered as a pinnacle of computing technology. Business and IT professional is progressively marching towards cloud generations as it consists of an internet-based ubiquitous computing technology and on-demand network model, which consists of software and provides the services like computation, data access, and storage service that don't have need of user's facts of the physical locality and configuration of the system that delivers the services. The paradigm of cloud computing is new, keeps everyone connected on-the-go and offers a plethora of services over the internet, hence, data security is paramount in cloud computing environment to have an access to these resources securely. Purloining an individual's legitimacy is the most insidious way of invading user's privacy; hence, authentication technique is need of the hour. Authentication technique is an unparalleled key technology for information security, which is a mechanism to ascertain proof of identities of the user to get access to information safely and mitigate the security risks in the cloud environment. In this paper, we focus on authentication techniques, which is the most challenging and promising component and there are three main approaches to user authentication are knowledge-based, possession-based, and biometric-based. In this paper, a comprehensive review and assessment of the various authentication approaches, strategies, and mechanisms has been done for a deep understanding of a secure and sturdy authentication process prevailing in a cloud environment.
Keywords: Cloud computing, Authentication, Encryption algorithm, security.

1. Introduction

Technology in its broadest sense is now more significant than ever in the field of computing. The pinnacle of technological advances in the computing field is that of Cloud Computing. Cloud offers a myriad of services and benefits to the people of the computer industry and business world. A potential application of Cloud computing ranges from Business applications, Data Storage & Backup, Management applications, Social applications and Entertainment and Art applications. The following companies are providing numerous services from the cloud. Some notable examples are *Google:* delivers various services to users, including document applications, text translation, email access, maps and much more. *www.salesforce.com. Microsoft*: Microsoft office online service-this provides platforms to build customized cloud services.

Cloud computing has a variety of characteristics such as Shared infrastructure, dynamic provisioning, Network access and managed metering. Cloud offers different kinds of services and models which make the cloud computing feasible and accessible to end users. Deployment model includes public, private, community and hybrid model and service model includes Platform-as-a-Service (PaaS), Infrastructure-as-a-Service (IaaS), and Software-as-a-Service (SaaS). Cloud computing is a newly sprouting and prominent

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model for the deliverance of platforms, software, and infrastructure to the outer world. The cloud computing-based services and applications offer the following benefits to its users, including Cost savings, scalability/flexibility, reliability, maintenance and mobile access. Cloud computing is playing a significant role in changing the computer industry.

The impact of cloud computing is immeasurable but, as with any new technology, there are potential downsides, such as: *Making the correct choice:* Cloud offers various services like PaaS, SaaS, IaaS. So, it is very imperative to select the best one based on our business needs. *Lack of Standards:* Cloud does not have a single platform for standardization. Clouds are fully documented interface but no standard has been defined so far. *Continuously Evolving:* Clouds are dynamic in nature and so user requirements, interfaces, networking, and storage. *Vendor Lock-in:* Data portability is very essential. Moving from one service provider to another is a daunting task on the cloud. *Service Provider availability:* The availability of a service provider is paramount and also his/her sustainability and reputation. *Security and Privacy:* These two are the main factors in cloud computing as huge data is located in different places. The user can access the shared resources which reside on cloud and nowadays, terabytes of data are stored in the cloud for an easy way of access and providing 24 hours availability at anywhere, any place and any time. It is a tedious task to maintain the security of the data on the cloud; hence, it is the responsibility of CSP (Cloud Service Provider) to ensure adequate authentication mechanism to access the data in cloud environment securely.

A well-established authentication is need of the hour as it enables cloud service provider to keep all their resources secure by allowing only authorized user to access or process the protected resources. As Cloud server and services are travel in open network, which causes the breach in the authentication process. Hence cloud service provider uses authentication to control which users have access to which resources and also this mechanism enables a remote user to securely access their applications and data's. User authentication is one of the approaches to achieve foolproof security. But still, there is a need of strong authentication mechanism through which users of cloud can be validated.

2. Authentication in Cloud Environment

Cloud computing environment has a rich set of distributed resources and enriched with good characteristics as we delineated above. Cloud permits the user to Store and to access the data available on cloud environment requires well-versed authentication mechanism which performs the following tasks such as, it is a process of proving or showing something to be true, genuine, or valid and it verifies the identity of a user or process. Before providing access to shared resources, user's identity must be verified by an authentication process. The security concerns mentioned above happens in the absence of a proper authentication mechanism and it can be avoided by adopting various authentication mechanisms. Cloud allows its customer to store their data but they are unaware of the location of data, where is it stored? So it requires transparency and every cloud service accessed by the customer, requires exchanging the authentication information. Hence, to prevent false accessing of cloud resources, a well-established authentication mechanism is paramount. After a successful authentication, a user's process is usually subjected to an authorization process as well, to verify whether the authenticated entity should be permitted access to a protected resource or system. Though user validity is authenticated, they could not access a resource if that user was not granted a permission to access it.

In a cloud computing environment three different types of authentication factors are used. The authentication factor consists of some portion of facts or trait that can be used to authenticate a user to access a system and its resources. Three important factors to be considered to authenticate users are knowledge factor, the possession factor and the inherence factor. **Knowledge factor:** defines "Something you know", means the credentials such as personal identification number (PIN), a username, a

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password or the respond to a secret question. **Possession factor:** which consists "Something you have" that is the credential that the user's hardware device like a cellular (smart) phone are capable to receive a authentication message or one-time password or PIN which are generated by authentication app. **Inherence factor:** means "Something you are" is typically based on some form of biometric identification, including finger or thumbprints, facial recognition, retina scan or any other form of biometric data. Apart from these factors, the following authentication protocols are used to ensure and support authentication process in a cloud environment such as Extensible Authentication Protocol (EAP), Challenge-Handshake Authentication Protocol (CHAP), Lightweight Directory Access Protocol (LDAP) and Single Sign-on Protocol (SSO). These protocols consist of various strategies to verify user's identity and ensure the secure access to resources in a cloud environment.

2.1 Authentication Issues

The cloud computing environment offers the massive range of services and it is helpful for users who in search of a precise cloud computing resource, Despite the numerous advantages of this technology, there are a few issues such as security and privacy that inadvertently affect the reliability of this technology, hence user authentication is the most important security issues in cloud computing environment. Here is some security issues are listed out in the cloud computing environment such as:

Cloud service providers may request customers to use and store their account credentials in the cloud; this information can be accessed by cloud service providers. This presents a privacy issue to the customer's private information. The privacy of the sensitive information is specified by Cloud service providers, so it is difficult for customers to make sure the proper rules are enforced. There is a lack of transparency in the cloud environment that allows the customers to supervise their own private information. Multiple cloud service allows the customer to store his/her password in multiple clouds, the more cloud service the customer is subscript to, the more replica of the user's credentials will be. This is definitely a security issue for the customers and the cloud service providers. The multiple replicas of account credentials will cause different authentication processes. Exchanging of authentication information for every cloud service may lead to an exploit of the authentication mechanism. Cloud service providers may use diverse authentication technologies for authenticating users; this may have a lesser amount of impact on SaaS than PaaS and IaaS.

2.2 Security Breaches in the Cloud Environment

A security breach is an act that contravenes security policies, practices, or procedures by unauthorized persons in a cloud environment and also security breaches are garnering more attention recently in the field of cloud environment causes heavy losses to the organization. This section presents various infamous security breaches happened global level. They are as follows:

- **WannaCry Ransomware**: happened in May 2017, was a type of malevolent software from cryptvirology that intimidates to reveal the user's data or perpetually obstruct access to it unless a requested ransom is made. This attack is carried out by Trojan that is disguised as a legitimate file that forces the user to download or open to hacking the system. This attack was targeted Microsoft Windows operating system and affected more than 2,00,000 systems across 150 countries with total damages ranging from hundreds of millions to billions of dollars. This attack was stopped within a few days of its discovery due to the immediate release of patches by Microsoft.
- **Yahoo!:** Two major data breaches shattered the services of the Internet Service providing company Yahoo. Both breaches registered a huge impact on the history of the Internet. Breaches that happened in 2014 and 2016 where hackers developed web cookies to falsely access the credentials of users and hacked their account

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without a password. Later, Yahoo confirmed that over 3 million users account were hacked.

- **LinkedIn:** Profession networking site faced worst data breaches in the year of 2012. LinkedIn lost over 167million Account credentials in that breach. Authorities of LinkedIn immediately invalidated the password of the accounts impacted, and they advised the customer to reset their passwords. Later security officials of the LinkedIn have beefed up the security mechanism of password databases with a new technique called hashing and salting.
- **Uber:** A massive hack that hit Uber in 2016, to delete users credentials and keep breach quiet firm paid $1,00,000. They hacked 57million users and drivers credentials; however sensitive information such as location data, birth date, and bank information had not been compromised.
- **Facebook:** The Facebook-Cambridge Analytica data scandal involves the collection of credentials of 87 million users of Facebook. The way that Cambridge Analytica gathered user's information was called inappropriate. The stolen data was used in the 2016 presidential election for influence voter opinion by the political representatives who hired Cambridge Analytica. This scandal paved the way for ethical standards for social media sites and many demanded greater security to prevent such an incident.

2.3 Authentication techniques in the cloud environment

Cloud computing authentication technique hinges on two important goals such as 1) Ensuring authorized persons can access the resources, 2) keeping an unauthorized person away from gaining access to resources in this environment. There is a number of components involved in accomplishing these tasks. This section expounds the various authentication techniques in the cloud environment as follows.

- Password based Authentication
- 2-Factor Authentication
- Multi-Factor Authentication
- Single Sign-on
- Key Stroke Analysis
- Graphical Authentication
- Remote User Authentication
- Shared Authority based Authentication
- **Password-Based Authentication:** Single-factor authentication scheme allows the user to define username and password to login to the cloud to access the data. This scheme is still in practice but it is not considered as best practice as a password leak leads to data breaches.
- **2-Factor Authentication:** It is referred to as 2-way verification and provides an extra layer of security to safeguard the user's data. It strengthens the security layer so the hacker cannot access the data available in the cloud.
- **Multi-Factor Authentication:** It supports two or more credentials for better security transaction. It improves the authentication status by including physical characteristics of user's biometrics such as fingerprint, voice, typed characters, eyries, and patterns in keypress intervals etc. It is more robust against the unauthorized person and illegal access.
- **Single sign-on:** Is a session and authentication process, by which user can access multiple applications with a single set of login credentials. Exchanging of user credentials across cloud are done by the SSO protocols such as Kerberos and SAML (Security Assertion Markup Language).
- **KeyStroke Analysis:** It is also called Keystroke biometrics that is used for an authentication process. Keystroke biometrics includes fingerprints, retinal scans, and DNA to verify the user's authentication to access the resources from the cloud.

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- **Graphical Authentication:** A graphical password authentication system allows a user to select an image from the graphical user interface and verifies the user's identity before allowing them to access the resources from the cloud.
- **Remote User Authentication:** A kind of authentication mechanism to verify the legitimacy of a user based on a smart card or biometric attributes and allows them to communicate with the cloud server to access the shared resources.
- **Shared Authority based Authentication:** To encounter the privacy issue for cloud storage, a Shared Authority based privacy-preserving authentication protocol has been implemented.

3. Related work

Technological advancement as well as the growing popularity of online banking, customers face potential risks like hacking of account, loss of money etc. Hence it is very essential to have a strong authentication mechanism. Chao Li et al[1], proposed a strong "A Two-Factor Authentication Design of Fingerprint Recognition System Based on DSP and RF Card". In this paper, fingerprint recognition combined with Smart card verification is proposed. The author proposed a strong authentication algorithm processing unit called TI's (Tenas Instrument) TMS320VC5510 (DSP). This architecture consists of five significant modules: Image gathering, Image processing, storage devices, Synchronous and asynchronous communication, and human-machine interaction. FPC1011C is a novel leading-edge capacitive fingerprint sensor based on the Certus Sensor Platform provides a strong fingerprint authentication system. In this system, the RF card component contains a universal asynchronous receiver/transmitter (UART) interface, which is designed to receive and transmit the data in a secure way. This algorithm exhibits high performance with low computation.

The proliferation of e-commerce facilitates user to perform a transaction through systems which are connected to the remote server to access the given services. To ensure remote authentication Tsague et al[2], implemented a good authentication scheme called "An Advanced Mutual-Authentication Algorithm Using 3DES for Smart Card Systems". In this paper, the author proposed an advanced mutual authentication scheme using 3DES to enhance smart cards based mutual authentication schemes security levels and to circumvent the different attacks. This scheme provides mutual authentication of identity, verification of the authenticity of the remote server, smart card reader and secures session key agreement, providing much security protection for smart card users and the server. In this scheme, Smart card details are encrypted using 3DES encryption technique. The design of this scheme consists of three phases such as registration; authentication and Password update which works well against various attacks.

The Internet has turned-up almost all the field and so online shopping and one can buy anything on-the-go. As online transactions are done by credit card, there may be various attacks are possible. To perform any transactions securely Muhammet et al[3], devised a system called "Combining Biometric ID Cards and Online Credit Card Transactions". This proposed system uses the concept of Turkish e-ID pilot system to verify user identification using strong multi-factor authentication which combines Biometric ID as well as Credit Card parameters. Turkish e-ID system offers, various Identity Verification Package (IVP), which include different parameters such as timestamp, biometric details, and demographic data to provide strong authentication system. This approach protects both end-user and business people from unauthorized services.

With the increase of heterogeneous information system in the enterprise, there are different access mechanisms and access policies require powerful authentication mechanism. Having this in mind, Yang et al[4], implemented a well-authenticated system called "The Optimization Mechanism Research of Distributed Unified Authentication Based on Cache". This paper proposes a unified authentication mechanism based on a cache with distributed architecture. It can manage multiple nodes

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and deal with high concurrent requests, so as to ensure the stability of the authentication system and improve the response time. The author also designs and implements a multi-factor cache replacement algorithm based on Hybrid (Hybrid-MF). The results show that the cache-based distributed unified authentication mechanism proposed in this paper can effectively improve the response time and deal with concurrent pressure better.

Jaspher Willsie, et al[5], introduced an Elliptic Curve cryptographic algorithm used for secure key generation and exchange. To raise the security level Smart Card based authentication scheme is proposed. Login & mutual authentication scheme verifies the client's authentication with an authentication server (AS) using Biohash which develops a hashed password and provides a biometric template. This scheme uses ECC based encryption, verification, and Signing. One of the advantages of ECC based scheme key size is low (i.e) 160 bits when compared to 1024 of RSA. This secure framework enables secure user authentication in cloud computing and mobile cloud users. The proposed scheme ensures low communication cost and biometric framework provides secure storage and transmission.

To secure organizational data from hackers, R.Nikam, et al[6], implemented a novel approach called "Cloud Storage Security using Multi-Factor Authentication". This paper proposes CP-ABE (Ciphertext-Policy – Attribute-Based Encryption and Multi-Factor Authentication (MFA) ensures the sharing of data between peer organization keeping the identity anonymous. In this paper, static username and password ensure initial level authentication followed by OTP based on Token (TOTP Algorithm) generator technique that is considered as credentials for users. This multi-level security system provides better cloud storage security using MFA, Encryption technique and this security check keep the hacker away from accessing the cloud environment.

The proliferation of ubiquitous computing environment and internet technology needs strong authentication technique. As authentication plays a very vital role in the field of a cloud environment, Salman H.Khan et al[7], proposed "Multi-Factor Authentication on Cloud". This paper proposes a novel authentication verification mechanism that combines human inherence factor (handwritten signature biometrics) with standard knowledge factor (traditional user-specified password) to have an enhanced security mechanism. In this proposed scheme, GAE (Google Application Engine) is used as cloud service provider, a hierarchical approach is used to perform signature matching and to ensure the authenticity of biometric, decision forest classifier is used. Easily scalable, low cost and resource allocation makes this scheme works well in smaller groups environment, but not suitable for large groups.

To address the security vulnerabilities of the traditional e-voting system, Oke B.A et al[8], introduced a system called "Developing Multifactor Authentication Technique for Secure Electronic Voting System". A well-known Multifactor Authentication scheme using Biometric fingerprint and a smart card that is cryptographically secured is proposed in this paper. This paper will focus on authentication, issues in verifying and validating the legitimate voters. Enhanced Feistel block cipher and First-moment feature extraction technique makes this system more secure and provides better confidentiality. The disadvantage of this system is, the Multifactor authentication is not integrated with the cryptographic model.

Cloud computing environment is known for providing various services over the internet; hence data security plays a very vital role. To have a secure access to cloud services one needs a good authentication system. R.K.Banyal et al[9], implemented a novel framework called "Multi-factor Authentication Framework for Cloud Computing". In this proposed framework, the user is authenticated using multifactor's which includes a Secret key, One Time Password and IMEI Number. Arithmetic Captcha Expression is also used to enhance the authentication process to access the cloud services and resources. A novel approach called Secret Splitting of Authentication Factor boost the security mechanism and provides the additional layer of security for this trusted environment. This scheme mitigates almost all possible attacks in a cloud environment.

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Y.Shah et al[10], coined a well-known authentication system called "Multi-Factor Authentication as a Service". In this scheme, Multi-Factor authentication architecture combines the features of Identity Federation and Single-Sign-on methodologies such as the OpenID framework which provides for the modular integration of various factors of authentication. MFAaaS aggregates authentication factors and exposes them to services. This improved federated identity management framework to provide enhanced secure MFA, which consists of all forms of authentication factors including both on-device and in-network authentication.

As most of the organization started to utilize the functionality of cloud, security plays an important concept to safeguard the data access and services. Niharika Gupta et al [11], implemented an authentication scheme called "Implementing High Grade Security in Cloud Applications using Multifactor Authentication and Cryptography". The author proposed a robust authentication scheme based on the Ticket-based one-time password to achieve a high-security mechanism for restricting the unauthorized access. This work uses multiple hashing encryptions methodology to thwart an unauthorized user from gaining the access rights. This paper also packed with different methodologies such as DoS Attack and Brute Force Attack Prevention, Prevent Phishing Attacks using Image Verification, SQL Injection Prevention and SMS based OTP Approach to have sturdy authentication mechanism and for storage of data in the cloud securely.

The integration of Internet of Things with Cloud paved a new concept called Cloud of Things which provides scalability, virtualized control and access to services provided by IoT. Here security plays a very crucial role in deploying the CoT. To address this Rohan et al[12], introduced a concept called "A Multifactor Authentication System using Secret Splitting in the perspectives of CoT". This paper clearly explains the Multifactor Authentication Scheme in three phases. Using multiple factors such as a smart card, biometric and encryption algorithm for authentication increases the security level.

Cloud computing model consists of the data owner, service provider and users. To have a secure transmission of data between these entities needs a strong security mechanism. The data in the cloud should always be kept confidential, maintain its integrity and above all accessed by an authenticated person. To achieve these, Nalini.S et al[13], devised an authentication mechanism called "MLA Scheme: Multi-Level Authentication for data in Cloud using NTP-Server and Biometric". The author proposed a multi-level Authentication scheme such as password-based authentication at initial level then biometric and timestamp-based authentication using NTP server. The crypto-system model ensures a strong authentication process which combines the functionality of biometric (fingerprint), and NTP time-stamps generated by NTP Server, data confidentiality, and integrity thus strong multi-level authentication is ensured.

Cloud computing field are faced an exponential growth in recent years that leads to many challenges and issues. One of the important challenges is ensuring the user authentication. To provide a strong authentication scheme B. S. Al-Attab et al[14], proposed a novel authentication scheme called "Authentication Scheme for Insecure Networks in Cloud Computing". The proposed authentication scheme combines the functionalities of USB Token based on Hash function and Diffie-Hellman key exchange. This two-factor authentication protects the network and also data with less cost and without needing any additional device. Yet, the user's data is still susceptible to various attacks.

The proliferation of cloud computing environment paves a way for many enterprises and government agencies to get their job done with ease. Yet, the strong authentication mechanism is required for accessing the data in the cloud environment. Hence, J. P. Singh et al[15], introduced a new kind of mechanism called "Authentication and Encryption in Cloud Computing" This paper proposes strong authentication mechanism based on tree structure which improves the user's authentication process and Elliptic Curve Digital Signature Algorithm ensures the data integrity. This method increases the efficiency of storage and retrieval of data. This scheme takes less time for key generation and signature verifying process.

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Yassin et al[16], proposes the two- factor authentication scheme which consists of password authentication and canny's edge detection to encrypt/decrypt the image. In this work, mutual authentication and one-time password between user and service provider are done at the first phase. During the second phase, Canny[s edge detection feature are utilised to encrypt/decrypt image which ensures mutual authentication, session key agreement, defends from replay attack, impersonation attack, forgery attack, reflection attacks, and parallel session attacks. It has lower transmutation cost with high security. A Valid user can select the valid password.

S.M.Barhate et al[17], had a review on "User Authentication issues in Cloud Computing". This paper highlights the interoperability environment in the cloud, which throws security challenges and privacy. Four different kinds of interoperability use cases are proposed to address the security challenges. This paper clearly explains interoperability, authentication algorithms such as RSA, AES, MD5, OTP Password Generation Algorithm, DES, Rijndael Encryption Algorithm, user authentication and authorization techniques and authentication protocols such as LDAP, EAP, and SSO protocols were also studied.

The proliferation of CRAN (Cloud Radio Access Network) requires a good authentic mechanism to guarantee the secure access to resources and services. Hence Hui Yang et al[18], devised a novel authentication technique called "Blockchain-based trusted authentication in cloud radio over fiber network for 5G". In this paper, the author proposed a kind of authentication scheme called BAA (Blockchain-based Anonymous Access) for a blockchain-based trusted authentication (BTA) architecture in C-RoFN for 5G. Network access authentication can be done by BTA with a tripartite agreement between manufacturer, uses and network operator. Blockchain-based Authentication provides better security, credibility and accessing of the network with low network cost.

Justin LeJeune, et al[19], proposed a new approach called "An Algorithmic Approach to Improving Cloud Security - The MIST and Malachi Algorithms". This paper proposes two different kind's algorithms called MIST and Malachi which ensures strict measures and methodologies for strengthening the cloud against the security attacks. This MIST algorithm works well against the weak password and account breach by incorporating highly user-specific questions. The Malachi algorithm safeguards the account information and protects the hackers from accessing the login credentials. Less computation makes this algorithm weak and it provides less security.

Emerging and popular cloud environment provide huge storage provision and it requires distinctive encryption and decryption algorithm to protect the data. Ali AZOUGAGHE, et al[20], proposed a new algorithm called "An Efficient Algorithm for Data Security in Cloud Storage". This paper proposed asymmetric encryption of Elgamal encryption scheme and symmetric encryption of AESC algorithm. The author also viewed as an extension of the Diffie-Hellman key exchange protocol. This algorithm implemented for file upload and file download phases. The author compares two algorithms such as RSA and Elgamal algorithms. The efficient way of algorithm carried out experiments on text file sizes. Two algorithm key sizes of 1024 for RSA and 160 bits for Elgamal.AES algorithm provide a fast and safe symmetric algorithm. The advantage of this paper, an unauthorized user never gets the data accidentally because two keys coming from two different locations.

Table 1. Comparisons of various authentication factor

Name of the Work	Authentication Factor	Verification Scheme, Methodology, Advantage, and Limitations
A Two-Factor Authentication Design of Fingerprint Recognition	Two Factor Authentication	– Fingerprint recognition – Five modules: Image gathering, Image processing, storage devices, Synchronous and asynchronous communication, and human-machine interaction

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System Based on DSP and RF Card		– Universal Asynchronous Receiver/Transmitter (UART) Interface. – Certus Sensor platform provides a strong authentication system – Speed optimization causes less quantizing accuracy – Low image processing
An Advanced Mutual-Authentication Algorithm Using 3DES for Smart Card Systems	Mutual Authentication	– Proposes authentication algorithm using 3DES – Smart Card details are encrypted using 3DES – It consists of three phases like registration, authentication and password update – Good authentication scheme – Heavy computational process
Combining Biometric ID Cards and Online Credit Card Transactions	Multi Factor Authentication	– Biometric ID and Credit Card parameters – Propose Multi-factor authentication combines biometric-ID and Credit Card parameters – Turkish-ID consists Identify Verification Package – Ready to use Security and Identification Infrastructure – Fraud possibility is very high
The Optimization Mechanism Research of Distributed Unified Authentication Based on Cache	Multi Factor Authentication	– Cache-based Unified authentication mechanism – Manage multiple nodes and deals concurrent requests – Multifactor cache replacement algorithm – The Hybrid-MF algorithm can deal with complex user better – Good response time – Improved cache hit ratio – Maintaining cache is a time-consuming task
A Secure framework for Enhancing User authentication in Cloud Environment Using Biometrics	Mutual Authentication	– Provides secure user authentication, mutual authentication, session key issue and proxy issue – Smart card based user authentication – Highly used in the mobile cloud – Secure key generation and exchange algorithm – Insecure Biohash technique – No clear explanation for computing the Nonce of value
Cloud Storage Security using Multi-Factor Authentication	Multi Factor Authentication	– Static Username and Password ensures initial level authentication and OTP based on (TOTP) Algorithm – Authentication and Encryption (CP-ABE) provides the safest environment – Provides a good authentication system. – Ensures better storage security – Computational cost is high
Multi-Factor Authentication on Cloud	Multi Factor Authentication	– Combines Human inherence factor and traditional password schemes – Hierarchical based Signature matching and Decision forest classifier are used to verify the authenticity – Low cost and good resource allocation scheme – Not fit for large groups
Developing	Multi Factor	– Biometric Fingerprint and a Smart Card based

Multifactor Authentication Technique for Secure Electronic Voting System	Authentication	authentication scheme – Feistel Block Cipher – First-moment feature extraction technique – More secure and provides better confidentiality – Not integrated with the cryptographic model
Multi-factor Authentication Framework for Cloud Computing	Multi Factor Authentication	– Multi-factors such as Secret key, One Time Password and IMEI Number – Arithmetic Captcha Expression is also used to enhance the authentication process – Secret splitting of Authentication Factor – Mitigates the hacker's effect – Too many authentication parameters
Multi-Factor Authentication as a Service	Multi Factor Authentication	– Identity Federation and Single Sign-on methodology such as OpenID Framework – MFAaaS aggregates all authentication factors – Secured MFA provides better security – Unpopular
Implementing High Grade Security in Cloud Applications using Multifactor Authentication and Cryptography	Multi Factor Authentication	– Ticket-based One Time Password – Multiple hashing encryption methodology – Image verification, SQL Injection and SMS based OTP for sturdy authentication scheme – Prevents unauthorized attacks such as DDoS and Brute Force – Heavy computational process
A Multifactor Authentication System using Secret Splitting in the perspectives of CoT	Multi Factor Authentication	– Multiple factors such as smart card, biometric and encryption algorithm provide better security – This system uses Ex-OR operations, Encryption and Diffie-Hellman key exchange algorithm – Multiple factors increase the security level – Requires a number of hardware devices
MLA Scheme: Multi-Level Authentication for data in Cloud using NTP-Server and Biometric	Multi Level Authentication	– Multi level authentication combines the Password-based authentication, Biometric and Time-stamp based authentication using NTP Server – Exhibits strong authentication process – Only suitable for UNIX environment
Authentication Scheme for Insecure Networks in Cloud Computing	Two Factor Authentication	– Two-factor authentication combines the functionalities of USB Token based on Hash function and Diffie-Hellman key Exchange scheme – Protects data and network with low cost – User's data is susceptible to various attacks
Authentication and Encryption in Cloud Computing	Single Factor Authentication	– Tree structure based authentication keeps the unauthorized person at the bay – Elliptic Curve Digital Signature algorithm ensures data integrity – Takes very less time for key generation and signature verifying process – Simple authentication scheme

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Cloud Authentication Based on Encryption of Digital Image Using Edge Detection	Two Factor Authentication	– Authentication phase consists of Image and User's password – A biometric method with Canny's edge detection – Edge Pixels of Image is encrypted using a stream cipher – Works well against the various attacks – Low transmutation cost – Use of MD5 may exhibit poor security scheme
User Authentication issues in Cloud Computing	Authentication Algorithm MD5	– Interoperability – RSA, AES and MD5 algorithm for authentication and encrypting the files – Authentication protocols such as LDAP, EAP, and SSO are explained – Insist the Security, Privacy measure, and Interoperability – Security issues are not addressed fully
Blockchain-based Trusted Authentication in Cloud Radio over Fiber Network for 5G	BTA (Blockchain-based Trusted Authentication)	– Authentication scheme called BAA (Blockchain-based Anonymous Access) for a blockchain-based trusted authentication (BTA) architecture in C-RoFN for 5G – Provides better security with the low-cost network – Agreement amount tri-party may be weak
An algorithmic approach to Improving Cloud Security: The MIST and Malachi Algorithms	MIST and Malachi Algorithms	– Data integrity and strong security – Users data protected through account security – To eliminate the weak passwords and account recovery vulnerability – MIST-Implementation of the question and answer system – Malachi-Different approach to account security – An innovative method for account recovery – To protect accounts in regular logins. – Statistical analysis not efficient
An Efficient Algorithm for Data Security in Cloud Storage	AES Algorithm	– Symmetric (AES) and Asymmetric (Elgamal encryption scheme) encryption – 128-bits keys for 10 cycles of repetition, 192-bit keys for 12 cycles and 256-bit keys for 14 cycles, in symmetric key encryption with rotation – Includes file upload and file download phases – AES is safe and fast in both directions (upload and download) – Takes more time for execution
An Enhanced Hybrid Data Security Algorithm for Cloud	AES, ECC and SHA-256	– Combination of hybrid data security cryptographic algorithm such as DES, AES, RSA, ELgamalMD5, SHA – Has a fine security concept – Large message size not applicable
Data Storage Security Algorithms for Multi-Cloud Environment	Public Auditability Algorithm, Data Dynamics Algorithm, Integrity Proof Algorithm,	– Need advanced cryptographic technique to secure data – Auditability - for storing and securing data – TPA (Third Party Auditor) – for privacy and secrecy of data and manages data based on SLA's (Service Level Agreements) – Ensures integrity and confidentiality of data

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	Privacy-Preserving Algorithm	– Auditing uses low-cost communication and computing technology – Data consistency is checked by TPA without downloading the data – Too many algorithms make the work cumbersome
Private Cloud Security: Secured User Authentication by using Enhanced Hybrid Algorithm	Enhanced Hybrid Algorithm	– Elliptic Curve Authentication Algorithm verifies the Authentication – Hybrid of AES and Blowfish provides good security – Key Generation and Exchange are done by Elliptic Curve – Diffie Hellman – MAES (Modified Advanced Encryption Standard) with 256-bit – Enhanced authentication scheme. – Attack by XSL (Extended Sparse Linearization) is possible
Proposal and Implementation of Cloud Security Algorithm to Enhance the Security of the Layers	Honey Encryption Algorithm	– Honey Encryption Algorithm combined with DES provides supplementary Security layer – Honey Encryption Algorithm increases the probability of deciphering the key – Data is more secure and free from all types of attacks – Level of complexity is high
Providing Security, Integrity and Authentication Using ECC Algorithm in cloud storage	Electronic Curve Cryptographic Algorithm	– Electronic Curve Cryptographic Algorithm for better security, authentication – Data Integrity is verified by Metadata – Metadata are created and encrypted by ECC – Less CPU power and Processing time – Not fit for the big environment
A Two-Factor Authentication Design of Fingerprint Recognition System Based on DSP and RF Card	Two Factor Authentication	– Fingerprint recognition – Five modules: Image gathering, Image processing, storage devices, Synchronous and asynchronous communication, and human-machine interaction – Universal Asynchronous Receiver/Transmitter (UART) Interface – Certus Sensor platform provides a strong authentication system – Speed optimization causes less quantizing accuracy – Low image processing
An Advanced Mutual-Authentication Algorithm Using 3DES for Smart Card Systems	Mutual Authentication	– Proposes authentication algorithm using 3DES – Smart Card details are encrypted using 3DES – It consists of three phases like registration, authentication and password update – Good authentication scheme – Heavy computational process
Combining Biometric ID Cards and Online Credit	Multi Factor Authentication	– Biometric ID and Credit Card parameters. – Propose Multi-factor authentication combines biometric-ID and Credit Card parameters – Turkish-ID consists Identify Verification Package

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Card Transactions		– Ready to use Security and Identification Infrastructure – Fraud possibility is very high
The Optimization Mechanism Research of Distributed Unified Authentication Based on Cache	Multi Factor Authentication	– Cache-based Unified authentication mechanism – Manage multiple nodes and deals concurrent requests – Multifactor cache replacement algorithm. – The Hybrid-MF algorithm can deal with complex user better – Good response time – Improved cache hit ratio – Maintaining cache is a time-consuming task
A Secure framework for Enhancing User authentication in Cloud Environment Using Biometrics	Mutual Authentication	– Provides secure user authentication, mutual authentication, session key issue and proxy issue – Smart card based user authentication – Highly used in the mobile cloud – Secure key generation and exchange algorithm – Insecure Biohash technique – No clear explanation for computing the Nonce of value
Cloud Storage Security using Multi-Factor Authentication	Multi Factor Authentication	– Static Username and Password ensures initial level authentication and OTP based on (TOTP) Algorithm – Authentication and Encryption (CP-ABE) provides the safest environment – Provides a good authentication system – Ensures better storage security – Computational cost is high
Multi-Factor Authentication on Cloud	Multi Factor Authentication	– Combines Human inherence factor and traditional password schemes – Hierarchical based Signature matching and Decision forest classifier are used to verify the authenticity – Low cost and good resource allocation scheme – Not fit for large groups
Developing Multifactor Authentication Technique for Secure Electronic Voting System	Multi Factor Authentication	– Biometric Fingerprint and a Smart Card based authentication scheme – Feistel Block Cipher – First-moment feature extraction technique – More secure and provides better confidentiality – Not integrated with the cryptographic model
Multi-factor Authentication Framework for Cloud Computing	Multi Factor Authentication	– Multi-factors such as Secret key, One Time Password and IMEI Number Arithmetic Captcha Expression is also used to enhance the authentication process – Secret splitting of Authentication Factor – Mitigates the hacker's effect – Too many authentication parameters
Multi-Factor Authentication as a Service	Multi Factor Authentication	– Identity Federation and Single Sign-on methodology such as OpenID Framework – MFAaaS aggregates all authentication factors – Secured MFA provides better security – Unpopular

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Implementing High Grade Security in Cloud Applications using Multifactor Authentication and Cryptography	Multi Factor Authentication	– Ticket-based One Time Password – Multiple hashing encryption methodology – Image verification, SQL Injection and SMS based OTP for sturdy authentication scheme – Prevents unauthorized attacks such as DDoS and Brute Force – Heavy computational process
A Multifactor Authentication System using Secret Splitting in the perspectives of CoT	Multi Factor Authentication	– Multiple factors such as smart card, biometric and encryption algorithm provide better security – This system uses Ex-OR operations, Encryption and Diffie-Hellman key exchange algorithm – Multiple factors increase the security level – Requires a number of hardware devices
MLA Scheme: Multi-Level Authentication for data in Cloud using NTP-Server and Biometric	Multi Level Authentication	– Multi level authentication combines the Password-based authentication, Biometric and Time-stamp based authentication using NTP Server – Exhibits strong authentication process – Only suitable for UNIX environment
Authentication Scheme for Insecure Networks in Cloud Computing	Two Factor Authentication	– Two-factor authentication combines the functionalities of USB Token based on Hash function and Diffie-Hellman key Exchange scheme – Protects data and network with low cost – User's data is susceptible to various attacks
Authentication and Encryption in Cloud Computing	Single Factor Authentication	– Tree structure based authentication keeps the unauthorized person at the bay – Elliptic Curve Digital Signature algorithm ensures data integrity – Takes very less time for key generation and signature verifying process – Simple authentication scheme
Cloud Authentication Based on Encryption of Digital Image Using Edge Detection	Two Factor Authentication	– Authentication phase consists of Image and User's password – A biometric method with Canny's edge detection – Edge Pixels of Image is encrypted using a stream cipher – Works well against the various attacks – Low transmutation cost – Use of MD5 may exhibit poor security scheme
User Authentication issues in Cloud Computing	Authentication Algorithm MD5	– Interoperability – RSA, AES and MD5 algorithm for authentication and encrypting the files – Authentication protocols such as LDAP, EAP, and SSO are explained – Insist the Security, Privacy measure, and Interoperability – Security issues are not addressed fully
Blockchain-based Trusted	BTA (Blockchain-	– Authentication scheme called BAA (Blockchain-based Anonymous Access) for a blockchain-

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Authentication in Cloud Radio over Fiber Network for 5G	based Trusted Authentication)	based trusted authentication (BTA) architecture in C-RoFN for 5G – Provides better security with the low-cost network – Agreement amount tri-party may be weak
An algorithmic approach to Improving Cloud Security: The MIST and Malachi Algorithms	MIST and Malachi Algorithms	– Data integrity and strong security – To eliminate the weak passwords and account recovery vulnerability – MIST-Implementation of the question and answer system – Malachi-Different approach to account security – An innovative method for account recovery – To protect accounts in regular logins – Statistical analysis not efficient
An Efficient Algorithm for Data Security in Cloud Storage	AES Algorithm	– Symmetric (AES) and Asymmetric (Elgamal encryption scheme) encryption – 128-bits keys for 10 cycles of repetition, 192-bit keys for 12 cycles and 256-bit keys for 14 cycles, in symmetric key encryption with rotation – Includes file upload and file download phases – AES is safe and fast in both directions (upload and download) – Takes more time for execution

4. Authentication Algorithm

Many organizations and enterprises store their important data on the cloud and these data are also accessed by many people (end user). To verify the user identity or credential for accessing the data or valuable resources available in the cloud environment, the cloud service provider employs an authentication process which involves in the process of validating a user's identity and allows them to access the needed resources from the cloud environment. It is the mechanism of verifying an incoming request with a set of identifying credentials and gives them permission to access the same. Encryption involves the process of transforming data from one format into another so that it is unreadable by anyone who does not have proper credential information. To provide better security to cloud users, a number of authentication algorithm and encryption algorithms are designed with distinctive features. To verify the authenticity of the cloud environment, the authentication algorithm uses the concept of a shared key. Two famous authentication algorithms are widely used in the cloud environment, they are MD5 and SHA1.

MD5 (Message Digest – Algorithm) is a widely used hash function, capable of producing a 128-bit fixed-length message digest which is typically represented as a sequence of 32 hexadecimal digits. To have an additional level of hashing MD5-HMAC (Hashed Message Authentication Code) can be used. The security of MD5 is severely compromised, as the size (128 bits) is small enough. The MD5 exhibits the poor security against the collision attack and not suitable for applications like SSL Certificates and Digital Signatures.

SHA-1 (Secure Hash Algorithm-1) a strong authentication algorithm is capable of producing 160-bit message digest from 264-bit of input message, which ensures that data has not been altered and begins from intended source. SHA-1 HMAC (Hashed Message Authentication Code) provides additional level hashing. SHA-256, SHA-384, and SHA-512 are variants of SHA-1 which are capable to process of Advanced Encryption Standard (AES), Data Encryption Standard (DES) and Triple (3DES) DES Encryption. SHA-1 Algorithm is widely used in security applications and protocols which includes TLS

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and SSL, PGP, SSH, S/MIME, and IPSec. SHA-1 provides better security against various attacks.

Table 2. The comparison between MD5 and SHA

Key for Comparison	MD5	SHA
Security	Less secure than SHA	High secure than MD5
Message Digest Length	128 Bits	Upto 512 Bits
Attackers effect	2^{128} bit operations required to break	2^{160} bit operations required to break
Attacks to find two messages producing the same MD	2^{64} bit iteration needed to break	2^{80} bit iteration needed to break
Speed	Faster, only 64 iterations	Slower, required 80 iterations
Success rate	Vulnerable	More secure than MD5

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5. Conclusion

As explained, Cloud undoubtedly is the best thing in the computing world. Cloud computing is also known as distributed computing since it has the capability to run many applications over a single resource on a network. The Cloud environment is known for a wide range of computing resources such as networks, storage, servers, and services. These precious resources can be accessed and processed by the valid users from the cloud server at anytime and anywhere via the secure channel of the Internet with great flexibility and ease. Yet, the ever-growing computing technology still faces major problem related to the authenticity of the user, where authentication is a process which ensures and verifies the legitimate user's identity before allowing them to access the data from the cloud and also it prevents the unauthorized user's from accessing the data. From the above discussion, it is evident that various existing authentication schemes have many problems and disadvantages, so, it is imperative to have well-defined authentication schemes to allow the legitimate users to access the various resources from the opulent cloud computing environment. In our proposed scheme, it is planned to implement strong multi-factor authentication based on three basic requirements such as something the user knows, something users have and the user is, for allowing the legitimate user to access the cloud storage server and data's with less computational cost and time.

6. Reference

1. Chao, Li., & Jin, Qi. (2010). A Two-Factor Authentication Design of Fingerprint Recognition System Based on DSP and RF Card. *The 2nd International Conference on Computer and Automation Engineering (ICCAE),* Singapore, 441-445.
2. Tsague, H. D., Nelwamondo, F., & Msimang, N. (2012). An Advanced Mutual-Authentication Algorithm using 3DES for Smart Card Systems. *Second International Conference on Cloud and Green Computing*, Xiangtan, 660-666.
3. Yildiz, M., & Göktürk, M. (2010). Combining Biometric ID Cards and Online Credit Card Transactions. *Fourth International Conference on Digital Society*, St. Maarten, 20-24.
4. Yang, D., & Kai, F. (2017). The Optimization Mechanism Research of Distributed Unified Authentication Based on Cache. *14th Web Information Systems and Applications Conference (WISA)*, Liuzhou, 297-300.
5. Kathrine, G. J. W. (2017). A Secure Framework for Enhancing User Authentication in Cloud Environment using Biometrics. *International Conference on Signal Processing and Communication (ICSPC)*, Coimbatore, 283-287.
6. Nikam, R., & Potey, M. (2016). Cloud Storage Security using Multi-Factor Authentication. *International Conference on Recent Advances and Innovations in Engineering (ICRAIE)*, Jaipur, 1-7.
7. Khan, S. H., & Akbar, M. A. (2015). Multi-Factor Authentication on Cloud. *International Conference on Digital Image Computing: Techniques and Applications (DICTA)*, Adelaide, SA, 1-7.
8. Oke, B. A., Olaniyi, O. M., Aboaba, A. A., & Arulogun, O. T. (2017). Developing Multifactor Authentication Technique for Secure Electronic Voting System. *International Conference on Computing Networking and Informatics (ICCNI)*, Lagos, 1-6.
9. Banyal, R. K., Jain, P., & Jain, V. K. (2013). Multi-Factor Authentication Framework for Cloud Computing. *Fifth International Conference on Computational Intelligence, Modelling and Simulation*, Seoul, 105-110.
10. Shah, Y., Choyi, V., & Subramanian, L. (2015). Multi-factor Authentication as a Service. *3rd IEEE International Conference on Mobile Cloud Computing, Services, and Engineering*, San Francisco, CA, 144-150.

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11. Gupta., Niharika., Rani., & Rama. (2015). Implementing High Grade Security in Cloud Application using Multifactor Authentication and Cryptography. *International Journal of Web & Semantic Technology.* 6. 09-17.
12. Shah, R. H., & Salapurkar, D. P. (2017). A Multifactor Authentication System using Secret Splitting in the Perspective of Cloud of Things. *International Conference on Emerging Trends & Innovation in ICT (ICEI)*, Pune, pp. 1-4.
13. Nalini, S., & Andrews, J. (2016). MLA Scheme: Multi-Level Authentication for Data in Cloud using NTP-server and Biometric. *IEEE International Conference on Computational Intelligence and Computing Research (ICCIC),* Chennai, pp. 1-3.
14. Al-Attab, B. S., & Fadewar, H. S. (2016). Authentication Scheme for Insecure Networks in Cloud Computing. *International Conference on Global Trends in Signal Processing, Information Computing and Communication (ICGTSPICC),* Jalgaon, pp. 158-163.
15. Singh, J. P., Mamta., & Kumar, S. (2015). Authentication and Encryption in Cloud Computing. *International Conference on Smart Technologies and Management for Computing, Communication, Controls, Energy and Materials (ICSTM),* Chennai, pp. 216-219.
16. Ali Yassin, A., Abullah., Hussain, A., & Keyan Abdul-Aziz Mutlaq. (2015). Cloud Authentication Based on Encryption of Digital Image Using Edge Detection. *International Symposium on Artificial Intelligence and signal Processing (AISP).*
17. Barhatel, S. M., & Dhore, M. P. (2016). User Authentication Issues in Cloud Computing. *National Conference on Recent Trends in Computer Science and Information Technology,* pp. 30-35.
18. Yang, H., Zheng, H., Zhang, J., Wu, Y., Lee, Y., & Ji, Y. (2017). Blockchain-Based Trusted Authentication in Cloud Radio over Fiber Network for 5G. *16th International Conference on Optical Communications and Networks (ICOCN)*, Wuzhen, pp. 1-3.
19. LeJeune, J., Tunstall, C., Yang, K. P., & Alkadi, I. (2016). An Algorithmic Approach to Improving Cloud Security: The MIST and Malachi Algorithms. *IEEE Aerospace Conference*, Big Sky, MT, pp. 1-7.
20. Azougaghe, A., Kartit, Z., Hedabou, M., Belkasmi, M., & El Marraki, M. (2015). An Efficient Algorithm for Data Security in Cloud Storage. *15th International Conference on Intelligent Systems Design and Applications (ISDA)*, Marrakech, pp. 421-427.

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Survey on Internet-of-Things based Wireless Sensor Network nodes in Cloud for Environmental Protection

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Abstract:

Wireless sensor nodes are used to measure and transmit the environmental data to a station for monitoring, analytical and controlling purposes. A network with a number of wireless sensor nodes is called Wireless Sensor Network (WSN). Internet-of-Things (IOT) is an innovative technology that is used to connect plenty of tiny electronic devices and gadgets with the internet in a cost-effective way. Modern Wireless sensor networks uses this advantage of IoT. If a device has the internet connectivity, then the device can be a member of a cloud service and it can access the services provided by the cloud with proper subscription. This work is targeted to make a clear analysis on how IoT based WSN nodes with cloud connectivity are used to monitor and protect environmental pollutions.

1. Introduction

Environmental safety is the hottest topic circulates around the world. Global warming and natural calamities tuned the heads of all nations towards them and make them to reckon their stand about protecting the nature. One side there are plenty of inventions to make the living as more comfortable and luxurious. But manufacturing those gadget requires much of raw materials taken from the earth and the taming process of those materials pollute the fundamental elements as earth, water, air and even space in recent days. To overcome these issues, many governments have their policies and guidelines in protecting the nature. The problem is monitoring the adoption of these guidelines in all corners of the country is complicated. Some industries are following the environmental guidelines

Strictly in the installation stage, but the machineries lax their rigidity over the time due to wear and tear.

To monitor the environment constantly in a periodical manner, a dedicated low-cost network to connect the sensor devices is required. The low cost refers here both initial instalment cost as well as the running cost. IoT provides an easy way to connect any device with the internet. Cloud provides ample of computational resources in cheaper cost. Combining the both with wireless sensor nodes can be used to construct a cost-effective environmental monitoring system. This survey is destined to search and analyse the existing methods which serves this requirement. The analysis is performed based on the standard network assessment metrics[1] such as throughput, packet delivery ratio, mobility, communication delays and power consumption

2. Existing Works

While searching for IoT-WSN in cloud platform for environmental pollution monitoring, there are three categories of works available as of now. They are, using IoT for environmental monitoring, connecting IoT devices with cloud and Cloud based environmental monitoring system. Some of the notable existing works are An Integrated System for Regional Environmental Monitoring and Management Based on Internet of Things[2], Constructing the Green Campus within the Internet of Things Architecture[3], Combining Cloud and sensors in a smart city environment[4], A Generic IoT Architecture for Smart Cities[5], An Integrated Cloud-Based Wireless Sensor Network for

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Monitoring Industrial Wastewater Discharged into Water Sources[6] and Polluino: An Efficient Cloud-based Management of IoT Devices for Air Quality Monitoring[7]. Since the topic "IoT-WSN Cloud based Environment monitoring and protection" is relatively new, there are only a limited number of articles found through the sources and they are mentioned above.

2.1. An Integrated System for Regional Environmental Monitoring and Management Based on Internet of Things (ISREMM-IoT)

This work introduces an innovative Integrated Information System (IIS) that combines Cloud Computing, Remote Sensing (RS), Geographical Information System (GIS), Global Positioning System(GPS) and IoT. Multiple sensors are used to collect the data and webservices are used to communicate the collected data among other nodes and control units. Application Gateway (AG), Application Software for different platforms and tasks (APPs), IoT Application Infrastructure (IoT-AI), Extraction Transformation Loading (ETL), Online Analytical Processing (OLAP), Real-time Operational Database (RODB) and Relational Online Analytical Processing (ROLAP). The middleware layer of IIS is used to implement the Application Program Interfaces (APIs).

Several operational blocks of Environmental Informatics (EI), Integrated Information System (IIS) and Internet-of-Things are discussed in the related works of ISRMM-IoT. The system architecture of ISREMM-IoT is based on major network modules such as Perception Layer, Network Layer, Middleware layer and Application layer.

The perception layer is responsible for gathering detailed information from physical world which is noted as targets. This layer usually works with real-time datasets, data processing methods and predefined cumulative knowledge.The real-time data is collected by a number of IoT based sensors and RS platforms as Aircrafts, Balloons, Radars and Satellites. These sensors are connected by existing mobile network technologies such as 2G, 3G and LTE using different IEEE 802.xx protocols like WiFi, Bluetooth and ZigBee. The perception layer has the capability of connecting lots of sensors and devices in a comfy way.

Fundamental functions of data interchanging are performed by the network layer. Access and Transport networks are the main entities of network layer. Short range wireless networks like Sensor Area Networks (SAN) are comes under the category of Access networks which is responsible to connection of things like sensors and devices those are used for monitoring and managing environmental constrains. Wide Area Networks (WAN) of both wired and wireless networks that operate based on Internet Protocol are comes under the transport network category. Third Generation Partnership Project (3GPP), Constrained Application Protocol (CoAP), Hypertext Transfer Protocol (HTTP), Internet Protocols (IPv4, IPv6), Transmission Control Protocol (TCP), Machine to Machine communication (M2M), User Datagram Protocol (UDP) are the frequently used standards in transport networks.

Middleware layer is a collection of sublayers of data management, software tools, models and platforms. This is the intermediate layer between network layer and application layer. This layer makes use of Real-time Operational Database (RODB) to handle the monolithic data collected by the huge number of sensors and devices. This layer is also responsible for gathering information from collected data and knowledge discovering. ETL is used to extract and process data from RODB to support the data in demand. The formatting processes of slicing, dicing and pivoting are processed by OLAP and ROLAP. AG, APPs, and IoT-AI are implemented in the middleware layer for applications and their services. Data as a Service (Daas), Infrastructure as a Service (IaaS), Platform as a Service (PaaS) and Software as a Service (SaaS) are used in IIS. ISREMM-IoT proposes a Service Oriented Architecture (SOA) for middleware in IIS to dismantle the complicated systems into simpler and well-defined subsystems for ease of execution.

Application Layer of proposed ISREMM-IoT's ISS is used to access the application support platforms and cloud computing platforms by IoT. The accessing to different platforms is used to

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perform organizing and sharing the environmental data among sensors, actuators, devices and webservices. The sharing process with webservice is used to contrive well designed resource monitoring, management, pollution monitoring, disaster prediction, disaster monitoring and weather forecasting. Aggregating millions of data collected from the sensors and process the data for prediction of certain events requires massive computational resources – and this is where the cloud takes advantage. Since cloud permits extreme virtualizations, the prediction process can be deployed in a well suited virtual machine to handle the complicated prediction tasks seamlessly.

ISREMM-IoT contributes its services for industrial and scientific purposes. The purpose of IoT is here to collect data and the cloud computing environment is to process the collected data. By integrating these two technologies, ISREMM-IoT is constructed and tested in real-world environment. Xinjiang - a place located in China, where the natural resources are sensitive to climate changes and human activities is taken for the study. The author of ISREMM-IoT used a collection of datasets that includes 50 years of hourly measured data of meteorological elements in the study place with the help of distributed sensors.

This work majorly concentrates on the accuracy of prediction of the environmental variables rather than how they can be collected cost effectively. Their entire model is tested with the pre-recorded data. Results are produced based on Gross Primary Production (GPP), Net Primary Production (NPP) and Leaf Area Index (LAI). It is clearly understood that the authors are projecting a model to monitor environmental data using IoT based sensors and processing of the collected data in a cloud environment. Even though the idea of collecting data from IoT and process the data in cloud is used in this paper, the evaluations are carried out in proving the prediction accuracy of the proposed model.

Graphs are provided for Annual Total GPP, Annual Total NPP and annual precipitation but the network metrics like throughput and communication delays are not evaluated in the implementation which is vital to measure the efficiency of IoT-WSN data collection in cloud platform.

2.2. Constructing the Green Campus within the Internet-of-Things Architecture (CGC-IoT)

As per the perspective of the author, applying IoT technology in resource conservation to support the global population growth in the earth. If we could design a well monitored campus then the same is applicable for the entire world. As global population growth entering the sky-high situation, maintaining the environment and resources are complicated process. Fortunately, the technological growth makes a way to maintain the balance in resources and pollution controls.

The "Smart Campus" concept is to adopt the standardization of Information Communication Technologies (ICT) to monitor complete facilities of the campus automatically. This monitoring process is intended to improve the efficiency of the facility utilization and to minimize the resource wastage.

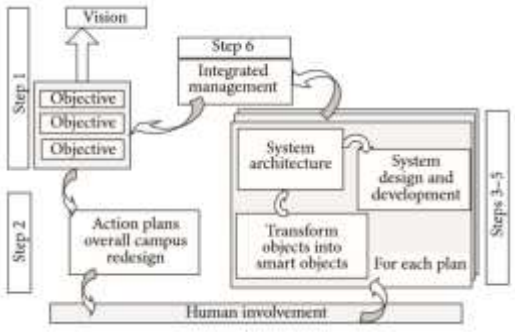

Figure 1. CGC-IoT procedure

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The smart campus which can save power and resource wastages is called as Green Campus. The IoT, Cloud computing and Wireless sensor networks are used in CGC-IoT. The followed procedure is given in Figure 1. There are three segments used in the CGC-IOT Project. They are the hardware segment, middleware segment and the presentation segment.

The hardware segment consists of RFID technology for students' identification, IoT devices are connected to centralize the controls of computers and Air conditioning units. The temperature sensors are connected with the ZigBee network boards with 2.4 GHz radio frequency with 10 meters coverage.

The middleware segment consists of a cloud server. This is used to maintain the database in which the campus data collected by the sensors. The database has information about the students' presence based in the RFID tags, temperature in different laboratories and the running status of the computers and air conditioning units. The cloud server is used to store, monitor and analyse the data.

The presentation segment is used to connect the database with the rich user interfaces for comfortable accessing of data by the students and by the General Affair Controller. This facility is used to get current availability of the labs and the General Affair Controller can monitor the utilization of the labs periodically. The overall architecture of CGC-IoT is illustrated in Figure 2.

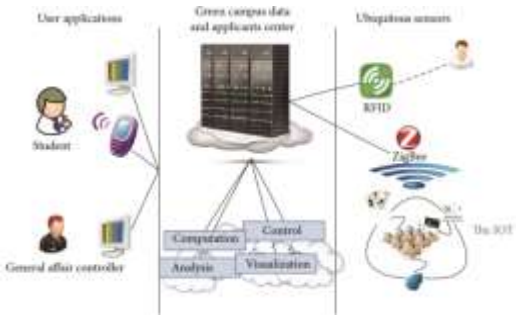

Figure 2: CGC-IoT Architecture

As per the study on CGC-IoT, the use of latest technologies as IoT, WSN and Cloud computing are integrated in this work to construct the Green Campus is a successful project. Based on the screenshots provide by the author make it clear that their mission on controlling campus environment with user friendly interface is a real success.

The presence of IoT, WSN and Cloud computing is clear but the efficiency of using these technologies for the dedicated purpose is not analysed in this work. The author used all readymade protocols like IEEE 802.11 b/g/n and ZigBee to develop this project. Therefore, the performance of the constructed network is not measured using the standard network metrics as throughput, packet delivery ratio, communication delays and power consumption of these network devices. Developing a legacy protocol to combine all these communication technologies for the environmental monitoring purpose will reduce the power consumption and increase the overall throughput by minimizing the communication delays.

2.3. Combining Cloud and sensors in a Smart City Environment (CCSSCE)

This work states that the number of devices connecting with internet in this modern Information and Communication Technology (ICT) trend is enormous. These connected devices are not operating on the same architecture thus they are heterogeneous in nature. A proposal is provided in CCSSCE work to connect a number of heterogeneous sensor nodes with the cloud to construct and monitor a smart city environment.

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A novel thing introduced in this work is the Zero Configuration (referred as ZeroConf in the paper) provision while adding new sensor to the network. This will improve the scalability of the heterogeneous network to connect different types of sensor in a dynamic way. To understand the smart city concept, different terminologies are used in this work. The representation of the term 'site' can be a building, road, factory or the entire city. Each site should be an Anonymous System (AS) with the provision for interaction between services. The AS should have dedicated data producers and data consumers. The input sensor or a collection of sensors are represented as SI. Database DB is used to store the data collected by the SI from the site. The DB is distributed to make the data available for required consumers. The coherent establishment of CCSSCE is given in Figure 3.

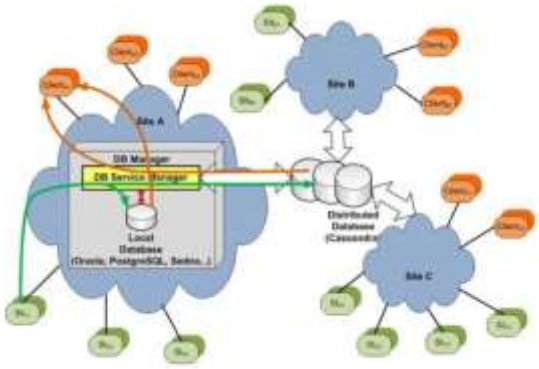

Figure 3: Coherent establishment of CCSSCE

The three main cloud components used in CCSSCE are Autonomic Enforcer, Hypervisor and Volunteer Cloud Manager. The hypervisor represents an embedded sensor of a personal device or a standalone sensor which works in the level of single node. The important tasks carried out are abstracting devices and their capabilities, telecasting commands, data retrieval, virtualization and labelling. The adapter is used to establish communication directly between sensing and actuating devices and tracks the connectivity among the resources. It translates the commands from the application into the native commands of the devices using native communication protocol of the devices.

The Autonomic Enforcer works as a bridge between the SaaS Clouds and virtual nodes. This is the process that is responsible for allowing a new node to connect with the cloud service thus allows the node to access the resources as a service through internet. It also manages the node resources in local based on both the higher-level cloud policies and local requirements. This Automatic Enforcer is designed in such away that distributed and taking decisions by communicating the nearest nodes and acquiring autonomic functionalities. The automatic enforcer is deployed in all nodes of the cloud to enforce the policies of the Volunteer Cloud Management module.

The Volunteer Cloud Management module is used to unify ad-hoc, volatile, dynamic resources and services in a cloud environment. The main advantage of this module is providing the resources in a dynamic nature based on the demands for the required periods. The Volunteer Cloud Management module is also responsible for creating and levying management strategies at the cloud level based on the continuous communication with all devices of the cloud constitution individually. The combination Automatic Enforcer and the Volunteer Cloud Manager are used to integrate the member nodes of the cloud for maintaining service subscriptions, directing policies and overlay installations. They also used to convert the accessing process of users, nodes and provider as the service.

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The implementation of CCSSCE, they used two light weight operating system. The first one is Contiki[8], an open source highly configurable memory efficient operating system for embedded systems. The memory requirements for Contiki is 40KB ROM and 2KB RAM. The main feature of this operating system is the light weight implementation of IP-Stack names as uIP which supports 6LoWPAN. Contiki also supports C style programming which is considered to be most optimal programming architecture for embedded systems.

The another OS used in this implementation is TinyOS[9]. TinyOS is well know for its versatility in implementing Wireless sensor network-based automations. It is written in nesC and released as open source under Berkeley Software Distribution (BSD) Licence. The functionalities of this OS are sensing, actuations, packet communication and routing. The TinyOS development toolchains are developed mostly in C language. This is one of the operating systems with fully non-blocking single call stack. The tasks are executed in this OS as first-in first-out order which is suitable for simple I/O centric applications.

The implementation of CCSSCE is done to serve the purpose of combining sensors under the cloud facility to monitor smart city environment. But the performance analysis is not yet provided with in the presented article. A clear analysis is required with various input output criteria is required to analyse the performance of the proposed model in CCSSCE.

2.4. A Generic IoT architecture for Smart Cities (GIoTA)

The IoT is defined as an integrated part of future internet by Cluster of European Research Projects (CERP). Based on this definition, the authors of GIoTA proposed a model to establish and maintain eGovernment of a smart city. Two types of platforms are discussed in this article, they are eGovernment related IoT platform and Enterprise-based and Company based IoT platforms. In GIoTA, the core element of eGovernment smart city system is the Integrated Information Centre (IIC). This centre is responsible for essential services such as electrical energy distribution, Water Supply, Gas Supply, Smart Transportation Services, Concerted Medical Services, Mobile communication Services, Internet services, Fire protection services, Security Services, Tax payment services, Commercial services and tourism services. A cloud computing-based data server is recommended to connect with IoT based sensors and actuators to maintain records for monitoring and future analysis purposes. The eGovernment based smart city can conserve resources in an optimized way and the environmental pollutions can be kept in control with the help of IoT based sensor devices to raise an immediate alarm in case of anomaly activity detection. Typical eGovernment smart city services are given in Figure 4.

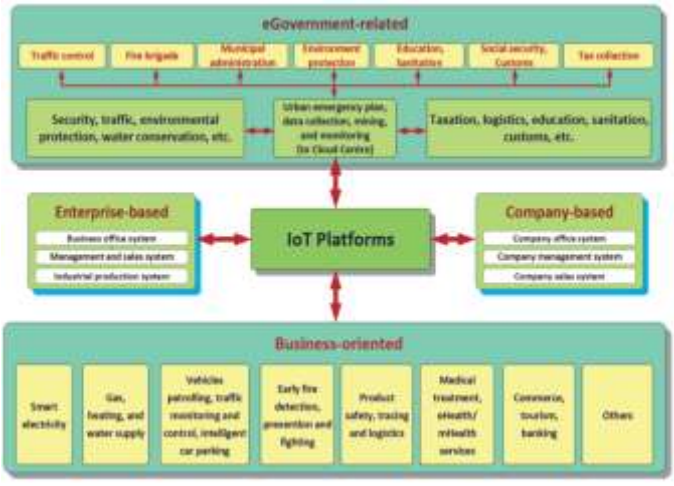

Figure 4: eGovernment Service of a Smart City

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This work also discusses about the utilization of available cloud network and IoT components as they are. The resources required to deployment of eGovernment and the power consumption required to run a lot of sensors, communication devices and centralized cloud server is not discussed here. The advantages of combining these latest technologies in environmental monitoring system is given tangible where the legacy design and performance improvements are left in the discussion.

2.5. An Integrated Cloud-Based Wireless Sensor Network for Monitoring Industrial Wastewater Discharged into Water Sources

This work presents a new epitome of Integrated Cloud-based WSN (ICWSN) to monitor wastewater contaminations discharged to the water sources. The IoT sensor devices are used to measure the conductivity, pH and dissolved oxygen in the wastewater drains. The measured data are communicated by the IoT devices to ThingSpeak[10] cloud using GPRS internet connectivity by using AT commands and HTTP GET. The water pollution details are provided with alert messages through the Telerivet message service.

The main components of ICWSN system are the wireless sensor node, the WSN Gateway node, Cloud platform for IoT and SMS gateway. The wireless sensor node is used to measure the conductivity, pH and dissolved oxygen from the wastewater. A wireless sensor node consists of the sensors devices, sensor interface shield, Arduino microcontroller and Wireless Communication module. The Arduino mega 2560 R3 open source board is used as the logical control unit of the wireless sensor node. Wireless communication is added to this node using XBee modules that uses IEEE 802.15.4/ZigBee protocol. This protocol is used because of its low power consumption nature. Two Li-Ion 3.7v, 3200mAh rechargeable batteries are used here as power source for wireless sensor nodes. The prototype block diagram is provided in Figure 5.

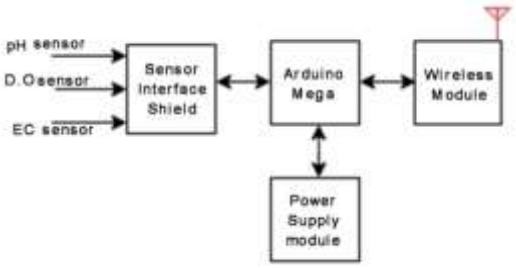

Figure 5: ICWSN node prototype block diagram

The WSN gateway node has Arduino Uno[11] as the central processing unit, XBee module to communicate with wireless sensor nodes and SIM800C Arduino GSM/GPRS module to connect with internet. An optional SD Card module is also provided to store data in case of unavailability of a proper internet connection. The hardware block diagram of the WSN Gateway not is given in Figure 6.

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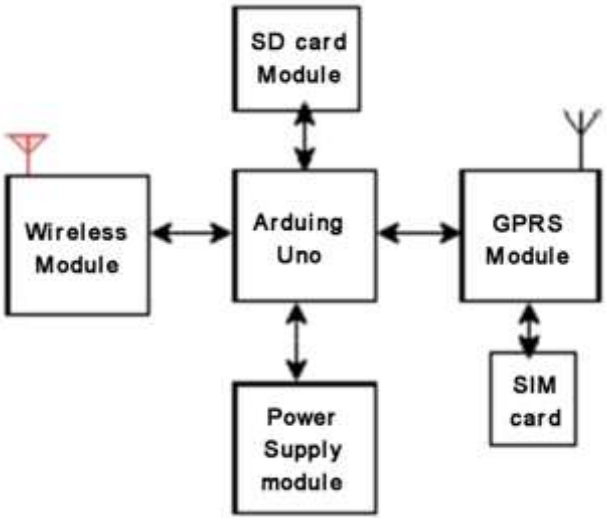

Figure 6: ICWSN Gateway Node hardware block diagram

The ICWSN architecture is implemented to serve the purpose with existing free open source cloud service and SMS gateway service where the security of the data is not assured. Moreover that, there is no specific IoT hardware used in the implementation. A conventional XBee module is used to provide the connectivity between the sensor nodes and Gateway node. This can be optimized by using dedicated low-cost power efficient IoT interface modules asEspressifESP8266. All the protocols used in this implementation are existing one prepared for general purpose IoT communication rather than communicating environmental data communication service. Even though the project operates well for monitoring water pollution, a dedicated IoT based sensor node with legacy communication protocols may reduce the power consumption significantly.

2.6. Polluino: An Efficient Cloud-based Management of IoT Devices for Air Quality Monitoring

This work targets on measuring impurities of air using IoT based sensors and connect to cloud for monitoring and analysing purpose. In this work the merits and demerits of different protocols such as Constrained Application Protocol (CoAP), Message Queuing Telemetry Transport (MQTT), Representational State Transfer Application Program Interface (RESTful API), Websockets and Extensible Messaging and Presence Protocol (XMPP).

The CoAP protocol is a web transfer protocol for limited or constrained hardware devices with IoT. This is a Machine-to-Machine (M2M) type protocol. CoAP works based on request and response code operations with structured messages. The CoAP message structure is given in Figure 7.

Byte								Byte								Byte								Byte							
0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	7
VER		TYPE		TKL (Token Length)				CoAP Request/Response Code								Message ID															
Token (If any in TKL bytes) (Maximum of 8 bytes)																															
Options (If Available)																															
1	1	1	1	1	1	1	1	Payload (If Available)																							

Figure 7: CoAP Message Structure

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Operations are performed using predefined methods like EMPTY, GET, POST, PUT, DELETE, FETCH, PATCH and iPATCH with response message as Created, Deleted, Valid, Changed, Content and Continue. CoAP message frames follows IEEE 802.15.4 standard to avoid fragmentations.

MQTT[12] is a very light weight protocol used to perform M2M communications. MQTT is developed for low power and low computational resource devices to work with low-bandwidth networks. It also supports unreliable communications with higher communication delays. The MQTT provides a little security by providing independent access to SSL encryption. But while using SSL security, the protocol will consume more resources and increases the communication overheads. The SSL security is accessed through independent calls, it means SSL is not built-in type security in MQTT. The MQTT message map is given in Figure 8.

Figure 8: MQTT Messages and Descriptions

The latest version of MQTT v3.1.1. is now listed under Organization of Advancement of Structured Information Standards (OASIS). Most cloud service providers like Microsoft Azure, AWS, Google cloud and IBM BlueMix supports MQTT.

RESTful API is an application program interface webservice with Hyper Text Transfer Protocol (HTTP) operations GET, PUT, POST and DELETE. The RESTful API splits a large transaction into small modules and perform the communication. It uses stateless calls and microservices which makes it optimum to use in cloud environment.

Websockets is one of the advanced technologies used to initiate and interactive communication sessions between the client and server. This API is used to send messages to a server. The responses are event-driven which reduces the burden of periodical polling of the server for response. The default port settings for Websocket is port 80 for HTTP and port 443 for HTTPS. Websocket requires adoption of SSL/TLS for security purpose anyway the usage of security layers will increase the resource consumption.

XMPP protocol is one of the most secured messageing protocols. This is declared as an open standard for presence and messaging. XMPP is widely used in IoT and WebRTCdomains. It is also free and open source. XMPP uses the TCP as base protocol with extended period TCP connections. It is also compatible with HTTP for firewall restricted networks. Secure Authentication and Secure Layer (SASL) and transport layer security are built-in security provisions of XMPP.

The Polluino hardware has a set of sensors to measure the impurities in the air. The sensor units are governed by a ATMega2560 based Arduino board. ESP8266-01 Wi-Fi module is used to establish the IoT communication. The authors analysed latency and communication overheads to measure the performance of the protocols discussed and as per the observations, the winner is MQTT protocol. The power consumptions for different protocols are not provided in this work. Other important delays like IP-Delay, Jitter and End-to-End delay are not measured for performance comparison. Based on sources collected from web-resources, this work perfectly implements the IoT based Sensor network with cloud computation. The project is done with available protocols and MQTT is stated as the best match for this kind of IoT-WSN-Cloud network. Still the MQTT protocol has the options to be customized to save power and to provide improved performance.

3. Table of Summery

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Work	Evaluation	IoT	WSN	Cloud	Network Metrics
ISREMM-IoT	IMP	Y	Y	Y	N
CGC-IoT	IMP	Y	Y	Y	N
CCSSCE	IMP	P	Y	Y	N
GIoTA	PRO	Y	Y	Y	N
ICWSN	IMP	Y	Y	Y	N
Polluino	IMP	Y	Y	Y	PA

IMP: Implemented, PRO: Proposed, Y: Yes, N: No, PA: Partially Analysed

4. Conclusion

There are some attempts to combining the technologies of IoT, WSN and Cloud for environmental monitoring and a few of them are successfully utilize these emerging technologies. Still there is a requirement for providing a dedicated IoT-WSN-Cloud architecture with legacy protocol to perform the environmental monitoring task more effectively with optimized power utilization. Zeroconf limitless new node adoption is one of the most required characteristics of a new conception.

5. References

[1] DivyaChitimalla, KoteswararaoKondepu, Luca Valcarenghi, Massimo Tornatore, Biswanath Mukherjee, "5G fronthaul-latency and jitter studies of CPRI over ethernet", IEEE/OSA Journal of Optical Communications and Networking, IEEE 2017

[2] ShifengFang,Li Da Xu, Yunqiang Zhu, JiaerhengAhati, Huan Pei, Jianwu Yan, Zhihui Liu, "An Integrated System for Regional Environmental Monitoring and Management Based on Internet of Things", IEEE TRANSACTIONS ON INDUSTRIAL INFORMATICS, IEEE 2014

[3] Hsing-IWang, "Constructing the Green Campus within the Internet of Things Architecture", International Journal of Distributed Sensor Networks, Hindawi Publishing Corporation - 2014

[4] Nathalie Mitton, SymeonPapavassiliou, Antonio Puliafito, Kishor S. Trivedi, "Combining Cloud and sensors in a smart city environment", EURASIP Journal on Wireless Communications and Networking, Springer - 2012

[5] Ivan Ganchev, Zhanlin Ji, M´airt´ınO'Droma, "A Generic IoT Architecture for Smart Cities", https://www.researchgate.net/publication/268191223, ResearchGate - 2014

[6] Yona Zakaria, Kisangiri Michael, "An Integrated Cloud-Based Wireless Sensor Network for Monitoring Industrial Wastewater Discharged into Water Sources", Wireless Sensor Networks, Scientific Research - 2017

[7] Giovanni B. Fioccola, Raffaele Sommese, ImmaTufano, Roberto Canonico, Giorgio Ventre, "Polluino: An efficient cloud-based management of IoT devices for air quality monitoring", 2016 IEEE 2nd International Forum on Research and Technologies for Society and Industry Leveraging a better tomorrow (RTSI), IEEE - 2016

[8] Maximilien Charlier, Bruno Quoitin , Sébastien Bette, Jens Eliasson, "Support for IEEE 802.15.4 ultra wideband communications in the Contiki operating system", 2016 Symposium on Communications and Vehicular Technologies (SCVT), IEEE 2016

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[9] Muhammad Amjad, Muhammad Sharif, Muhammad Khalil Afzal, Sung Won Kim, "TinyOS-New Trends, Comparative Views, and Supported Sensing Applications: A Review", IEEE Sensors Journal (Volume: 16, Issue: 9, IEEE - 2016

[10] Ahmed I. Abdul-Rahman, Corey A. Graves, "Internet of Things Application Using Tethered MSP430 to Thingspeak Cloud", Service-Oriented System Engineering (SOSE), IEEE 2016

[11] Yusuf Abdullahi Badamasi, "The working principle of an Arduino", International Conference on Electronics, Computer and Computation (ICECCO), IEEE 2014

[12] Meena Singh, M.A. Rajan, V.L. Shivraj, P. Balamuralidhar, "Secure MQTT for Internet of Things (IoT)", Communication Systems and Network Technologies - IEEE 2015

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AN ENHANCED FRAMEWORK FOR CLASSIFIYNG CANCER GENE DATA USING DECISION TREE AND SUPPORT VECTOR MACHINE

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Abstract: **In microarray analysis one of the most important challenges is to find out the genes or group of genes that are well expressed in cancer cells but not in normal cells. To improve the diagnostic of different diseases the supervised machine learning techniques are used with microarray datasets. In this work, an enhanced framework has been developed for classifying cancer gene data. Generally in datasets and classification methods, Gain ratio attribute selection slightly enhanced the classification accuracy due to the concentration on the most promising genes having the effective information gain that discriminate the dataset. This enhanced framework uses three stages such as attribute selection, choosing appropriate predictor and produced model for classifying cancer gene data using decision tree and support vector machine. The proposed framework achieves higher accuracy with attribute selection and Gain ratio attribute selection for classifying cancer gene data.**

Keywords: **Microarray analysis, cancer cells gene, supervised machine learning and classification.**

I. INTRODUCTION

Cancer is a disease involving irregular cell growth with the possible to attack or spread to other parts of the body[9]. This dissimilarity with benign tumors, which do not spread to other parts of the body [2]. Potential signs and symptoms include a lump, irregular bleeding, extended cough, unexplained weight loss and a change in bowel movements. At the same time as these symptoms may point out cancer, they may have other causes. Over 100 types of cancers affect humans. The major challenges in microarray analysis in cancer gene expression are to find out genes or groups of genes that are highly expressed in cancer cells but not in normal cells. To build classification models that improves the diagnostic of different diseases using supervised machine learning techniques with microarray datasets. Inside almost every cell in our body is a structure called the nucleus? Inside the nucleus there are 23 pairs of chromosomes. These are long strings of DNA. DNA stands for deoxyribonucleic acid. Each string of DNA looks like a twisted ladder. Scientists call this a double helix. We have more than 2 meters of DNA inside every cell, but it is very tightly coiled up so it all fits. DNA is similar to a code containing all the instructions that tell a cell what to do. It is made up of genes [1]. Humans have around 25,000 genes in total. Some genes control how much each cell grows and divides. Occasionally people take over certain faulty genes from their parents. This can give them an increased risk of cancer. Generally, cells can restore faults in their genes. When the damage is extremely bad, the cell may identity destruct as a substitute. Or the immune system may recognize them as abnormal and kill them. This helps to protect us from cancer. At times alteration in significant genes causes a cell to no longer understand instructions. The cell can start to multiply out of control. It doesn't repair itself properly, and it doesn't die when it should. This can lead to cancer [7].

II. CANCER CLASSIFICATION CHALLENGES

The major cancers by molecular criteria rather than their tissue of origin can provide patients with further exact diagnoses, researchers have reported in Cell. The cluster examined the molecular characteristics of more than 3,500 samples of twelve different cancers and reclassified them based on the new information [3]. For five of the cancer types with acute myeloid leukemia, the molecular classification mainly matched the tissue-of-origin classification. This genomic learning not only challenges our existing model of classifying cancers based on tissue type, but also provides a massive new data resource for further exploration, as well as a comprehensive list of the molecular features distinguishing each of the newly described cancer classes"[8]. Gene classification as domain of research poses a some challenges due to its unique problem nature [6]. Initial challenge comes from the unique nature of the available gene expression dataset; where most of these datasets has sample size below 200, vs. thousands to hundred thousands of genes presented in each tuples. Next, only a few numbers of

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these presents relevant attributes to the investigated disease. Third, comes from the presence of noise inherent in the dataset. Final challenge occurs from the application area, for instance accuracy is an important criterion in cancer classification task, but it is not the only goal, in cancer domain we want to achieve, biological relevancy as well as classification accuracy [4].

III.PROBLEM DEFINITION

There is no single classifier superior over the rest, for instance the classification accuracy is depend on the classification method, gene selection method, and dataset. Let $x_1;x_2; \ldots ;x_m$ be random variables for genes $g_1;g_2; \ldots ;g_m$ respectively, where x_i has domain dom (x_i) which is the range of expression values for gene g_i.. Table 3.1 illustrates the confusion illustrates the confusion matrix for positive and negative tuples matrix for positive and negative tuples.

Table 3.1 Confusion Matrix

	Predicted class		
Actual class		C1	C2
	C1	TP	FN
	C2	FP	TN

Let C be the random variable for the class labels, and dom$(C) = \{1,2,\ldots,K\}$, where K is denotes a total number of classes. Let t $=\{t.X_1, t.X_2, \ldots, t.X_m\}$ denotes a size m tuple of expression values for m genes. Let T = $\{(t_1, c_1), (t_2, c_2) \ldots (t_n,c_n)\}$ Denoting a training set of n tuples, where i ={1; 2; . . . ; n}, $c_i \in$ 2 dom(C) is the class label of tuple t_i. Let the test set be $S=\{t_1; t_2; \ldots ; t_l\}$ where l is the size of the test set. To find a classification function Class, which gave maximal classification accuracy on S, where the classification accuracy calculated by divide number of correct classified instances on total number of instances.

$$\boldsymbol{Accuracy = \frac{TP + TN}{TP + FN + FP + TN}}$$

Where

- TP is the True positive number of predicted positive cases that are actually positive.
- TN is the True negative the number of predicted negative cases that are actually negative.
- FP is the False positive number of predicted positive cases that are actually negative.
- FN is the False negative number of predicted negative cases that are actually positive.

IV. DECISION TREE, SVM AND MICROARRAY CLASSIFICATION

Decision trees belong to the most popular predictive models in machine learning. They are usually easy to understand and interpret. People who are not acquainted with decision theory are able to understand them and find an explanation for their decisions. They are two broad categories of decision trees which vary in the kind of tests used in non-terminal nodes[5]. The decision tree is called axis parallel, if each test is based on a single attribute. If, on the other hand, tests are based on more than one feature, such a tree is called multivariate. Oblique decision trees exemplify the particular kind of multivariate trees in which hyper planes are used as tests. Because the problem of inducing an optimal decision tree is very difficult (NP-complete), greedy heuristics are mostly employed. The most widely used approach is based on the splitting criterion. It is known as the top-down induction of decision trees and is used in most well-known algorithms. A decision tree is learned by recursive splitting the subset of examples based on the test (univariate or multivariate) in the current node. The procedure is recursively repeated for obtained subsets. Certain algorithms that follow this framework vary mainly in the stopping criterion and the way the tests are chosen. The proposed framework used as a renowned example of the axis parallel decision trees inducers, and OC I for oblique decision trees induction. These algorithms can be treated as the de-facto standards in empirical evaluations of decision trees. Even though the aforementioned

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approach is robust and performs well with large data in a short time, it fails for certain problems and finds the local optimum.

There are two drawbacks that also be a most important weakness in using decision tree for microarray analysis problems in the ancient times. The first one is their insecurity that is tightly connected with the second disadvantage – i.e., difficulties to stemming the trees when the number of samples is too low. Unsteadiness of decision trees was successfully solved by ensembles way where multiple trees built from different subsets of the initial dataset were built to get better the robustness of the final classifier. Unfortunately, ensembles of classifiers groups very small level of their knowledge understand-ability and are not suitable for analysis of the acquired knowledge. Due to high cost per experiment in microarray studies it is nowadays still acceptable for studies with 100 or yet less samples to represent benchmarking datasets for evaluation of the most complex classifiers.

Most of the microarray data today is collected in centralized repositories containing large numbers of samples like Gene Expression Omnibus (GEO) by National Center for Biotechnology Information (NCBI) or Array Express by European Bioinformatics Institute (EBI) . Unfortunately such repositories are too large and contain data coming from various sources using different protocols to serve as a benchmarking collection of datasets. The proposed framework takes advantage of one of the largest in public available repositories of gene expression measurements that were collected by EBI. This is presently one of the most suitable collections of gene expression samples for evaluation of classification methods. Calculate the expected information required to classify a tuple from D based partitioning by A.

$$Info_A\ (D) = \sum_{i=1}^{m} \frac{D_j}{D}\ X\ Info_A\ (D)$$

The term $\frac{D_j}{D}$ acts as the weight of the j[th] partition.

- Calculate information gain of attribute A.

$$Gain(A) = Info(A) - Info_A\ (D)$$

- Calculate split information of attribute A

$$SplitInfo_A\ (D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|}\ \text{X}\ \log_2\left(\frac{|D_j|}{|D|}\right)$$

- Calculate gain ratio

$$GainRation(A) = \frac{Gain(A)}{SplitInfo_{A\ (D)}}$$

The attribute with the maximum gain ratio is selected as best splitting attribute. Classification and Regression Tree (CART) is a binary decision tree, which split a single variable at each node. CART approach can also generate classification trees, which depends on the type of the dependent variable CART Similar to C4.5 but use Gini index as split criteria that calculated by using the following equation

$$Gini(D) = 1 - \sum_{i=1}^{m} {p_i}^2$$

The Gini index of attribute A for a binary spilt is calculated by following equation

$$Gini_A(D) = \frac{|D_2|}{|D|}\ \text{Gini}\ (D_1) + \frac{|D_2|}{|D|}\ \text{Gini}(D_2)$$

Random Tree

- It is constructing a tree that randomly selected attributes at each node. It makes no pruning.

Decision Stump

- It is an algorithm builds simple binary decision 'stumps' for both nominal and numeric classification task.
- It deal with missing values by extending a third branch from the stump or treating 'missing' as a separate attribute value.
- It does regression or classification based on entropy.

REP Tree

- It is a fast decision tree algorithm
- It is based on C4.5 algorithm and can produce classification.
- It builds a regression/decision tree using information gain/variance and prunes it using reduced-error pruning.

ADTree

- It is an applied alternating decision trees.

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- This algorithm improving procedures to decision tree algorithms to produce precise classifiers.
- The classifiers are in the form of a best part to choose over a number of decision trees but having a smaller and easier to understand classification rules.

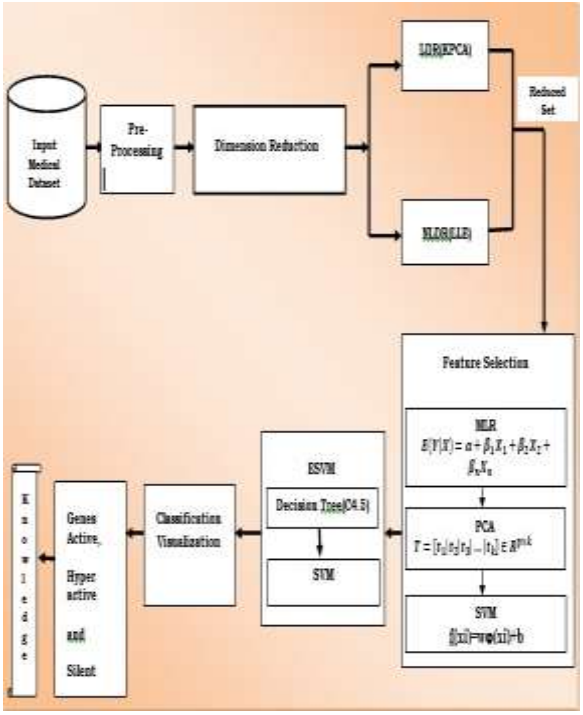

Figure 4.1: Overview of the Enhanced Framework

Random Forests

- It ensemble decision tree methods by resampling attributes.
- The early random decision trees technique merges bagging and random feature selection methods to produce multiple classifiers.
- It is based on CART method.

Bagging

- It uses a bootstrap technique to resample the training data sets D.
- To form a resampled data set Di. Each sample in D has a probability of 1/n of being drawn in any trial.
- The most often predicted class label will be the final classification result.

AdaBoost

- It was first developed by Freund.
- The initial classifier is constructed from the original data set where every sample has an equal distribution ratio of 1.
- In this method training data set D*i*, the distribution ratios are made different among samples depending on their prediction accuracy in the previous data set D_{i_1}.
- If a sample has a lower prediction accuracy rate in D_{i_1}, it will be given a higher weight in Di and therefore get a higher possibility to be selected in D_i.

V. RESULTS AND DISCUSSIONS

The experiments have been carried out and the results obtained have been encouraging. The results for the proposed framework are presented in Table 6.1. The table 5.1 shows the classification accuracy using gain ratio and chi-square model**.** Table 5.1 shows the classification accuracies of the proposed model has performed quite well compared to C4.5 and SVM. For this dataset, the proposed method using gain ratio attributes

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selection method to improve accuracy of gene classification. This establishes the efficiency of the proposed framework with respect to classification accuracy.

Table 5.1: Accuracy of the C4.5, SVM and Proposed Method

Dataset	Accuracy (%)		
	C4.5	SVM	Proposed Method
Breast1	95	95	97
Breast	94	82	96
Colon	88	81	97
Lung2	95	93	96
Prostate	92	90	95
Prostate2	82	74	93
Prostate3	80	82	87
Multi tissues	89	86	92
Leukemia	91	90	94
Lymphoma	92	94	97

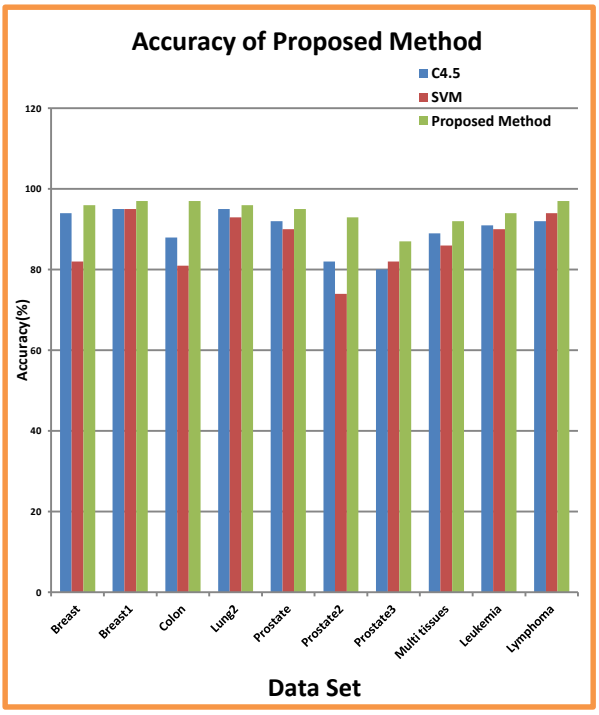

Figure 5.1: Classification Accuracy

VI.CONCLUSION

This enhanced framework improves classification performance of decision tree algorithms microarray datasets. In addition, the effect of attribute selection on building decision tree based models is investigated. Gain ratio

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attribute selection significantly improves classifier accuracy on majority of the dataset and classification method. Lastly, decision trees are particularly attractive for biologists due to their interpretability, being able to highlight which genes are actually influencing the classification task as well as results show that decision tree classifiers might play an important role in microarray analysis in the future. But small numbers of genes shared to constrict tree from microarray dataset is still critical criteria because missing such genes mean missing classification result.

REFERENCES

[1] Heling Jiang, An Yang , Fengyun Yan and Hong Miao, "Research on Pattern Analysis and Data Classification Methodology for Data Mining and Knowledge Discovery" , International Journal of Hybrid Information Technology Vol.9, No.3 (2016), pp. 179-188.

[2] Abhishek Bargaje, Shubham Lagad, Ameya Kulkarni, and Aniruddha Gokhale, "Review of Classification algorithms for Brain MRI images" , International Research Journal of Engineering and Technology (IRJET), Volume: 04 Issue: 01 , -ISSN: 2395 -0056Jan-2017.

[3] Vijayan A, Kareem S, and Kizhakkethottam JJ., "Face recognition across gender transformation using SVM classifier", Procedia Technology, 24: 1366-1373, (2016).

[4] Dang, Thuy Hang, Trung Dung Pham, HoaiLinh Tran, and Quang Le Van. "Using dimension reduction with feature selection to enhance accuracy of tumor classification."In Biomedical Engineering (BME-HUST), International Conference on, pp. 14-17.IEEE, 2016.

[5] Onur Inan ,Mustafa Serter Uzer and Nihat Y lmaz"A New Hybrid Feature Selection Method Based On Association Rules And PCA For Detection Of Breast Cancer", International Journal of Innovative Computing, Information and Control ICIC International conference 2013 ISSN 1349-4198 Volume 9, Number 2, February 2013.

[6] A.K. Shafreen Banu and Dr.S.Hariganesh," A Hybrid Approach For An Efficient Classification Using Decision Tree and SVM", International Journal of Computer Science and Mobile Computing(IJCSMC), ISSN 2320–088X, Vol.7 Issue.2, February- 2018, pg. 42-48.

[7] Karegowda1 AG, Manjuna AS, Jayaram MA., "Comparative study of attribute selection using gain ratio and correlation based Featuer selection" . Int J Inform Technol Know Manage 2010;2(2):271–7.

[8] K.S.NS. Goal Krishna, B.L.S. Suraj, M. Trupthi, "Classification Of Breast Cancer Into Benign and Malignant Using Support Vector Machines", International Journal of Advance Research in Science and Engineering (IJARSE), ISSN 2319-8354, Vol.6, Issue 10, October 2017.

[9] M. Kumar, S. S. Tomar and B.Gaur, "Mining based Optimization for Breast Cancer Analysis: A Review",International Journal of Computer Applications, vol. 19, no. 13,(2015).

International Journal of Management, IT & Engineering
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A NOVEL SECURED APPROACH FOR MINING CLOSED HIGH UTILITY ITEMSETS

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Abstract: The formulated High Utility Itemset is compact in size that's why the efficiency will degraded of mining process. Closed High Utility Itemset proposed to achieve the efficiency. Data privacy problem occur because one party never honesty on another party. Outsourcing of this task may have privacy leak problems. In this work we proposed an efficient system to handle the load of computation, storage and processing to another party with preservation of privacy of outsourced high utility mining and also to produce the concise representation of High Utility Itemsets using existing work. Each item in item set is assigned with numerical id randomly. Output of this process is converted dataset, Map of the Items and their numerical ids. This converted database and encrypted utility threshold given to third party for high utility mining in a secured way. The proposed system to improve the process of generation of compact HUIs to another party and also fulfilled the aroused need of security and privacy of the data as outsourced party may not be trusted one.

Keywords: *High Utility Itemset, Closed High Utility Itemset, Privacy and Compact HUI.*

I. INTRODUCTION

To get useful and important information for the knowledge the huge mining of data required. Some organization required such data for their purpose. Market basket analysis is popular method in the market is FIM[5]. Some issues are in the FIM model it generated itemset having low importunacy item and low selling tags. It may generate high importunacy itemset. FIM possess same importunacy, weights to all itemsets. Utility mining is established to overcome this problem to reduce cost weight and height. Some itemset are useful in data mining for the market base analysis purpose [9]. Current model of market base analysis produces low important itemsets and frequent lose selling value. To avoid this utility mining is essential. Extraction high itemset from database is very difficult task. So formulated HUI is compact in size that's why the efficiency will degraded of mining process. Some ideas are available to achieve the efficiency are free sets closed itemsets etc. Closed high utility itemset (CHUI) proposed to achieve this goal. To outsourced the party for mining of data. Data privacy problem occur because one party never trustworthiness on another party. Outsourcing of this task may have privacy leak problems[6.7].

Some applications such as website click stream analysis, mobile commerce environment, biomedical applications etc has high utility mining. If the utility of an itemset is greater than user-specified minimum utility threshold then it is said to be high utility itemset otherwise it is considered as a low utility itemset. Large amount of high utility itemsets causes difficulty to user for result analysis of HUI. Require more memory and more processing time causes less efficiency [8]. For more efficiency to reduce cost overhead and mining task and to provide concise representation, different approaches like Freeset, Non derivable sets, Closed Itemset are introduced in FIM. But applying these techniques to HUI produced several challenges:

- How to recover all HUI's from the concise representation.
- Lossy representation of all HUI which is not meaningful to user.
- Algorithms may not be efficient.
- Significant reduction in the extracted patterns may not be achieved

The concept of closed itemset into high utility itemsets mining to address mentioned challenges and named iii) Closed+ High Utility itemset Discovery proposed by algorithm. Information and storage require extra

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memory and processing them to get a CHUI is very difficult step. Organizations have huge data and tend to outsource the task of HUI mining to another party for the analysis [10]. In given system outsourced party will have information and does not have privacy module to protect data privacy. Proposed system is to reduce the load of computation, storage and processing to another property with preservation of privacy of outsourced high utility mining.

II. REVIEW OF LITERATURE

Itemset mining is popular application to generate frequently purchased itemsets in market basket analysis. But it can generate high amount of frequent itemsets if the data is highly correlated and set minimum support threshold is very low[8]. Instead of mining all frequent items the solution is to construct concise representation of frequent itemsets.

T. Calders et.al. [1] proposed to recognize the redundancy of frequent itemsets to diminish the result of mining process. The proposed deduction rules consents the minimal depiction of all frequent itemsets. Non Derivable Itemsets measured for brief illustration. The experiment results showed that mining concise illustrate first and then from this creating frequent itemsets give improved results than existing algorithms.

U. Yun et.al. [2] to reduce the search spaces which are not satisfying certain condition several algorithms used a support constraint. This method permits for simple pruning but the resulting patterns have weak attraction after mining datasets for obtaining frequent patterns. The proposed efficient mining algorithm called Weighted Interesting Pattern mining. The proposed algorithm created on mining weighted frequent patterns. This algorithm decide the concept of a weighted hyper clique form that uses a new measure, called weight-confidence, to consider weight affinity and prevent the generation of patterns with substantially different weight levels. The experimental results showed the proposed algorithm is efficient in weighted frequent pattern mining and it produces less but more powerful patterns for users.

W. Cheung et.al. [3] proposed the compressed and arranged transaction sequences tree algorithms. After the tree is created, it can be used for multiple frequent pattern mining with different supports. Also, the proposed tree algorithms allow single pass frequent pattern mining and transaction stream mining. In addition, transactions can be added to or removed from the tree at any time. The proposed tree covers the idea of FP-Tree to recover storage compression and permit frequent pattern mining without generation of candidate itemsets.

S.F. Shie et.al. [4] proposed an approach that shuns the expensive generation of a large number of candidate sets and repeated database scans. Updates to the transaction database could invalidate existing rules or introduce new rules. The difficult of updating the association rules can be condensed to find out the novel set of frequent itemsets in the updated database. A proposed solution to the update problem was to re-mine the frequent itemsets of the whole updated database.

III. SYSTEM DESIGN

3.1System Modules

System aims at mining closed high utility Itemsets in privacy preserving manner. Mining concise representation of Closed High Utility Itemsets, methodology is used. Privacy in outsourced task is achieved by this system. Secured techniques are applied before the mining task and then outsourced the further process to another party. Our system contains following modules:

3.1.1 Preprocessing

First, every item in the transaction table is substituted with its respective numerical Id. Each item in item set is assigned with numerical id randomly. Output of this process is converted dataset D', Map of the items and their numerical ids. This map is kept securely at client side and converted dataset D' is forwarded to next step. For example Item I1 is converted to random numeric id 78 and so on for other items. Then we applied Homomorphic encryption on weights of the data. Weights of the items are encrypted using homomorphic cryptosystem. The system is using homomorphic encryption because it allows mathematical operation on encrypted data and when results of this mathematical encryption are decrypted then decrypted results reflects the

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mathematical operation's effect. The proposed system used one of the homomorphic cryptosystem called Pallier cryptosystem, which exhibits following properties:

i. Homomophic addition

$$\text{Dsk (Epk (a+b))} = \text{Dsk(Epk(a) * Epk(b) mod } N^2)$$

ii. Homomorphic Mulitplication

$$\text{Dsk(Epk (a*b))} = \text{Dsk(Epk } (a)b \text{ Mod } N^2)$$

Where Epk is encryption function with Key public key Pk derived using N and g where N is product of two prime numbers of similar two lengths and g is generator in $ZN2$. Also, let Dsk be the decryption function with secret key sk.

iii. Semantic Security: It is impossible to figure out information about plain text using cipher text.

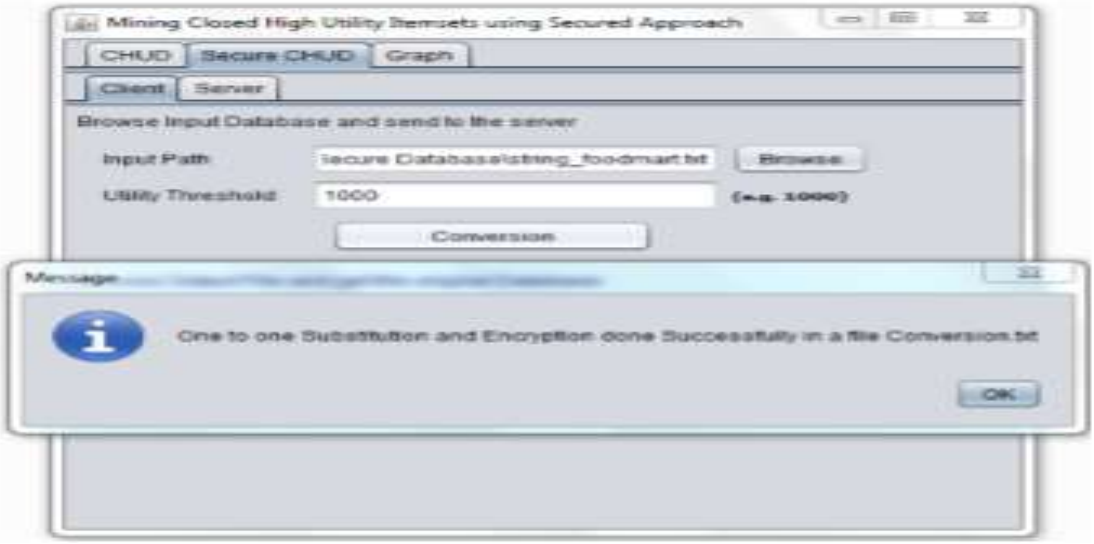

Figure 3.1. GUI of converting database into secured form.

Encrypted weights of the items, Converted Database. This converted database and encrypted utility threshold given to third party for high utility mining. To achieve CHUI mining task, the algorithm used is Closed High Utility Item sets Discovery.

3.1.2 Mining of CHUIs in secured way

- Converts the database in vertical database and simultaneously calculates the utility for each transaction and also its transaction weighted utility of items. When transaction is fetched, its Tid and transaction utility are stored into global table named Global Utility Table (GUT).
- In this algorithm process to scans database and collect promising items having estimated utility greater than abs_min_utility into ordered list which is sorted in increasing order of support. Then utilities of unpromising items are removed from Global Utility table (GUT).
- In this step CHUD generates candidates in recursive manner starting from candidates containing a single promising item and recursively joining items to them to form larger candidates.
- Here performs Subsume check on X (item) which verifies if there exists an item which is included in a closed item set that has already been found and supersets of X do not need to be implemented.
- Then next computes the closure of an Xc = C(X) of X. Then the estimated utility is calculated.
- After that DCM strategy (Discarding candidates with maximum utility less than minimum utility threshold) is applied i.e. it computes the maximum utility of Xc. It discards the candidate whose estimated or maximum utility is less abs_min_utility; otherwise Xc is outputted with its estimated utility.

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- Here a node N (Xc) is created and the procedure Explore is called for finding candidates that are supersets of Xc i.e. potential CHUIs. Then RML strategy is applied to remove minimum utility items from local transaction utility table.
- This step consists of taking each candidate X and calculates its utility. Each candidate of low utility is discarded and candidate with high utility than absolute minimum utility is outputted.

First the converts the database vertically and gives Global Utility Table of all promising items and then extracts candidate closed high utility itemsets and finally extracts the final closed+ high utility itemsets.

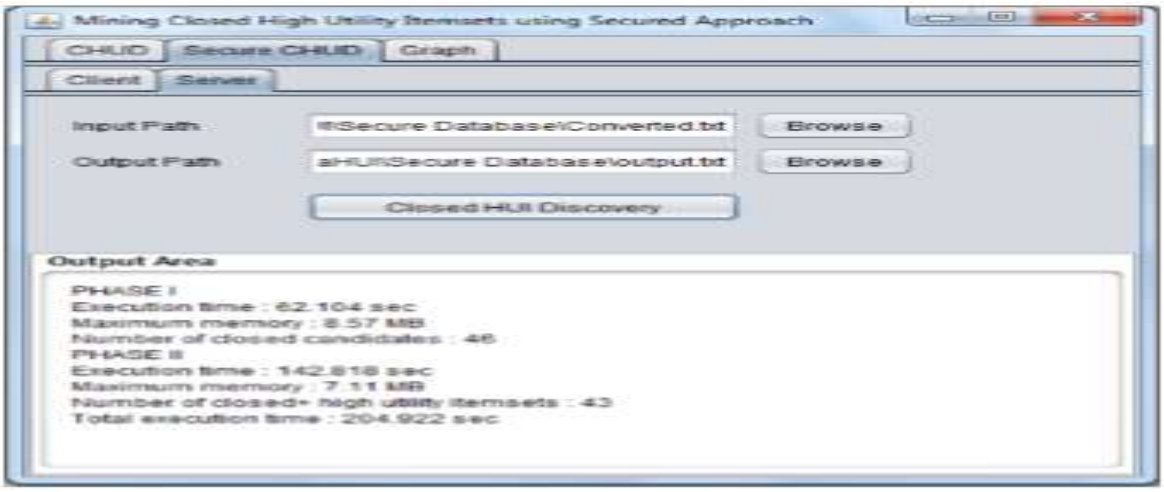

Figure 3.2 GUI of extraction of CHUIs.

3.1.3 Conversion of encrypted output:

This phase converts the encrypted output back to plain text readable to the client side which includes number of closed high utility itemsets, containing transaction and item utilities Complete set of CHUIs which is in substituted form, is obtained using locally kept map of items and its numerical Ids.

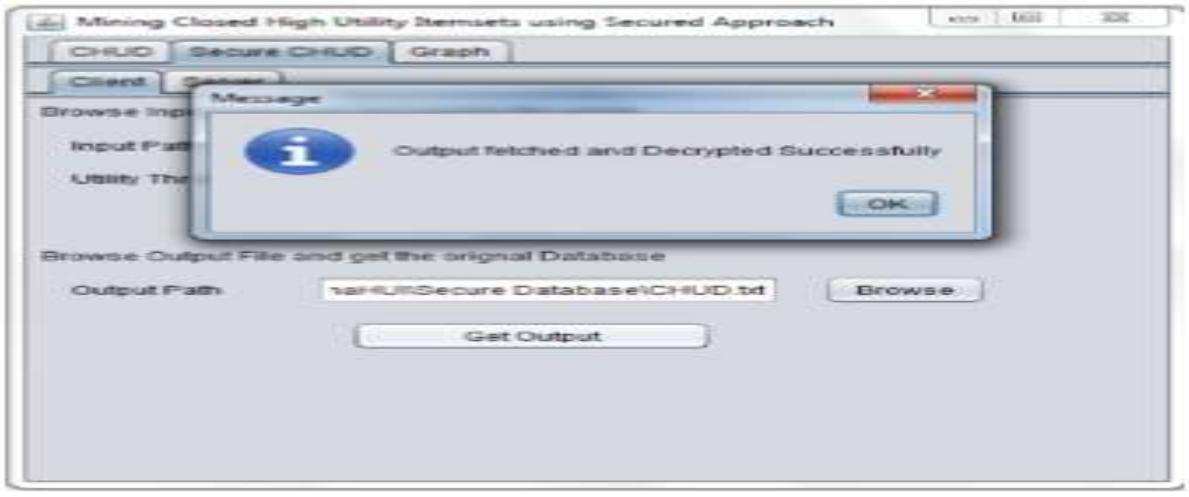

Figure 3.3 GUI of converting output from server

IV. COMPARISON OF EXISTING AND PROPOSED SYSTEM

Existing system does the task of mining Closed High Utility Itemsets. There is no difference between number of extracted candidate CHUI and final CHUIs by existing and proposed system respectively. The main difference is, the CHUI mining task is outsourced to another party and is in secured way i.e. with privacy preservation. Though the items and utility information is given to another party the privacy is preserved. This is achieved by

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applying items substitution and homomorphic encryption technique. The figure 4.1 shows the time required for the existing system and proposed system for CHUI generation

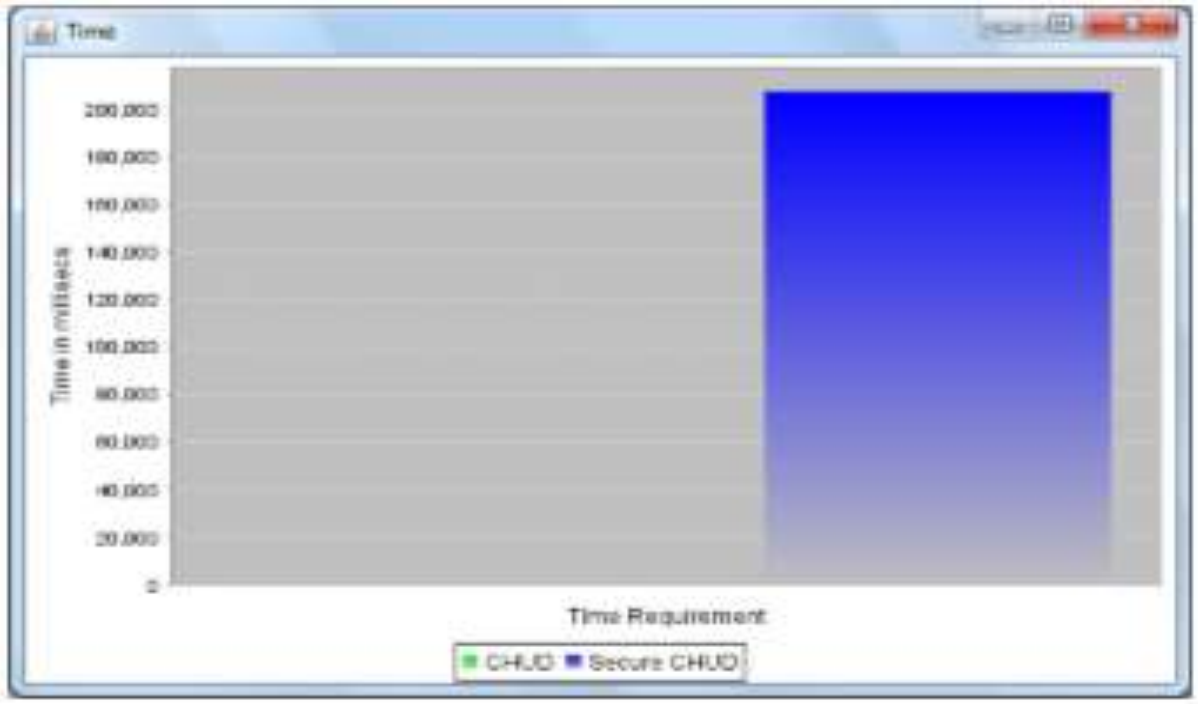

Fig. 4.1 GUI of comparison of time requirement of CHUD and Secure CHUD

V. CONCLUSION

In this work we discussed several techniques of creating HUIs and addressed the issues in high utility mining and also improve the efficiency and accuracy of process that generating the set of high utility itemsets. This work also discussed certain compact representations available for High utility itemsets. Amongst them Closed High Utility Mining showed better and efficient. But conducting all these processes, organizations have to carry the storage and computation overhead. We proposed a system to ease the process of generation of compact High utility itemsets to another party and also satisfied the aroused need of security and privacy of the data as outsourced party may not be trusted one. In current CHUI discovery, the user gives the minimum utility threshold and with reference to that the CHUIs generated. It can be possible that the user may provide minimum utility threshold lesser and thus can be result in more number of and unnecessary high utility itemsets. Also the user may provide very high minimum utility threshold and fewer number of CHUIs may generated and important Itemsets may get ignored. In future work we can apply the technique which will select the optimum minimum utility threshold to avoid the mentioned problem.

REFERENCES

[1] T. Calders and B. Goethals, "*Mining all non-derivable frequent itemsets*", in Proc. Int. Conf. Eur. Conf. Principles Data Mining Knowl. Discovery, 2002, pp. 74–85.
[2] U. Yun, "*Efficient Mining of Weighted Interesting Patterns with a Strong Weight and/or Support Affinity*", Information Sciences, vol. 177, pp. 3477-3499, 2007.
[3] W. Cheung and O.R. Zaı̈ane, "*Incremental Mining of Frequent Patterns without Candidate Generation or Support Constraint*", Proc. Seventh Int'l Database Eng. and Applications Symp. (IDEAS '03), pp. 111-116, 2003.
[4] J.L. Koh and S.F. Shie, "*An Efficient Approach for Maintaining Association Rules Based on Adjusting FP-Tree Structures*", Proc. Ninth Int'l Conf. Database Systems for Advanced Applications (DASFAA '04), pp. 417-424, 2004.

International Journal of Management, IT & Engineering
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[5] Ahuja, Bharti Ahuja and Rupali Bhartiya, "*Novel Technique for Mining a High Utility Itemsets from Transactional Database*", International Journal of Mechanical and Production Engineering (IJMPE), ISSN: 2320-2092, Volume- 5, Issue-2, Feb.-2017.

[6] Arwa Altameem and Mourad Ykhlef, "*Hybrid Approach for Improving Efficiency of Apriori Algorithm on Frequent Itemset*", International Journal of Computer Science and Network Security (IJCSNS), VOL.18 No.5, May 2018.

[7] Anusha N ,Rajashree and Srikanth Bhat K , "*A Survey on Medical Data by using Data Mining Techniques",* International Journal of Science, Engineering and Technology Research (IJSETR), ISSN: 2278 -7798,Vol. 7, Issue 1, January 2018.

[8] Ms.Archana Kisan Dere and S .A.Kahate , "*Mining Closed High Utility Itemsets using Secured Approach*",International Journal of Scientific Engineering and Applied Science (IJSEAS), ISSN: 2395-3470 Vol.2, Issue-6,June 2016.

[9] Dewarde D.H. and Kahate S, "*A High Utility Itemsets Mining from Incremental Databases*", International Journal of Scientific Engineering and Applied Science (IJSEAS), ISSN: 2395-3470 Vol. 1, Issue 4, July 2015.

[10] K. Uma Maheswari and S. Kalaimathi, "*Finding Frequent Item Set using Apriori Algorithm for Online Shopping (Ekart)*", International Journal of Engineering Science and Computing (IJESCC),Vol. 8 Issue No.3, March 2018.

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A Survey on Handover Mechanisms in Mobile Ad hoc Networks

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ABSTRACT

In modern years, mobile equipments are becoming the most primary platforms for many users who always around and access the mobile computing applications. The rapid growth and development of mobile communication network system has accelerated data transfer speed. Such technologies are expected for providing the various services such as voice, data, web browsing, video conferencing, video streaming and telemetry along with mobility of the end users. The mobility is provided by different handoff mechanisms for all types of services. However, the most challenging issues in wireless mobile communication networks are providing the seamless handover while mobile equipment moves between different access networks. Making a transition from one network to another and moving between heterogeneous networks are not the issues that worried scientific and mobile operators; the concern of Quality-of-Service (QoS) requirement is the most significant for handoff. The requirements such as capability of the network, network conditions, handoff latency, power consumption, network cost and user preferences should be considered during handoff. Hence, different adaptive mechanisms have been required for implementing the handoff mechanisms in wireless networks and producing an effective service for the user by considering different handover parameters. This paper covers a detailed study and analysis on different handover mechanism for transmitting the data in wireless networks technology. In addition, comparison analysis is also presented based on their merits, demerits and performance metrics. Hence, this study helps to reveal the requirement for a novel approach which satisfy the most essential requirements and reduce the handover failure probability and the number of unnecessary handover.

Keywords: Wireless Networks, Mobile computing, Handover, Mobility, Quality-of-Service.

I. INTRODUCTION

In wireless mobile telecommunications, handover is defined as the process of transferring an ongoing call or data session from one channel to the other channel which is connected to the access network. It is also referred as handoff which is mostly utilized within International and European organizations like ITU-T, IETF, ETSI and 3GPP, and standardized within European originated standards like GSM and UMTS [1]. The handover is used for the following purposes:

While the mobile is moving away from the coverage region by one cell and entering the coverage region by another cell, the call is transferred to the second cell for avoiding the call termination when mobile gets outside the range of the first cell.

When the channel utilized by the mobile becomes interfered by another mobile which uses the similar channel in the different cell, the call is transferred to the different channel in the same cell or to the different channel in another cell for avoiding the interference.

The most fundamental concept of handover is when the call in progress is redirected from its current cell named source to the new cell called target [2]. In terrestrial networks, the source and the target cells may be served from two different cell sites or from one and the same cell site. Such handover is known as inter-cell handover.

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The source and the target may be served from same cell sites and only the used channel is changed during handover is known as intra-cell handover. Moreover, handover is also classified into hard handover and soft handover.

Hard handover is one where the channel in the source cell is released and only then the channel in the target cell is engaged. Therefore, the connection to the source is broken before or as the connection to the target is made so such handovers are also called as break-before-make. These are intended to be instantaneous for minimizing the disruption to the call. When the mobile is between base stations, the mobile may switch with any of the base stations. Hence, the base stations bounce the connection with the mobile back and forth which is called as ping-ponging. Soft handover is one where the channel in the source cell is retained and used for a while in parallel along with the channel in the target cell. For this, the connection to the target is established before the connection to the source is broken so these are also called as make-before-break [3]. Handover may be classified based on the handover techniques used. Such classification includes three types like network controlled handover, mobile phone assisted handover and mobile controlled handover.

Furthermore, it is categorized as horizontal and vertical handoff according to the types of access networks. Handoff between homogeneous networks is called as horizontal handoff in which one type of network is considered. Alternatively, a handoff between different types of networks such as heterogeneous environment is known as vertical handoff. The horizontal handoff takes place between two cellular base stations whereas vertical handoff happens between an access points of a Wireless Local Area Network (WLAN) and a base station of a cellular base station. Vertical handoff is implemented across heterogeneous cells of access systems which may differ in various QoS parameters such as bandwidth, data rate, operation frequency, etc. Such various characteristics of the networks make vertical handoff more challenging as compared with horizontal handoff [4].

Generally, WLAN connections provide higher speeds when cellular technologies provide more ubiquitous coverage. Hence, the mobile equipments may require to use the WLAN connection whenever one is available and to fall over to the cellular connection when WLAN is unavailable. Therefore, vertical handover is used for automatic fall over from one technology to the other for maintaining the communication. This is different from the horizontal handover between various wireless access points which use the same technology in that a vertical handover involves the data link layer modification for accessing the network. However, several issues are observed in the implementation of handover mechanism in wireless technologies such as QoS, optimization, rerouting connection, etc. The primary objective of this article is to present the detailed survey on different handover mechanisms in wireless technologies and notice the issues during handover process using such mechanisms.

II. LITERATURE SURVEY

Context-aware load balancing in WiFi-WiMAX heterogeneous network environment [5] was proposed for deciding the handover points. Initially, the bandwidth management and admission control scheme were proposed for proper distribution of total network traffic over an integration of WiFi-WiMAX environment. The load-imbalance problem in WiFi base station was mitigated by distributing the traffic load among the overlapping access points in a WiFi hotspot. According to the bandwidth management mechanism, a handover policy was designed which instructs the users when to do a handover between WiFi and WiMAX interfaces other than normal handover performed due to mobility for maintaining QoS and Quality-of-Experience (QoE) of the end-users when preferring WiFi interface for communication.

A utility-based fuzzy TOPSIS method was proposed [6] for energy efficient network selection in heterogeneous wireless networks. A novel method was proposed which considers the user preferences, network conditions, QoS and energy consumption requirements for selecting the optimal network which achieves the best balance between performance and energy consumption. The utilization of parameterized utility functions was incorporated by the proposed network selection method for modeling the diverse QoS elasticity of different

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applications and adopting the different energy consumption metrics for real-time and non-real-time applications. User preferences were easily configured for different application and situation contexts through the utilization of linguistic assessments and their representation as triangular fuzzy numbers. The aggregation of multiple criteria for the determination of the overall rating of the networks was performed by using Fuzzy Set Representation TOPSIS method which is used for solving the inconsistency issues related to conflicting decision criteria. Moreover, the utility functions were employed for removing the ranking abnormality problem.

Energy and quality of service aware fuzzy technique [7] was proposed for heterogeneous wireless network. A novel energy efficient vertical handover decision algorithm was designed which is called as fuzzy technique for order preference by similarity to ideal solution (FUZZY-TOP). The proposed algorithm was obtained by combining a fuzzy-rule based mechanism including with the mechanism for order preference by similarity to ideal solution approach. Fine tuning of fuzzy membership values was performed by increasing the number of membership regions and also reduced the rule set. Moreover, the detailed evaluation of the proposed handover decision mechanism was investigated by using the generalized Markov chain.

An optimized seamless dual-link handover mechanism [8] was proposed for High-Speed Rail (HSR) with Long Term Evolution (LTE) technology. The seamless handover was performed based on the utilization of two antennas on the front and rear of the train respectively for proper handover and communication. A novel algorithm was considered for optimizing the handoff opportunity by investigating not only the hysteresis exceeding level but also the signal strength of both antennas. The parameter selection was analyzed according to the probability model. Moreover, a set of new handover mechanisms were introduced for optimizing the utilization of two antennas which reduce the overhead and improve the QoS of the users. Thus, the proposed algorithm was used for avoiding untimely and unstable handover and unnecessary bi-casting while train comes to the base station.

An adaptive membership function for handover decision mechanism [9] was proposed in wireless mobile network. The proposed approach was incorporated by self-tuning of fixed Fuzzy Membership Functions (FMF) which dynamically modifies the membership functions for matching the requirements of the service option requested. Therefore, a single FMF set was maintained. In the proposed dynamic FMF mechanism, the adaptive behavior of a fuzzy engine was realized by dynamically modifying its FMF sets. This may be accomplished by the FMF self-tuning process. The performance was compared with the static adaptive membership based handover decision algorithm.

A hybrid Network or Mobile Controlled Handover (N/M-CHO) [10] was proposed for heterogeneous wireless networks. Initially, soft or hard vertical handover mechanism was proposed by using the mathematical analysis model based on the six-dimension (6-D) Markov chain model. Then, a reward and cost model was defined according to the data rate and the bandwidth allocated to the incoming calls. This model was used by mobile users operating in MCHO mode for deciding whether perform a hard or soft handover or not. Moreover, a soft or hard handover decision mechanism was also defined which may work by using either MCHO or NCHO depending on the decision metric used. The performance was evaluated based on the handover gain, handover frequency.

An enhanced fast handover with seamless mobility support [11] was proposed for next generation wireless networks. The proposed handover protocol was named as enhanced Seamless Mobile IPv6 (e-SMIPv6). Bidirectional tunnels were established among access routers before actual handover accordingly mobile users may utilize their previous care-of-address within the new visiting network. The delay related to duplicate address detection was reduced by each access router which maintains the pool of duplicate-free addresses. Moreover, the packet loss was also minimized by access router which performs bicasting for roaming node. In addition, the mobility signaling during handoff was minimized by the proposed mechanism which is used for presenting an ideal solution for fast moving and ping-pong moving mobile users.

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A hybrid artificial intelligent based handover decision algorithm [12] was developed in wireless mobile communication. The developed model was based on the hybrid of Artificial Neural Network (ANN) based prediction model and fuzzy logic. Initially, the Received Signal Strength (RSS) was obtained over the time period during the network access to form a time series data. Then, the data was fed into the proposed k-step ahead ANN-based RSS prediction system for estimating the prediction model coefficients. The synaptic weights and adaptive coefficients of the trained ANN were used for computing k-step ahead ANN-based RSS prediction model coefficients. The predicted RSS value was codified as fuzzy sets and in conjunction with the other measured network parameters were fed into the fuzzy logic controller for finalizing the handover decision process. Thus, the proposed system was having the capability for reducing the ping-pong effect associated with other handover mechanisms.

An optimization performance was studied [13] for minimizing the impact of the handover for mobile users in WLAN. A performance model was developed for a set of networked 802.11 based WLAN access points which is based on the Mixed Integer Linear Program (MILP). The objective function was used simultaneously for maximizing the total system rate and also minimizing the number of handovers for a configurable handover signaling rate. Since, the conflicting behavior of the two objective functions such multi-objective optimization was complex for exploration. In addition, the weighted sum method for understanding the impact of higher weights on one of the two objective functions. Finally, the proposed approach was evaluated based on the different scenarios which consider different number of users and mobility parameters like user speed.

Handover management model was developed [14] for WiMAX point-to-multi-point networks. A distributed base station cooperation-based handover management mechanism was proposed for providing the quality of service to handover nodes. Furthermore, a delay reduction approach was proposed for reducing the packet delivery delays during handover process. Then, Call Admission Control (CAC) algorithm was introduced for handling handover calls of different service classes accurately based on their priorities. In addition, a bandwidth borrowing method was proposed for reducing the handover call dropping probabilities of different service classes when not starving the ongoing calls of lower priority service classes. Finally, a Markov model was developed for analyzing the proposed CAC method and obtaining the approximate handover call dropping probabilities of different service classes.

Simplified and improved multiple attributes alternate ranking method [15] was proposed for vertical handover decision in heterogeneous wireless networks. The proposed approach was referred as SI-MAAR for eliminating the attribute normalization and weight calculation methods so the rank reversal problem resulting in reliable network rank for seamless handover was solved. This may be achieved by a simple closeness index matrix which is computed based on the network's attributes and their expectations. Furthermore, new positive and negative ideal solutions were proposed based on the benefit and cost attributes for overcoming the rank reversal problem.

Application of renewal theory was proposed [16] for call handover counting and dynamic location management in cellular mobile networks. The three fundamental strategies such as distance-based, time-based and movement-based and their corresponding optimization cost were described for location management. Moreover, counting the number of wireless cell crossings or handovers occurring in the call duration time during the inter-call times was emphasized which is a basic issue for mobility management analysis. Finally, the renewal theory was developed for modeling the probabilistic structure of such optimization problems.

Guaranteed handover schemes were described [17] for a multilayer cellular system. In the proposed approach, two guaranteed handover schemes were designed in which overflow and take-back operations with geographical information were utilized for assisting in guaranteeing the handover. A system model was established for illustrating the proposed two schemes. In the system model, the cell layer with smaller sized cells (CL I) were overlaid by the Cell Layer with larger sized cells (CL II). The call blocking probability for the system with Time-based Channel Reservation Algorithm (TCRA) was reduced by the overflow operation. Moreover, by

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considering the overflowed traffic back to the CL I, the call blocking probability in the CL I can be further improved and the overflowed traffic may be reduced.

III. DISCUSSIONS

This section illustrates an overview of merits and demerits of different handover mechanisms whose functional scenarios are discussed in brief in literature survey. From the following Table 1, the most challenging issues in handover mechanisms are observed and an ideal solution is identified for overcoming such issues during handover process in wireless network scenarios.

Table.1 Comparison of Different Handover Mechanisms

Ref. No.	Merits	Demerits
[5]	Less communication cost, Improved network capacity	The total handover delay was increased
[6]	High QoS performance and Less energy consumption	Ping-pong effect was not removed and handover triggering was not examined
[7]	Improved availability and QoS requirements	Requires better optimization method for selecting fuzzy membership functions
[8]	Reduced handover failure probability and communication interruption probability	High complexity and Requires an effective approach for parameter selection and QoS requirements
[9]	Improved network selection	Performance of QoS requirements was not analyzed
[10]	Improved throughput, bandwidth saving percentage and handover latency	High handover latency for hard vertical handover method
[11]	Reduced total cost and packet losses	When router does not have enough buffer, the packet loss was high and QoS parameters were not analyzed
[12]	Improved handover decision and removed ping-pong effect	Requires optimization method for selecting the fuzzy input parameters
[13]	Reduced total download volume and handover failure probability	Requires robust optimization techniques for improving the performance
[14]	Reduced call dropping probability	The handover call dropping probability was increased due to increased call arrival rates
[15]	Reduced rank reversal problem and High network rank reliability	High complexity and Requires the analysis of handover delay and QoS parameters
[16]	Improved mobility management	Requires QoS parameter analysis
[17]	Reduced new call blocking probability and overflowed traffic	QoS parameters were not analyzed

IV. CONCLUSION

In this paper, a detailed survey on handover mechanisms in wireless and mobile computing technologies was encountered. It is obvious that all researches have tried in different handover mechanism under different network scenarios to achieve better results than the other handover mechanisms with different modifications based on the previous mechanisms. Even then, further improvement of handover mechanisms can make the estimation of QoS parameters based on the multiple services. Hence, the further research focus will be based on the reduction of handoff delay and QoS variability based on priority-based algorithm using location-aware adaptive applications.

REFERENCES

[1] Nisha, Kumar, S., & Bhatnagar, J., "Handoff strategies in cellular system", *International Journal of New Trends in Electronics and Communication, 1*(2), 22-28, 2013.

[2] Zekri, M., Jouaber, B., & Zeghlache, D., " A review on mobility management and vertical handover solutions over heterogeneous wireless networks", *Computer Communications*, *35*(17), 2055-2068, 2012.

International Journal of Management, IT & Engineering
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[3] Paul, L. C., "Handoff/Handover Mechanism for Mobility Improvement in Wireless Communication", *Global Journal of Researches in Engineering, Electrical amd Electronics Engineering*, *13*(16), 2240-4596, 2013

[4] El Fachtali, I., Saadane, R., & Koutbi, M., "A survey of handovers decision algorithms for next generation wireless networks", *International Journal of Advanced Research in Computer and Communication Engineering*, *4*(1), 159-165, 2015.

[5] Sarma, A., Chakraborty, S., & Nandi, S., "Deciding Handover Points Based on Context-Aware Load Balancing in a WiFi-WiMAX Heterogeneous Network Environment", *IEEE Transactions on Vehicular Technology*, *65*(1), 348-357, 2016

[6] Kantubukta, V., Maheshwari, S., Mahapatra, S., & Kumar, C. S., "Energy and quality of service aware FUZZY-technique for order preference by similarity to ideal solution based vertical handover decision algorithm for heterogeneous wireless networks", *IET networks*, *2*(3), 103-114, 2013

[7] Chamodrakas, I., & Martakos, D., "A utility-based fuzzy TOPSIS method for energy efficient network selection in heterogeneous wireless networks", *Applied Soft Computing*, *12*(7), 1929-1938, 2012

[8] Yu, X., Luo, Y., & Chen, X., "An optimized seamless dual-link handover scheme for high-speed rail", *IEEE Transactions on Vehicular Technology*, *65*(10), 8658-8668, 2016

[9] Thumthawatworn, T., "Adaptive Membership Functions for Handover Decision System in Wireless Mobile Network", *Procedia Computer Science*, *86*, 31-34, 2016

[10] Vegni, A. M., & Natalizio, E., "A hybrid (N/M) CHO soft/hard vertical handover technique for heterogeneous wireless networks", *Ad Hoc Networks*, *14*, 51-70, 2014

[11] Zhang, L. J., & Pierre, S., "An enhanced fast handover with seamless mobility support for next-generation wireless networks", *Journal of Network and Computer Applications*, *46*, 322-335, 2014

[12] Aibinu, A. M., Onumanyi, A. J., Adedigba, A. P., Ipinyomi, M., Folorunso, T. A., & Salami, M. J. E., "Development of hybrid artificial intelligent based handover decision algorithm", *Engineering Science and Technology, an International Journal*, *20*(2), 381-390, 2017

[13] Zola, E., & Kassler, A. J., "Minimizing the impact of the handover for mobile users in WLAN: A study on performance optimization", *Computer Networks*, *107*, 292-303, 2016

[14] Lakshmi, L. R., Ribeiro, V. J., & Jain, B. N., "Handover management framework for WiMAX Point-to-Multi-Point networks", *Computers & Electrical Engineering*, 2015

[15] Chandavarkar, B. R., & Guddeti, R. M. R., "Simplified and improved multiple attributes alternate ranking method for vertical handover decision in heterogeneous wireless networks.*Computer Communications*, *83*, 81-97, 2016

[16] Rodríguez-Dagnino, R. M., & Takagi, H., "Application of renewal theory to call handover counting and dynamic location management in cellular mobile networks", *European Journal of Operational Research*, *204*(1), 1-13, 2010

[17] Li, S., Grace, D., Wei, J., & Ma, D., "Guaranteed handover schemes for a multilayer cellular system", In *Wireless Communication Systems (ISWCS)*, 300-304, 2010

International Journal of Management, IT & Engineering
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Impact of Routing Issues in Mobile Ad Hoc Network

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ABSTRACT

Mobile Ad-hoc Network is a collection of autonomous mobile nodes with dynamic capabilities. Each node in the network acts as a router as well as host. The infrastructures less network mobile nodes transfer the data through wireless links. Due to dynamic behavior of nodes in the network routing becomes complicated. Routing is the process of finding an optimal path to reach the destination. This paper analyzes the various issues associated with routing and also emphasis on the network performance factors such as throughput and packet delivery ratio.

Keywords: **Routing, Mobility, Link Failure, Hidden Terminal, Interference.**

I. INTRODUCTION

A wireless ad hoc network is a collection of wireless devices which have a capability of communicating with each other through wireless links. These networks transfer the data in the absence of centralized administrator. Each node in the wireless ad hoc network carries the functions of both host as well as router [1]. The ad hoc nodes have self organizing capability in dynamic topology. The wireless device on the ad hoc networks has the ability to detect the presence of other devices which exist, within the transmission range and follows the single hop or the multi hop communication between the mutually reachable devices [4]. In the multi hop communication intermediate nodes are behaving as a router. The popularity of portable devices and the progress of

Wireless communication increases the importance of Mobile ad hoc network services. The Mobile Ad hoc network can be set up easily at any time and any place regardless of the geographical position without central administrator with less expensive. MANET applications are diverse widespread ranges because there is no need of running a cable, access point and installation of any hardware. The typical applications of MANET are Military Battle Field, Emerging technologies, Commercial sectors, Rescue operation and disaster area [2]. Even though MANET brings lot of advantages a number of issues remain to be addressed. The most important issue is Routing. In Mobile ad hoc network the nodes works in distributed fashion to transmit data from source to destination. Routing is the

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process of selecting a suitable path to reach the destination. Because of the lack of centralized control routing becomes a centralized issue [3]. Routing is concentrated as one of the critical issue in the research area of MANET, where it has the important role to determine the network QOS performance parameters such as throughput, packet delivery ratio, routing overhead, delay and jitter. Many Routing protocols such as reactive protocols and proactive protocols are invented specifically for MANET some of them are adopted from the wired network but each routing protocols does not solve all the issues associated with dynamic behavioral routing instead of it concentrated only on peculiar problems. The proactive protocols are maintains a shortest path between source and destination nodes
along with the protocols transmit data but eventually it has to update the routing table entries [3]. These periodic updates reflect the High Routing Overhead in the network environment. Reactive protocols are worked on On-demand basis; these protocols gather the information about the neighbor while transmitting the RREQ and RREP packets. Any failures obtained on the route leads to reroute discovery initiation. This manuscript mainly focus on the various issues and challenges related to routing and designing a routing protocols in highly dynamic ad hoc networks and analyses the impact of each issues on the performance of the network. Section II describes the characteristics of MANET. Section III focuses on the Issues of routing, Section IV reveals the Impact of each issues and Section V concludes the paper.

II. CHARACTERISTICS OF MANET

The following are the characteristics of MANET

Autonomous Behavior: In MANET each mobile node independently takes its own decision for transmission. Each node can have the functionality of both host and router[1].

Distributed Environment: The Mobile Ad hoc Networks does not have a backbone control for network operations; the control of the operation distributed among the nodes. The nodes involved in the network makes a communication among themselves, and relay the transmission [2].

Multi Hop Routing: When the communication nodes are out of transmission range, the packets can be forwarded through the intermediary router or host.[1]

Dynamic Topology: The Nodes in the Networks environment randomly wander in any direction with different speeds. The random movement of nodes (nodes can join or leave the network at any time) may change the network topology at unpredictable time. The nodes themselves are responsible for dynamically discovering other node communication.

Light Weight Terminal: The Mobile nodes are battery powered, holding small memory and less CPU processing power.[5]

Low Bandwidth: Compared to the fixed infrastructure network infrastructure less ad hoc networks have lower capacity and shorter transmission range. The through put of wireless communication is lesser than wired communication because of some impairment such as multiple access, fading, noise and interference [1][2].

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Unreliable Communication: The unstable channel quality of wireless links on the shared medium may result in high packet loss rate and re- routing instability, which is the common phenomena that leads to throughput, drops in multipath network [4].

III. ISSUES AND CHALLENGES OF MANET

Dynamic Topology: In Mobile Ad hoc networks the mobile nodes are free to move throughout the network. The mobile nodes may appear or disappear at any instant of time. The random movement causes unpredictable topology changes and disturbs the relationship among the nodes.

Routing Overhead: Unpredictable topology change, changes the node location, these location changes results extra overhead in route discovery process [4].

Hidden Terminal Problem: Figure 1 illustrates the hidden terminal problem. Suppose that node *A* wants to transmit to node *B* located at a distance *x* from *A*. By only sensing the medium, node *A* will not be able to hear transmissions by any node (*C*) in the dashed area denoted by *A*(*x*), and will start transmitting, leading to collisions at node *B*. This is the well known hidden terminal problem, where the hidden nodes are located in the area *A*(*x*)[7].

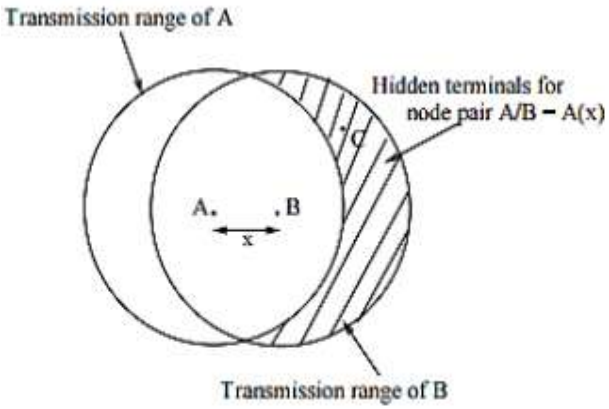

Figure 1. The hidden terminal problem

Unreliable Transmission: Due to the dynamic topology and frequent movements of nodes increased the collision within the presence of hidden terminal problem, interference, unidirectional links and repeated path breaks leads to packet loss and failed to transmit the data.

IV. IMPACT OF ISSUES

The Mobile ad hoc networks fail due to Interference, mobility, radio range, channel effects and battery limitation. Maintaining the node connectivity is one of the main problems in an ad hoc network. The Network fails when the node fails, the link between the node fails, and high mobility [10].

Impact of Node Mobility: Node Mobility has a larger impact on the behavior of ad hoc network. The problems are such as Network partition, Topology changes, transmission error,

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route breakages, routing overhead and break of average connected paths. The topology changes disturb the relationship among the nodes the impact of mobility increase degrades the TCP throughput causes link breakage result in route failure due to leads to route repair larger route repair bring high delay on the network performance[6,7].

Impact of Node Failures : The Node failure occurs when an intermediate node or device acting as a router, is not available due to software or hardware failure. The mobile nodes are battery powered in wireless transmission when transmitting, receiving, retransmitting, beaconing mobile node's battery consumes power. The consumption of energy results in battery depletion. The low energy node unable to participate in the routing process so the node failure is also leads to routing failure, packet loss collision and congestion[5,9].

Impact of Link Failure: Link failures occur when there is a obstacle between the communication nodes. Some protocols attempt to locally repair the link failure. In Certain Cases the node failed to attempt the local repair which increase the Path Length, time required for repairing the broken route and discovering the new route as well as maintaining the route. Unsuccessful local repair process will result in Increase Routing overhead; delay and unreliable data delivery create worst Network Performance [5].

Impact of Node Density: When the number of nodes increases changes occurs in the network connectivity. In proactive routing protocols every node maintains the routing information by periodically updating the routes. Change in the number of nodes reflects in more overhead for such network due to the updating of routing tables when new nodes are added or existing nodes are disconnected. In reactive protocols node density increment increase the hop count, increase the control packet generation for discovering of route and to maintain the route. This control packets generation results as extra overhead in the network. Both protocols suffer the incremental growth of routing overhead [7].

Impact of Hidden Terminal Problem: Hidden terminal problem arise when simultaneously transferring data to the nodes that are not within the direct transmission range of the sender but the nodes are within the transmission range of the receiver. This problem causes unsuccessful transmission of data due to collision. Collision cannot be detected which then corrupt the data received by the access point. RTS/CTS ACK and handshake mechanism used to overcome the hidden terminal problem but it is not the complete solution. This event leads to degradation of network throughput rate [8].

Impact of Interference: Ad hoc networks transmit the data over a wireless channel. If multiple transmissions take place at the same time over the same wireless channel the transmission may collide and the data will be lost. This interference limit the throughput, enhance the packet loss, Produce congestion on the network [11].

V. CONCLUSION

In this paper we have examined the main issues and challenges concerned with mobile ad hoc networks. Initially various characteristics that are unique to MANETS are discussed. The most important issues of routing due to mobility of MANETs are analyzed. Finally we have

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observed the impacts of network performance on various factors such as node mobility, Node density, hidden terminals, Node failure, and interference.

VI. REFERENCES

[1]. Sandeep Kumar, Dr.Suresh Kumar "Study Of Manet:Characteristics, Challenges, Application, Routing Protocols And Security Attacks. International journal of r&d in engineering, science and management Vol.2, Issue 5, July 2015, p.p.266-274, ISSN 2393-865X.

[2]. Jhum swain, Binod Kumar pattanayak, Bibudhendu Pati " Study and Analysis of Routing Issues in MANET", International conference on Inventive Communication and Computational Technologies @2017 IEEE.

[3]. R.Nismon Rio and P.Caladuwel Newton "AASOP : An Approach to Selelect Optimum Path for Minimizing Data Transfer Delay in Mobile Ad Hoc Networks Advances in Computer and Computational Sciences, 2017 Springer.

[4]. Jonathan Loo, Jaime Lloret Mauri, and Jesús Hamilton Ortiz" Mobile Ad Hoc NetworksCurrent Status and Future Trends" CRC Press.

[5]. Varun G Menon, Joe Prathap P M "Routing in Highly Dynamic Ad Hoc Networks: Issues and Challenges". Intrnational Journal of Computer Science and Engineeering. Vol.8 No.4, April 2016.

[6]. Vijaya, Amiya Kumar Rath, Bhagabat Puthal, Debahuti Mishra and S. Satapathy "performance analysis of qos parameters of manet on mobility and energy based model with different manet routing protocols, Indian Journal of Science and Technology, Vol 9(37), October 2016 ISSN (Print) : 0974-6846.

[7]. Haider Alani and Raed Alsaqour, " routing discovery scheme for high mobility in manet" ARPN Journal of Engineering and Applied Sciences vol. 12, no. 2, january 2017 issn 1819-6608.

[8]. Aruna Jayasuriya, Sylvie Perreau, Arek Dadej, Steven Gordon "hidden vs.exposed terminal problem in ad hoc networks",P1-8.

[9]. Atinder Preet Singh, Avneet Singh Chawla, Surjeet "Performance Issues and Behavioral Analysis of Routing Protocols in MANETs", Network and System Sciences, 2016, 9, 431-439, ISSN Print: 1913-3715.

[10]. Mohit jain, Sathis Chand, "Issues and Challenges in Node Connectivity in Mobile Ad Hoc Networks: a Holistic Review", Wireless Engineering and Technology, 2016,7,24-35.

[11]. Eman S.Alwadiyeh, Ala F A Aburumman " Interference-Aware Multipath routing Protocols for Mobile Ad hoc Networks", 13th Annual IEEE workshop on Wireless Local Networks, 2013.

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INTRUSION DETECTION TECHNIQUES FOR MOBILE CLOUD COMPUTING – A LITERATURE SURVEY

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ABSTRACT

Mobile cloud computing is employed in various businesses to get cloud-based services by leveraging mobile technologies. With the evolution of wireless networks, maintaining threats from wireless communications have been performing a conspicuous part in the Web security field. Intrusion Detection System (IDS) is an effective method for preserving wireless communications in the Fifth Generation (5G) context. In modern years, many of companies have selected their methods for allowing cloud-based computing to produce scalable, virtualized on-demand access to a distributed supply of computing sources such as networks, servers, storage, applications, and services. Mainly cloud computing technology allows users/enterprises to reduce the necessities for setting up valuable computing infrastructure and diminishes systems' working costs. As a consequence, this technology is utilized by a growing amount of end users. On another hand, current security deficiencies and vulnerabilities of underlying technologies can issue an open door for intrusions. Through this paper, an intrusion detection technique on cloud computing has reviewed and displayed the future direction in the mobile cloud against intrusion detection.

Keywords: Mobile Cloud Computing, Intrusion Detection System, Cloud computing, Attacks, Signature based Detection, Anomaly detection

1. Introduction

The fast improvement of cutting edge mobile advances has brought about an extraordinary bounce for the development of Mobile Cloud Computing (MCC) [1]. The core advantage of embracing MCC is to enhance the execution of mobile gadgets by coordinating three Internet-related innovations [2][3], including mobileInternet, mobile computing, and cloud computing. Thinking about the Fifth Generation (5G) setting sooner rather than later, MCC will have more elevated amount exhibitions in offloading calculations by relocating information handling and information storing to the cloud for enhancing mobile gadgets' abilities [4] by the reason of improved data transmission. In any case, propelled wireless networks will confront numerous difficulties in the security area [8], which has been investigated by earlier research from different points of view [10]. One of the difficulties is that beating dangers from intrusions is troublesome as a result of high-productivity wireless communications, shared impedances between flag cells, inappropriate client verifications, overseeing device impediments, and purposeful attacks. Progressed organizing pace may help assailants to shroud their intrusions. This paper tends to security issues in MCC and blends ongoing accomplishments in intrusion detection procedures so as to discover the methodologies that can successfully use the advances of heterogeneous networks. In the area of wireless security, keeping wireless communications from intrusions is an essential part. The intrusion detection-based methodology is considered

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as a methodology giving a protected communication condition to future networks. As a conventional security approach, an intrusion detection system (IDS) is a dynamic order that has been related with various strategies. With the particular highlights of every procedure, every detection technique has both prevalence and constraints. **Evolution of Intrusion Detection / Intrusion Prevention Technique**

Traditional IDS/IPS techniques such as signature-baseddetection, anomaly detection, artificial intelligence (AI) baseddetection etc. can be used for Cloud.

1.1 Signature based Detection

Signature-based intrusion detection endeavors to characterize a lot of principles or signatures or predefined learning base that can be utilized to choose that a given example is that of a gate-crashed. Subsequently, signature-based systems are fit for achieving elevated amounts of precision and an insignificant number of false encouraging points in distinguishing even exceptionally unpretentious intrusions. Little variety in realized attacks may likewise influence the examination if a detection system isn't appropriately arranged. In this way, signature-based detection is a productive answer for recognizing realized attacks yet neglects to identify obscure attacks or variety of known attacks. One of the persuading motivations to utilize signature-based detection is ease in keeping up and refreshing preconfigured rules. These signatures are made out of a few components that recognize the traffic. For instance, in SNORT the parts of a signature are the header (e.g. source address, goal address, ports) and its alternatives (e.g. payload, metadata), which are utilized to decide if the network traffic compares to a known signature.

1.2 Signature based Detection

Anomaly (or behavioural) detection is concerned about recognizing occasions that seem, by all accounts, to be odd as for typical system conduct [32]. A wide assortment of systems including information mining, measurable displaying, and concealed markov models have been investigated as various approaches to approach the anomaly detection issue. Anomaly-based methodology includes the accumulation of information identifying with the conduct of authentic clients over some undefined time frame, and after that apply measurable tests to the watched conduct, which decides if that conduct is real or not. It has the benefit of distinguishing attacks which have not been found beforehand. The key component for utilizing this methodology effectively is to create administers so that it can bring down the false caution rate for the obscure and additionally known attacks.

2. Literature Survey on Intrusion Detection Techniques in Mobile Cloud Computing Environment

Yan, Qiao, et al [3] In this paper, we discuss the new trends and characteristics ofDDoS attacks in cloud computing, and provide a comprehensivesurvey of defense mechanisms againstDDoS attacks using SDN.In addition, we review the studies about launching DDoS attackson SDN, as well as the methods against DDoS attacks in SDN. Tothe best of our knowledge, the contradictory relationship between SDN and DDoS attacks has not been well addressed in previousworks. This work can help to understand how to make full useof SDN's advantages to defeat DDoS attacks in cloud computingenvironments and how to prevent SDN

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itself from becoming avictim of DDoS attacks, which are important for the smoothevolution of SDN-based cloud without the distraction of DDoSattacks.

Gai, Keke, et al [3]propose a higher-level framework of implementing secure mobile cloud computing by adoptingIDS techniques for applying mobile cloud-based solutions in 5G networks. On the basis of the reviews and synthesis, the authorsconclude that the implementation of mobile cloud computing can be secured by the proposed framework because it willprovide well-protected Web services and adaptable IDSs in the complicated heterogeneous 5G environment.

Osanaiye, Opeyemi, et al [3] propose an ensemble-based multi-filter featureselection method that combines the output of four filter methods to achieve an optimum selection. We thenperform an extensive experimental evaluation of our proposed method using intrusion detection benchmarkdataset, NSL-KDD and decision tree classifier.

Iqbal, Salman, et al [4] in this research work, Cloud based attacks and vulnerabilities are collected and classify with respect to their cloud models. They also present taxonomy of cloud security attacks and potential mitigation strategies with the aim of providing an in-depth understanding of security requirements in the cloud environment.

Pandeeswari, N., and Ganesh Kumar [5] This work proposes an anomaly detectionsystem at the hypervisor layer named Hypervisor Detectorthat uses a hybrid algorithmwhich is a mixture of Fuzzy CMeansclustering algorithm and Artificial Neural Network(FCM-ANN) to improve the accuracy of the detection system.The proposed system is implemented and compared with NaïveBayes classifier and Classic ANN algorithm.

Chen, Zhijiang, et al [6] propose a cloud computing-based network monitoring and threat detection system to make critical infrastructure systems secure. The proposed system consists of three main components: monitoring agents, cloud infrastructure, and an operation center. Tobuild the proposed system, the authors use both Hadoop MapReduce and Spark to speed up data processing by separating and processing data streams concurrently.

Chiba, Zouhair, et al [7] This paperpresents an overview of different intrusions in cloud, variousdetection techniques used by IDS and the types of CloudComputing based IDS. Then, we analyze some pertinentexisting cloud-based intrusion detection systems with respect totheir various types, positioning, detection time and data source.The analysis also gives strengths of each system, andlimitations, in order to evaluate whether they carry out thesecurity requirements of cloud computing environment or not.

Chiba, Zouhair, et al [8] proposed a cooperative and hybrid network intrusion detection system (CH-NIDS) to detect network attacks in the cloud Environment by monitoring the network traffic, while maintaining the performance and service quality. The authors used Snort as a signature-based detection to detect known attacks, while for detecting the network anomaly, the authors used Back-Propagation Neural Network (BPN).

Saadi, Chaimae, and HabibaChaoui[9] This work proposes new cloud infrastructure architecture, whichcombines IDS based on mobile agent sand using three types of honeypots in order to detect attacks, to study thebehavior of attackers, increase the added value of Honeypot and IDS based mobile agents, solve systems limitationsintrusion detection, improve knowledge bases IDS thus increase the detection rate in our cloud environment.

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Puthal, Deepak, et al [10] any new applications in the smart healthcare, smartcity, and precision agriculture domains are collectingdata using Internet of Things (IoT) sensing devicesand shipping it to remote cloud datacenters foranalysis (fusion, storage, and processing). The big data analyticslifecycle, which starts with raw data collection and moves to dataanalytics and decision making, requires intelligent coordinationof activities between tiny IoT sensors, IoT gateways, and in-transitnetwork devices in an edge datacenter (EDC), with the big dataprocessing frameworks and hardware resources hosted in largecloud datacenter (CDC) farms.

Li, Yibin, et al [11] The proposed approach divides the file and separately stores the data in the distributed cloud servers. An alternative approach is designed to determine whether the data packets need a split in order toshorten the operation time. The proposed scheme is entitled Security-Aware Efficient Distributed Storage (SA-EDS) model, which is mainly supported by our proposed algorithms,including Alternative Data Distribution (AD2) Algorithm, Secure Efficient Data Distributions(SED2) Algorithm and Efficient Data Conflation (EDCon) Algorithm.

Guo, Jia, Ray Chen, and Jeffrey JP Tsai [12]approach is to classify existing trust computation models for service management in IoT systems based on five essential designdimensions for a trust computation model: trust composition, trust propagation, trustaggregation, trust update, and trust formation. The authors summarize pros and cons of eachdimension's options, and highlight the effectiveness of defense mechanisms against maliciousattacks. The authors also summarize the most, least, and little visited trust computation techniques inthe literature and provide insight on the effectiveness of trust computation techniques asapplying to IoT systems.

Inayat, Zakira, et al [13] Inthis article, we discuss the cloud-based IDRS in the contextof SMD and cloud resources in the MCC infrastructure. Thestringent security requirements are provided as open issuesalong with possible solutions. The article aims at providingmotivations for researchers, academicians, security administrators,and cloud service providers to discover mechanisms,frameworks, standards, and protocols to address the challengesfaced by cloud-based IDRS for SMD.

Gupta, B. B., and Omkar P. Badve [14] present an overview of DoS attack anddistributed DoS attack that can be carried out in Cloudenvironment and possible defensive mechanisms, tools anddevices. In addition, the authors discuss many open issues andchallenges in defending Cloud environment against DoSattack. This provides better understanding of the DDoSattack problem in Cloud computing environment, currentsolution space, and future research scope to deal with suchattacks efficiently.

Modi, Chirag N., and KamatchiAcha [15] investigate different vulnerabilities and attacks at virtualization layer of cloudcomputing. We examine the proposals of cloud intrusion detection system (IDS) andintrusion detection and prevention system frameworks.The authors recommend the cloud IDSrequirements and research scope to achieve desired level of security at virtualizationlayer of cloud computing.

Smara, Mounya, et al [16] propose a Fault Detection framework for the ComponentbasedCloud Computing by using Recovery Blocks' Acceptance Test. This framework aims to constructFail-Silent Cloud modules which have the ability of Self-Fault

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detection. In this, the detection processof transient hardware faults, software faults, and response-time failures is performed locally on eachcomputing machine in the Cloud system.

Lee, JeongKyu, SeoYeon Moon, and Jong Hyuk Park [17] propose an enhanced ransomware prevention system based on abnormalbehavior analysis and detection in cloud analysis system—CloudRPS. This proposedsystem can defend against ransomware through more in-depth prevention. It can monitorthe network, file, and server in real time. Furthermore, it installs a cloud systemto collect and analyze various information from the device and log information todefend against attacks.

Midi, Daniele, et al [18] In this paper, we introduce Kalis, a self-adapting,knowledge-driven expert Intrusion Detection System able todetect attacks in real time across a wide range of IoT systems.Kalis does not require changes to existing IoT software, canmonitor a wide variety of protocols, has no performance impacton applications on IoT devices, and enables collaborative securityscenarios. Kalis is the first comprehensive approach to intrusiondetection for IoT that does not target individual protocols orapplications, and adapts the detection strategy to the specificnetwork features.

Nezarat, Amin, and Yaser Shams [19] presented inthis paper, a group of mobile agents act as the sensors of invalid actions in the cloudenvironment. They start a noncooperative game with the suspected attacker and thencalculate the Nash equilibrium value and utility so as to differentiate an attack fromlegitimate requests and determine the severity of attack and its point of origin.

Lin, Hui, et al [20] In this paper, a Category-based Context-aware and Recommendation incentive-based reputation Mechanism (CCRM) is proposed to defend against internal attacksand enhance data veracity in MCC. In the CCRM, innovative methods, including a data category and context sensing technology, a security relevance evaluation model, and a VickreyClark-Groves (VCG)-based recommendation incentive scheme, are integrated into the processof reputation evaluation. Cost analysis indicates that the CCRM has a linear communicationand computation complexity.

Hatef, Mohammad Amin, et al. [21] present a comprehensive and accuratesolution to detect and prevent intrusions in cloud computing systems by using a hybrid method,called HIDCC. The implementation results of the proposed method show that the intrusion coverage,intrusion detection accuracy, reliability, and availability in cloud computing systems areconsiderably increased, and falsewarnings are significantly reduced.

Kumar, Neeraj, et al [22] In this paper, the problem of maintaining the QoS with respect to high communication cost, available bandwidth, and securityis investigated in a mobile cloud environment. A Bayesian cooperative Coalition Game (BCG) is formulated in which Learning Automata(LA) stationed at RFID readers are assumed as the players.

Bhushan, Kriti, and B. B. Gupta [23] In this paper,the author first discuss various essential features of SDN that makes it a suitable networking technology for cloud computing. Inaddition, the authors represent the flow table-space of a switch by using a queuing theory based mathematical model. Further, wepropose a novel flow-table sharing approach to protect the SDN-based cloud from flow table overloading DDoS attacks. Thisapproach utilizes idle flow-table of other OpenFlow switches in the network to protect the switch's flow-table from overloading.

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Kalpana, G., et al [24]In this paper, we suggest Shifted Adaption Homomorphism Encryption (SAHE), which is regarded as the better option for all the current research goingon. SAHE implements the smallest public key of 32 bit and is able to encrypt integer and real numbers. A major issue in this field of research is difficulty in protecting user's questions, which is addressed by conceiving a public key encryption technique which is based on thereversed index. Our schema preserves search efficiency using inverted index, by solving onetime only search drawback encountered in earlier research works.

Chen, Mingming, et al [25] The proposed work in this research paper addresses an efficientfuzzy clustering-based algorithm for intrusion detection of a MANET implementation in acloud storage environment. This paper has presented a model and experimental justificationsfor improving the efficiency.

Deshpande, Prachi, et al [26] The paper reports a host-based intrusiondetection model for Cloud computing environment alongwith its implementation and analysis. This model alerts theCloud user against the malicious activities within the systemby analysing the system call traces. The methodanalyses only selective system call traces, the failed systemcall trace, rather than all. An early detection of intrusionswith reduced computational burden can be possible withthis feature.

Liu, Hang, et al [27] This paper presents a comprehensivesurvey of Mobile Edge Computing (MEC) systems, including the concept, architectures,and technical enablers. First, the MEC applications are exploredand classified based on different criteria, the service models anddeployment scenarios are reviewed and categorized, and the factorsinfluencing the MEC system design are discussed. Then, thearchitectures and designs of MEC systems are surveyed, and thetechnical issues, existing solutions, and approaches are presented.

Modi, Chirag, and Dhiren Patel[28]Intrusion detection/prevention is the greatest security challenge at virtual network layer of Cloudcomputing. To address this challenge, there have been several security frameworks reported. However, still thereis a scope of addressing newer challenges. Here, we propose a security framework to detect network intrusionsin Cloud computing. This framework uses Snort and combination of different classifiers, viz Bayesian, Associativeand Decision tree. We deploy our intrusion detection system (IDS) sensors on each host machine ofCloud. These sensors correlate intrusive alerts from each region of Cloud in order to identify distributed attacks.

Al-Faifi, Abdullah Mohammed, et al [29] In this paper, the authors try to tackle this challenge by automating the selection processbased on actual workload pattern from Smart data and resource demand acquired from existing servicehistory data. An automatic performance prediction model based on Naïve Bayes classifiers is proposedto predict the performance metrics of cloud nodes with respect to different options for configuration oftheir resources.The authorsexamined Naïve Bayes classifier along with kernel density estimation to solve the zerovariance of feature distribution and enhance the accuracy of predictions.

Stergiou, Christos, et al [30] present a survey of IoT and Cloud Computing with a focus on the security issues of both technologies.Specifically, the authors combine the two aforementioned technologies (i.e Cloud Computing and IoT) in orderto examine the common features, and in order to discover the benefits of their integration.

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3. Research Direction

The number of questions for the future research direction has identified

(1) How can we prevent users' privacy leaks when adopting the cloud-based IDS?

(2) What is a secure data transmission method between end users and cloud-based IDSs?

(3) Whether we can develop an authentication model for mobile devices in order to ensure they can fully utilize the benefits o MCC.

4. Conclusion

This survey discussed about a few intrusions which can risk honesty, classification, and accessibility of Cloud administrations and MCC later on. One of the existing solutionsviz. the firewall may not be adequate to unravel Cloud security issues. Being the preparations of network advancements, MCC is wanted to bring clients more advantages. In any case, wireless communications won't be anchored except if the information transmissions are ensured by a proficient methodology. This paper looked into basic ideas and ongoing exploration about MCC, Intrusion Detection System (IDs). Based on the results of the survey, later on, an abnormal state structure to be suggested that utilizes cloud-based intrusion detection and gadget based IDS way to deal with increase secure wireless communication in MCC.

Reference

[1] Yan, Qiao, et al. "Software-defined networking (SDN) and distributed denial of service (DDoS) attacks in cloud computing environments: A survey, some research issues, and challenges." *IEEE Communications Surveys & Tutorials*18.1 (2016): 602-622.

[2] Gai, Keke, et al. "Intrusion detection techniques for mobile cloud computing in heterogeneous 5G." *Security and Communication Networks* 9.16 (2016): 3049-3058.

[3] Osanaiye, Opeyemi, et al. "Ensemble-based multi-filter feature selection method for DDoS detection in cloud computing." *EURASIP Journal on Wireless Communications and Networking* 2016.1 (2016): 130.

[4] Iqbal, Salman, et al. "On cloud security attacks: A taxonomy and intrusion detection and prevention as a service." *Journal of Network and Computer Applications* 74 (2016): 98-120.

[5] Pandeeswari, N., and Ganesh Kumar. "Anomaly detection system in cloud environment using fuzzy clustering based ANN." *Mobile Networks and Applications* 21.3 (2016): 494-505.

[6] Chen, Zhijiang, et al. "A cloud computing based network monitoring and threat detection system for critical infrastructures." *Big Data Research* 3 (2016): 10-23.

[7] Chiba, Zouhair, et al. "A survey of intrusion detection systems for cloud computing environment." *Engineering & MIS (ICEMIS), International Conference on*. IEEE, 2016.

[8] Chiba, Zouhair, et al. "A cooperative and hybrid network intrusion detection framework in cloud computing based on snort and optimized back propagation neural network." *Procedia Computer Science* 83 (2016): 1200-1206.

Vol. 9 Issue 1, (Special Issue HICAAMMC- January 2019)
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[9] Saadi, Chaimae, and HabibaChaoui. "Cloud computing security using ids-am-clust, honeyd, honeywall and honeycomb." *Procedia Computer Science* 85 (2016): 433-442.

[10] Puthal, Deepak, et al. "Threats to networking cloud and edge datacenters in the internet of things." *IEEE Cloud Computing*3.3 (2016): 64-71.

[11] Li, Yibin, et al. "Intelligent cryptography approach for secure distributed big data storage in cloud computing." *Information Sciences* 387 (2017): 103-115.

[12] Guo, Jia, Ray Chen, and Jeffrey JP Tsai. "A survey of trust computation models for service management in internet of things systems." *Computer Communications* 97 (2017): 1-14.

[13] Inayat, Zakira, et al. "Cloud-based intrusion detection and response system: open research issues, and solutions." *Arabian Journal for Science and Engineering* 42.2 (2017): 399-423.

[14] Gupta, B. B., and Omkar P. Badve. "Taxonomy of DoS and DDoS attacks and desirable defense mechanism in a cloud computing environment." *Neural Computing and Applications*28.12 (2017): 3655-3682.

[15] Modi, Chirag N., and KamatchiAcha. "Virtualization layer security challenges and intrusion detection/prevention systems in cloud computing: a comprehensive review." *the Journal of Supercomputing* 73.3 (2017): 1192-1234.

[16] Smara, Mounya, et al. "Acceptance test for fault detection in component-based cloud computing and systems." *Future Generation Computer Systems* 70 (2017): 74-93.

[17] Lee, JeongKyu, SeoYeon Moon, and Jong Hyuk Park. "CloudRPS: a cloud analysis based enhanced ransomware prevention system." *The Journal of Supercomputing* 73.7 (2017): 3065-3084.

[18] Midi, Daniele, et al. "Kalis—A System for Knowledge-Driven Adaptable Intrusion Detection for the Internet of Things." *Distributed Computing Systems (ICDCS), 2017 IEEE 37th International Conference on*. IEEE, 2017.

[19] Nezarat, Amin, and Yaser Shams. "A game theoretic-based distributed detection method for VM-to-hypervisor attacks in cloud environment." *The Journal of Supercomputing* 73.10 (2017): 4407-4427.

[20] Lin, Hui, et al. "Toward better data veracity in mobile cloud computing: A context-aware and incentive-based reputation mechanism." *Information Sciences* 387 (2017): 238-253.

[21] Hatef, Mohammad Amin, et al. "HIDCC: A hybrid intrusion detection approach in cloud computing." *Concurrency and Computation: Practice and Experience* 30.3 (2018): e4171.

[22] Kumar, Neeraj, et al. "Bayesian Cooperative Coalition Game as a Service for RFID-Based Secure QoS Management in Mobile Cloud." *IEEE Transactions on Emerging Topics in Computing* 6.1 (2018): 58-71.

[23] Bhushan, Kriti, and B. B. Gupta. "Distributed denial of service (DDoS) attack mitigation in software defined network (SDN)-based cloud computing environment." *Journal of Ambient Intelligence and Humanized Computing* (2018): 1-13.

International Journal of Management, IT & Engineering
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[24] Kalpana, G., et al. "Shifted Adaption Homomorphism Encryption for Mobile and Cloud Learning." *Computers & Electrical Engineering* 65 (2018): 178-195.

[25] Chen, Mingming, et al. "FCM technique for efficient intrusion detection system for wireless networks in cloud environment." *Computers & Electrical Engineering* 71 (2018): 978-987.

[26] Deshpande, Prachi, et al. "HIDS: A host-based intrusion detection system for cloud computing environment." *International Journal of System Assurance Engineering and Management* 9.3 (2018): 567-576.

[27] Liu, Hang, et al. "Mobile edge cloud system: Architectures, challenges, and approaches." *IEEE Systems Journal* 12.3 (2018): 2495-2508.

[28] Modi, Chirag, and Dhiren Patel. "A feasible approach to intrusion detection in virtual network layer of Cloud computing." *Sādhanā* 43.7 (2018): 114.

[29] Al-Faifi, Abdullah Mohammed, et al. "Performance prediction model for cloud service selection from smart data." *Future Generation Computer Systems* 85 (2018): 97-106.

[30] Stergiou, Christos, et al. "Secure integration of IoT and cloud computing." *Future Generation Computer Systems* 78 (2018): 964-975.

International Journal of Management, IT & Engineering
Vol. 9 Issue 1, (Special Issue HICAAMMC- January 2019)
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COLD START PROBLEM IN RECOMMENDER SYSTEMS: CLASSIFICATION AND ANALYSIS OF PROBLEMS AND SOLUTIONS

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Abstract— **Recommender systems are playing important role, by providing personalized information to users over the web. With the evolution of the web, the recommender systems have evolved from being based on user and item information into hybrid models capable of providing effective real time recommendation on a per user basis. However, the system can't recommend items that are relevant to the user due to lack of ratings or profile information that is cold-start user problem. This paper provides an overview of recommender system techniques, detailed discussion of Cold Start problems, exploring different kinds of solution to these problems in last decade and also discusses the various computational techniques to solve these problems. Similarity computing methods and the need for novelty in Cold Start have also been discussed.**

Keywords—Recommender Systems, Content Based Recommendation, Collaborative Filtering, Social Network, Cold-Start Problem.

Introduction

Recommender system provides or predicts the rating that a user might be preferred for an item. It is made possible by collecting information about the user preferences for a set of items. The preference information could be collected either explicitly or implicitly.

Early recommender systems approaches were based on common filtering methods listed by Breese et al [1] are Content based recommendation (CB), Collaborative Filtering and Demographic Filtering(DF). Hybrid Recommender system could be used to achieve better performance. Two or more basic recommendation techniques can be combined to create a hybrid technique that surpasses the individual technique in terms of prediction accuracy. The advent of social networking and the ability to harvest network followers and related data has enabled a new class of Social Filtering based Recommender Systems. Several challenges are encountered during the creation of recommender system like Sparsity, Cold Start problem, Over Specialization and Scalability.

Cold Start problem still needs to be addressed, as it does not have a feasible solution for recommendation systems. Cold Start refers to a scenario where it is not possible to provide reliable recommendations due to lack of ratings or user profile information. Though the new user problem is heavily discussed, the new community and new item problems are also found to be equally important, when it comes to the creation of a successful recommender system. New user problem represents the scenario in which the user would not have provided enough ratings to get personalized recommendations. New users often tend to feel that they are being ignored and may even leave the service. The new item problem is much more difficult to address as that leads to a cycle where new items will not be recommended and hence be ignored by a vast group of users and this in turns prevents those items from getting the required number of ratings to get noticed. This has less impact in domains where there are more than one way to discover new items (movies), but not so in other domains like e-commerce, blog posts, etc. Cold Start items can be classified to complete Cold Start items and incomplete Cold Start items by whether number of rating records is zero or not. Common practice is to combine Collaborative Filtering Technique with other techniques like CB or DF to overcome the challenges faced by Traditional Collaborative model like Cold Start, Data Sparsity.

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The rest of the paper is organized as follows. Section 2 explains the various approaches used to solve the Cold Start problems, Section 3 provides the list of similarity measures used for computing the Cold Start problems, Section 4 provides conclusion.

Cold-Start Problem solving approaches

In this below section we have identified the important approaches to solve Cold Start problems based on the recommendation techniques.

Content Based Recommendation (CB):

It utilizes the previous preferences of users to recommend services [2]. CB creates user profile based on the user preferred items or services and recommends items that are similar to those that a user liked in the past. CB recommender use knowledge about each product to recommend new products. CB approaches [3] perform better with new users where their taste is rapidly defined. They have the problem of lacking diversity and serendipity. It Enhanced in a better way to deal with a Cold Start problem.

Multi Layer Ensemble:

Bianchi et al.[4], approach is mainly content-based and consists in recommending jobs with features similar to those already interacted with by users, as well as those matching user profiles. We also resort to collaborative information, such as users' past interactions, in an indirect way. The goal of this method to tune a recommendation system able to predict past users' interactions, for the offline stage, and to provide recommendations pushed everyday to real users for the online stage.

Ranker Ensemble:

A Ranker ensemble [5] based on interaction profiles and content of data available in the system. It can be further divided into Interactive Based Ranker (IBR) and Content Based Ranker (CBR). In IBR creates an interaction profiles based on the job seekers and job ads with positive and negative semantics. It relies on Match based features and Latent sematic features. In CBR to compensate for job seekers who have less content information features. Several hybrid ranking algorithm [6] employing ranker ensemble methods in collaboration with other methods tends to be outperform the standalone models in terms of coverage.

Collaborative Filtering(CF)

It performs better in some domains adding diversity to the recommendations. However, those systems required an important amount of information gathered from the users behavior making them relatively weaker when dealing with the Cold Start problem. Goldberg et al.[7] classified CF into two categories: Memory based and model based. The memory based system [8] uses a heuristic modeling architecture where the similarity is identified by methods like correlation analysis. Hence the memory complexities are huge. Model based includes: Bayesian model[9],Matrix factorization[10],Markov Model [11].

IRCD :(Integrated Recommendation model with CF & Deep Learning)

A recommendation system is assumed with users as U and non Cold Start item as V [12]. In addition it is assumed there are J as no ratings items like complete Cold Start items, which receive no ratings from the users until the time of investigation, and few ratings items as I like incomplete Cold Start items, which receive only a few ratings from the users. This models combine a time aware collaborative filtering (CF) model and time SVD++ with a deep learning architecture SDAE(stacked by multiple denoising auto encoders). To compute the similarity between complete Cold Start items and incomplete Cold Start items using PCC (pearson correlation coefficient).

CBPF: Collaborative Bayesian Poisson Factorization:

Event based social network provides recommendation based on CBPF[13]. It integrates the user response, social relation, and event content information through Bayesian Poisson factorization. In CBPF model, event hidden factor is replaced by content latent factor because event hidden factor raising out-of-matrix prediction problem. Moreover, social relations are also considered in CBPF.

LFM: Light Factorization Model:

In LightFM[14], like in a collaborative filtering model, users and items are represented as embedding vectors . However, just as in a CB model, these are entirely defined by functions (linear combinations) of embeddings of the content features that describe each product or user. This model motivated by two considerations (i) it must be able to learn user and item representation from interaction data (ie) if the items are

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consistently liked by users, the model must be learn similar users.(ii)the model must be able to compute recommendations for new items and users. From a implementation standpoint, it can be construed as a special case of Factorization Machines [15].

Item K-NN:

K-NN method [16], recommend items to the user based on the user interest with the help of other group of item present in the system and easy to finds the K-closest item. It can be classified into Item Recommendation and Item Rating recommendation. In item Recommendation generate a ranking list for the items to the user based on the COS similarity whereas Item rating prediction is predicting the value to a particular item using COS and Pearson (CORR).

Fuzzy Set:

It deals with similarities based on the customer and product data [17][18].In a broader sense, fuzzy logic has been largely used for analyzing the vague situation in NLP(Natural Language Processing). In a narrow sense, it is characterized as a symbolic logic with the notion.

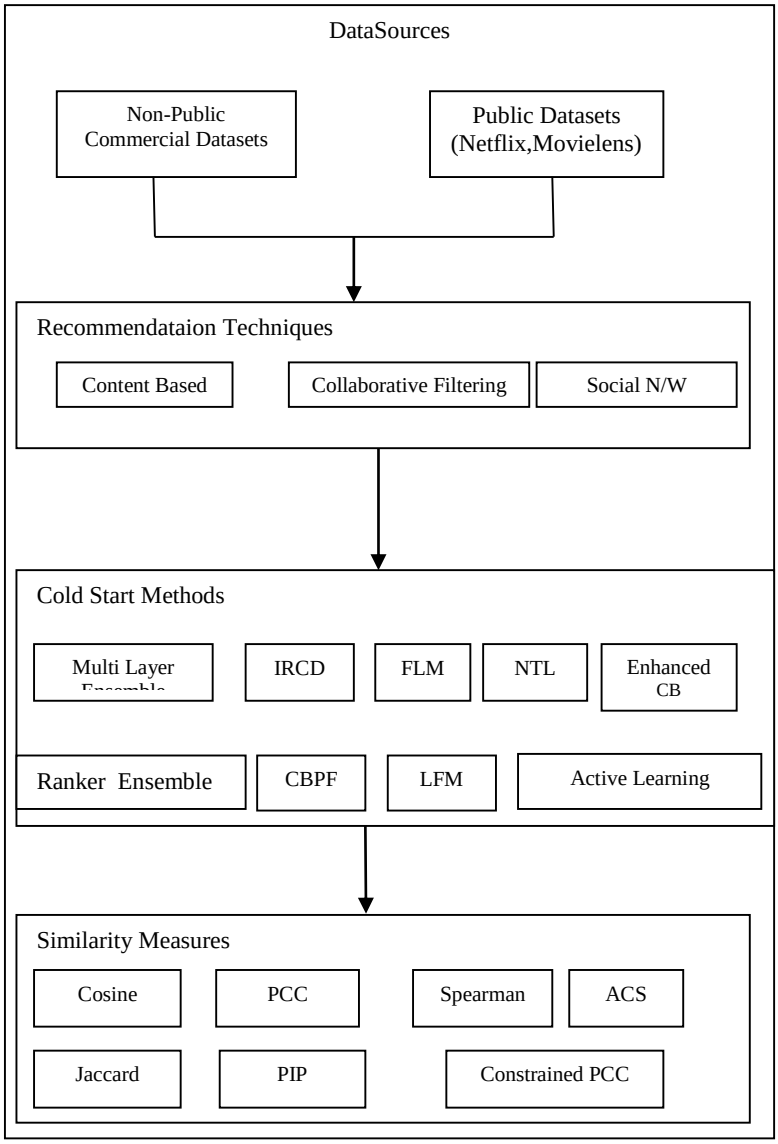

Figure 1. Cold Start Problem Methods

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FLM: (Fuzzy Linguistic Modeling)

From highlevel perspective, this approach that using bibliometrics aids[19] to soft or remove the necessity of interaction of users providing them with personalized profiles built beforehand,thus reducing the Cold Start problem.

Error Reflected Models:

This models are generated from explicit ratings and then collaborative filtering recommender systems model are applied. This method[20] predict the ratings and then identifies the error in each predicted rating. using this error information and pre computed models is collectively known as error reflected model.

CF Predictions:

This method [21]uses collaborative filtering of other users for prediction of new users. It works well when lot of related users are found for new user all items. But it fails when a unique kind of user or item arives.

Social Network recommendation Technique:

The unprecedented growth of social networking platforms and the availability of social interaction profiles led to the development of recommender systems based on Social Network Analysis (SNA). Most real-time systems, provide an opportunity to interact socially with fellow users and makes use of this information to provide better recommendations. This in collaboration with Collaborative Filtering models and CB models helps to overcome the Cold Start problems. Trust based models for Collaborative Filtering on Social Networks is a widely discussed topic. Several studies prove that there is a positive correlation between trust and user similarity in online communities [22]. Social bookmarks, tags, physical context and co-authorship relationships are currently being exploited to provide better recommendations. Social network based recommendation system works according to the social relationship between people. There are three primary objectives of this system (i) to improve the quality of prediction and recommendations (ii) to generate new data sources (iii) significant relationship between social and collaborative filtering.

NTL: Neighbourhood Transfer Learning

The idea of transefer learning[23] is to leverage the correlarions among the user and item.In most recommendation methods , similarity is a central concept between user-user or item –item, because the neighborhood can be constructed for like-minded users preference aggregation and the target user's preference prediction.In cross domain preference ,first we calculate cosine similarity for each user , we remove the users with a small similarity value and then take similar users to construct a neighbourhood.In item level preference ,we are not able to have score directly because items are new for each user. Thus we propose to approximate the item level preference using category preference. Specifically ,NTL to addresses the new user and new item by category preference.

Active Learing:

Neil et al [24], proposed friend recommender method. In this method, they used step process to make more accurate recommendations. In the first step on the basis of similarity, a list of possible friend is generated. Secondly, they used the assumption that friend of friend is also a friend. By using this approach they do co-clustering from the previous list and generate the final list of friends.

Enhanced CB Algorithm using Social Networking:

This method[25][26] makes use of content based algorithm and social networking techniques . We can get information of similar users using social networking techniques, then we can use content based algorithms or on the items created by them. The prediction ratings generated using this method solve the cold start problem.

SIMILARITY MEASURES

In this section some of the user based algorithms similarity measuring methods mentioned in the below table.

Name	Formula
Cosine Similarity	$S(u,v) = \frac{\vec{u}\cdot\vec{v}}{\left\|\vec{u}\right\|\left\|\vec{v}\right\|} = \frac{\sum_i r_{ui} r_{vi}}{\sqrt{\sum_i r_{ui}^2}\, X \sqrt{\sum_i r_{vi}^2}}$ Where r_{ui} and r_{vi} are rating vectors of user and item.

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Pearson Correlation	$S(u,v) = \frac{\sum_{i\in I}(r_{u,i}-\bar{r}_{u,I})(r_{v,i}-\bar{r}_{v,I})}{\sqrt{\sum_{i\in I}(r_{u,i}-\bar{r}_{u,I})^2}+\sum_{i\in I}(r_{v,i}-\bar{r}_{v,I})^2}$ Where I represented co-rated items
Constrained Pearson Correlation	$S(u,v) = \frac{\sum_{i\in I}(r_{u,i}-r_{med})(r_{v,i}-r_{med})}{\sqrt{\sum_{i\in I}(r_{u,i}-r_{med})^2}+\sum_{i\in I}(r_{v,i}-r_{med})^2}$ Where r_{med} –median value for rating scale
Adjusted Cosine Similarity	$S(u,v) = \frac{\sum_{i\in I}(r_{u,i}-\bar{r}_u)(r_{v,i}-\bar{r}_v)}{\sqrt{\sum_{i\in I}(r_{u,i}-\bar{r}_u)^2}+\sum_{i\in I}(r_{v,i}-\bar{r}_v)^2}$ Where r_i Average rating of user I for all itms rated by user
Jaccard Similarity	$S(u,v) = \frac{\lvert I_u \cap I_v\rvert}{\lvert I_u \cup I_v\rvert}$ Where I_u and I_v set of items rated by user u and v
PIP Similarity Proximity Impact Popualarity	$S(u,v) = \sum PIP(r_{u,i}-r_{v,i})$ $PIP(r_{u,I}, r_{v,i})=P(r_{u,I}, r_{v,i}).I(r_{u,I}, r_{v,i}).P(r_{u,I}, r_{v,i})$
Spearman Rank Correlation	$S(u,v) = 1-\frac{6\sum_{h_0}^{n_i} d_h^2}{n_i(n_i^2-1)}$ Where d_h difference ranks of item ,n_i is number of items co-rated by users.

TABLE I. Similarity Measures

Table 2: New Similarity Measures In Terms Of Similarity Measures In Table 1.

Name	Formula
Jaccard Mean Squared Differences	JMSD_S(u,v)=Jaccard_S(u,v).MSD_S(u,v)
PSS Similarity (Proximity Significance Singualrity)	PSS_S(u,v)=∑ PSS ($r_{u,I} - r_{v,i}$) PSS ($r_{u,I}$, $r_{v,i}$)=P($r_{u,I}$, $r_{v,i}$).S($r_{u,I}$, $r_{v,i}$).S($r_{u,I}$, $r_{v,i}$)
Jaccard PSS Simialrity	JPSS_S(u,v)=J'_S(u,v).PSS_S(u,v) J'_S(u,v)= $\lvert I_u \cap I_v \rvert$ / $\lvert I_u \rvert$ X $\lvert I_v \rvert$

CONCLUSION

Recommendation system helps us out of the information overload that is bestowed upon us by the information era.In this survey,we have presented an overview of the recommendation system techniques and also discusses the Cold Start problems. Several approaches for solving Cold Start problems has been discussed from an implementation perspective. Some of the techniques can be used in different domains. After considering all these points we can choose one of the method or combination of the method to solve Cold Start problem in any system.

References

[1] J.S. Breese, D. Heckerman, C. Kadie, Empirical analysis of predictive algorithms for collaborative filtering, in: 14th Conference on Uncertainty in Artificial Intelligence, 1998, pp. 43–52.

[2] J. Mooney, L. Roy, Content-based book recommending using learning fortext categorization, in: Proceedings of the Fifth ACM Conference on DigitalLibraries, 2000, pp. 195–204

[3] Lops, P., Gemmis, M., Semeraro, G.: Content-based recommender systems: State of the art and trends. In: Ricci, F., Rokach, L., Shapira, B., Kantor, P.B. (eds.) Recommender Systems Handbook, Springer 2011, pp. 73–106.

[4] Bianchi, M., Cesaro, F., Ciceri, F., Dagrada, M., Gasparin, A., Grattarola, D., Inajjar, I., Metelli, A. M., and Cella, L. (2017). Content-based approaches for cold-start job recommendations. In Proceedings of the Recommender Systems Challenge 2017, RecSys Challenge '17, pages 6:1–6:5, New York, NY, USA. ACM.

[5] Murat Yagci and Fikret Gurgen. A Ranker Ensemble for Multi-objective Job Recommendation in an Item Cold Start Setting. In Proceedings of the Recommender Systems Challenge 2017. ACM, 2.

[6] D. S. Banerjee and K. Kothapalli. Hybrid Algorithms for List Ranking and Graph Connected Components, in the Proc. of HiPC, 2011.

Vol. 9 Issue 1, (Special Issue HICAAMMC- January 2019)
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[7] Goldberg, K. , Roeder, T. , Gupta, D. , & Perkins, C. Eigentaste: A constant time collaborative filtering algorithm. Information Retrieval, 4 (2), 2001,pp. 133–151 .

[8] Sarwar, B. ,Karypis, G. , Konstan, J. , &Riedl, J. Item-based collaborative filtering recommendation algorithms. In Proceedings of the 10th international conference on World Wide Web, (2001,). (pp. 285–295)

[9] Y.H. Chien, E.I. George, A bayesian model for collaborative filtering, *Proc. of Inte'l Conf. Workshop Artificial Intelligence and Statistics*, 1999.

[10] J.D.M. Rennie, N. Srebro, Fast maximum margin matrix factorization for collaborative prediction, *Proc. of Inte'l Conf. Machine learning*, 2005,713-719.

[11] Q. Yang, J. Fan, J. Wang, L. Zhou, Personalizing Web Page Recommendation via Collaborative Filtering and Topic-Aware Markov Model, *Inte'l Conf. Data Mining*, 2010,1145-1150.

[12] J. Wei, J. He, K. Chen, Y. Zhou, Z. Tang Collaborative filtering and deep learning based recommendation system for Cold Start items Expert Systems with Applications, 69 (2017), pp. 29-39.

[13] W. Zhang, J. Wang, "A collective bayesian poisson factorization model for cold-start local event recommendation", 2015, ACM, pp. 1455-1464.

[14] Kula M (2015) Metadata embeddings for user and item cold-start recommendations. arXiv:1507.08439

[15] S. Rendle. Factorization machines. In Data Mining (ICDM), IEEE 10th International Conference on, 2010,pages 995-1000.

[16] Anshu Sang, Santosh K. Vishwakarma, "A Ranking based Recommender System for Cold Start & Data Sparsity Problem", Tenth International Conference on Contemporary Computing, August 2017,pp.10-12.

[17] Zhang Z, Lin H, Liu K,Wu D, Zhang G, Lu J, A hybrid fuzzy-based personalized recommender system for telecom products/services. Inf Sci (2013),117–129.

[18] Lu J, Shambour Q, Xu Y, Lin Q, Zhang G , A web based personalized business partner recommendation system using fuzzy semantic techniques. Comput Intell 29(1): (2013)PP:37–69

[19] Alvaro Tejeda-Lorente, Juan Bernabé-Moreno, Carlos Porcel, and Enrique Herrera-Viedma. 2017. Using Bibliometrics and Fuzzy Linguistic Modeling to Deal with Cold Start in Recommender Systems for Digital Libraries. In Advances in Fuzzy Logic and Technology 2017. Springer, 393–404.

[20] Heung-Nam Kim, Abdulmotaleb El-Saddik, and Geun-Sik Jo. Collaborative error-reflected models for cold-start recommender systems. Decision Support Systems 51, 3 (2011), 519–531.

[21] Xiaoyuan Su and Taghi M. Khoshgoftaar. 2009. A Survey of Collaborative Filtering Techniques. Adv. in Artif. Intell. 2009, Article 4 (Jan. 2009), 1 pages. https: //doi.org/10.1155/2009/421425

[22] K.Reka, T.N.Ravi, "Hybrid Recommender Systems: Process, Challenges, Approaches and Metrics", International Journal of Computer Sciences and Engineering, Vol.6, Issue.5,2018, pp.1024-1033.

[23] Jixiong Liu, Jiakun Shi, Wanling Cai, Bo Liu, Weike Pan, Qiang Yang, and Zhong Ming. Transfer Learning from APP Domain to News Domain for Dual Cold-Start Recommendation. In CEUR Workshop Proceedings. 38.(2017)

[24] Neil Houlsby, José Miguel Hernández-Lobato, and Zoubin Ghahramani.Cold-start active learning with robust ordinal matrix factorization. In International Conference on Machine Learning.2014, 766–774.

[25] Rahul Srivastava nitin mishra, saumya chaturvedi. Research Challenges in Recommendation System. In Proceedings of the CONIAPS (CONIAPS '14). IIIT Jabalpur, Jabalpur MP, India, 2014,pp:39–46.

[26] Nitin Mishra,Vimal Mishra,Saumya Chaturvedi.Tools and Techniques for solving cold start Recommendataion,ACM,2017.

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EFFECTS OF CHEMICAL REACTION IN CASSON FLUID FLOW WITH THERMAL AND MASS DIFFUSION

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Abstract

Effect of Chemical reaction in Casson fluid over a vertical plate with thermal and mass diffusion has been studied. A similarity analysis method was used to transform the system of partial differential equations describing the problem into an ordinary differential equations, Analytical solutions are obtained by solving the ODE to analyze the velocity and temperature fields. Variations of interesting parameters on the velocity are observed by plotting graphs. The effect of Casson fluid parameter, Schmidt number and Hall parameter suppress the fluid velocity field. Also, the influence of Chemical reaction parameter increase primary and decrease secondary velocity.

Key Words : Casson fluid, Hall effect, MHD, Thermal diffusion, Mass Diffusion

Introduction

Heat and mass transfer problems have received the attention of Scientists and Engineers in recent days, due to their practical importance. Heat and mass transfer occur simultaneously in processes such as drying, evaporation at the surface of water body, energy transfer in a cooling tower and flow in a desert cooler. In many of these processes, interest lies in the determination of the total energy transfer processes and in the overall mass transfer for moisture removal. A large amount of exothermic and endothermic reactions are accompanied by chemical reaction. In a lot of industrial processes, these characteristic can be seen often. Convective flow with simultaneous heat and mass transfer under the influence of a magnetic field and chemical reaction have attracted the attention of researchers because such process exists in many branches of Engineering field. Possible applications of this type of flow can be found in many industries, cooling of nuclear reactors and Magnetohydrodynamic power generators.

Heat pipe is one of the devices for enhancement of heat transfer. Heat and mass transfer problem on a continuous moving plate has been studied by Ericken et al. [1]. Even now, the concept of improving the heat capabilities, if thermally conducting fluids is of great concern to many Engineers and Scientists. The interest in analytical studies and experimental designs of heat pipes started growing rapidly in following the publication of Grover et. al.[2]. Thermal diffusion or the Soret effect is the flux of mass caused due to temperature gradient. Thermal diffusion is of smaller order in magnitude than the effects explained by Fick's law are often ignored in heat and mass transfer processes. Even though the effect is very small, the thermal diffusion effect was utilized for isotope separation and in mixtures between gases with very light and medium molecular weight, so it cannot be neglected in some certain cases [3]. The experimental investigations on mass transfer problems with thermal diffusion was

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first performed by the Charles Soret [4]. The flow along the vertical plate in the presence of heat and mass transfer was analyzed by Elbashbeshy [5]. The influence of heat and mass transfer with MHD free convection over a vertical surface was studied by Chen [6].

Makinde [7] presented the similarity solution of hydromagnetic heat and mass transfer over a vertical plate with a convective surface boundary condition. The combined effect of heat and mass transfer by natural convection from a semi-infinite plate submitted to a magnetic field with Hall current was investigated by Joaquin et. al [8]. Hayat [9] studies the effects of Soret and Dufour on MHD flow of Casson fluid. Mukhopadhyay [10] investigated the magnetohydrodynamic boundary layer flow of Casson fluid through an exponential stretching sheet. There is a great need from the industrial sector of different methods or techniques for augmenting the heat and mass transfer with chemical reaction in many areas so as to design reliable, economic and user-friendly devices that are specific to the required application. So far no attempt has been made to study explicitly the effect of heat, mass, chemical reaction, Soret number and Schmidt number in Casson fluid flow with Hall effect and rotation in an uniformly accelerated plate. Hence, the purpose of the present analysis is to study the combined effect of heat, mass, chemical reaction, Soret number, Schmidt number, Hall effect and rotation in Casson fluid flow.

Formulation of the Problem

Consider an incompressible electrically conducting, Casson fluid past an infinite vertical flat plate occupying the plane y $= 0$. The x-axis is taken in the direction of the motion of the plate and z – axis lying on the plate normal to both x and y – axis. Initially it is assumed that the plate and the fluid rotate in unison with a uniform angular velocity $\bar{\Omega}$ about the y - axis normal to the plate are at the same temperature T and concentration C everywhere in the fluid. At time $t > 0$, the plate starts moving impulsively with the uniform velocity in its own plane along the x-axis. Also the temperature of the plate is raised/lowered to T_∞ and the concentration is also raised/lowered to C_∞. A uniform magnetic field H_0, parallel to y - axis is imposed. The schematic representation of the problem is given in Fig 1.

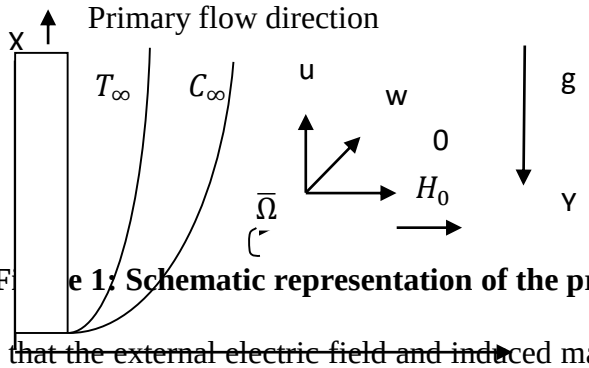

Fig. 1: Schematic representation of the problem

It is assumed that the external electric field and induced magnetic field are negligible. The effect of buoyancy and the density variation are taken in consideration in the momentum equation and the concentration of species far from the wall is infinitesimally small and the Joules and Viscous dissipation term in the energy equation is neglected. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is as follows [11]

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$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi < \pi_c \end{cases}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} are the $(i,j)th$ component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid and P_y is the yield stress of the fluid. Under these assumptions, the governing boundary layer equations with Boussinesq's approximation for casson fluid flow are given by:

Equation of continuity

$\nabla.\bar{q}=0$ (1)

where $\bar{q}=(u,0,w)$ is the velocity vector.

Equation of motion

$$\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{\mathrm{q}} + 2\bar{\Omega}\times\bar{\mathrm{q}} = -\frac{1}{\rho}\nabla \mathrm{P} + \nu\left(1+\frac{1}{\gamma}\right)\nabla^2\bar{\mathrm{q}} + \frac{1}{\rho}(\bar{\mathrm{J}}\times\bar{\mathrm{B}}) + \frac{1}{\rho}(\rho\bar{\mathrm{g}}) \quad (2)$$

Here $\bar{\Omega}= (0,\Omega_y,0)$ denotes uniform angular velocity, P is the pressure, γ is the Casson fluid parameter, ρ is the density, ν is kinematic viscosity,$\bar{H}= (H_x, H_0, H_z)$ is the magnetic induction, $\bar{\mathrm{J}}\times\bar{\mathrm{B}}$ is the Lorentz force and $\bar{\mathrm{g}}$ is the acceleration due to gravity. As the plate is infinite, all variables in the problem are functions of *y* and *t* only.

The Energy equation

$$\rho C_P\left[\frac{\partial T}{\partial t} + (\bar{q}.\nabla)T\right] = K\ \nabla^2 T \quad (3)$$

where C_p is the specific heat at constant pressure and K is the thermal conductivity.

The Species Concentration equation

$$\frac{\partial C}{\partial t} + (\bar{q}.\nabla)C = \mathrm{D}\nabla^2 C + D_T\nabla^2 T - k_1 C \quad (4)$$ where

D is the coefficient of chemical molecular mass diffusivity, D_T is the coefficient of chemical thermal diffusivity and $k_1$is the rate of chemical reaction.

The generalized Ohm's law is given by $\frac{\bar{\mathrm{J}}}{\sigma} = (\bar{\mathrm{E}} + \bar{q}\times\bar{\mathrm{B}}) - \frac{\bar{\mathrm{J}}\times\bar{\mathrm{B}}}{n.e}$ (5)

where σ is the electrical conductivity, $\bar{\mathrm{J}}$ is the current density, $\bar{E}= (E_x, 0, E_z)$ is the electro static field,ρ is density, ν is kinematic viscosity, e is the electric charge, τ is the mean collision time, n is the electron number density and m_e is mass of an electron.

Now using, $T(y,t) - T_\infty = \theta_T(y,t)$ and $C(y,t) - C_\infty = C'(y,t)$

where $T(y,t)$ is the temperature of the fluid in the boundary layer and T_∞ is the fluid temperature far away from the plate. $C(y,t)$ is the species concentration in the boundary layer and C_∞ is the species concentration in the fluid far away from the plate.

The initial and boundary conditions are

$u = 0,\ w = 0,\ \theta_T = 0, C' = 0$ for all $t \le 0$ and for all y

$u = U_0, w = 0, \theta_T(0,t) = ae^{i\omega t}, C'(0,t) = be^{i\omega t}$ for all $t > 0$ and $y = 0$

$u \to 0,\ w \to 0,\ \theta_T \to 0, C'(y,t) \to 0$ for all $t > 0$ and $y \to \infty$ (6)

Introducing the non-dimensional quantities:

$$y^* = \frac{U_0.y}{\nu},\ u^* = \frac{u}{U_0},\ w^* = \frac{w}{U_0},\ t^* = \frac{U_0^2 t}{\nu}, \theta_T{}^* = \frac{\theta_T}{a}, C^* = \frac{C'}{b},$$

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$$Gr = \frac{g\beta\nu a}{U_0^3}, Gc = \frac{g\beta^*\nu b}{U_0^3}, m = \omega\tau, M^2 = \frac{\sigma H_0^2 \nu}{\rho U_0^2}, Pr = \frac{\nu\rho C_p}{K}, K^2 = \frac{\nu\Omega_y}{U_0^2},$$
$$\xi = \frac{\nu\omega}{U_0^2}, Sc = \frac{\nu}{D}, S_0 = \frac{D_T\, a}{\nu\, b}, k_c = \frac{k_1\nu}{U_0^2} \quad (7)$$

By applying the non-dimensional forms the equations (1), (2) and (3) becomes

$$\frac{\partial u}{\partial t} = \left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} + Gr\,\theta_T + Gr\,C - \frac{M^2}{(1+m^2)}(u+mw) - 2wK^2 \quad (8)$$

$$\frac{\partial w}{\partial t} = \left(1+\frac{1}{\gamma}\right)\frac{\partial^2 w}{\partial y^2} + \frac{M^2}{(1+m^2)}(mu-w) + 2uK^2 \quad (9)$$

$$\frac{\partial^2 \theta_T}{\partial y^2} = Pr\frac{\partial \theta_T}{\partial t} \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + S_0\frac{\partial^2 \theta_T}{\partial y^2} - k_c C \quad (11)$$

The corresponding boundary conditions in non-dimensional form are

$$t \le 0: u(y,t) = w(y,t) = 0,\ \theta_T(y,0) = 0, C(y,0) = 0 \text{ for all } y$$
$$t > 0: u(0,t) = 1, w(0,t) = 0,\ \theta_T(0,t) = e^{i\xi t}, C(0,t) = e^{i\xi t}$$
$$t > 0: u(y,t) \to 0,\ w(y,t) \to 0,\ \theta_T(y,t) \to 0, C(y,t) \to 0 \text{ as } y \to \infty \quad (12)$$

Solution of the Problem

By using the complex function $q = u + iw$, equations (8) and (9) can be combined as

$$\frac{\partial q}{\partial t} = \left(1+\frac{1}{\gamma}\right)\frac{\partial^2 q}{\partial y^2} - \left[\left(\frac{M^2}{1+m^2}\right)(1-im) - 2iK^2\right]q + Gr\theta_T + Gc\,C \quad (13)$$

The boundary conditions (12) are transformed to

$$q(y,0) = 0, \quad q(0,t) = 1, \quad q(\infty,t) = 0$$
$$\theta_T(y,0) = 0, \quad \theta_T(0,t) = e^{i\xi t}, \quad \theta_T(\infty,t) = 0$$
$$C(y,0) = 0, \quad C(0,t) = e^{i\xi t}, \quad C(\infty,t) = 0 \quad (14)$$

Substitute $\theta_T(y,t) = e^{i\xi t} f(y)$ in equation (10), we get

$$f''(y) - i\xi\,\mathrm{Pr}\, f(y) = 0 \quad (15)$$

with the boundary condition

$$f(0) = 1, f(\infty) = 0$$

Hence the solution of the equation (15) is

$$f(y) = e^{-y\sqrt{i\xi Pr}} \quad (16)$$

Now $\theta_T(y,t) = e^{i\xi t}e^{-y\sqrt{i\xi Pr}}$ (17)

Separating into real and imaginary parts and taking the real part only we get

$$\theta_{T_r}(y,t) = e^{-yS_1}Cos(\xi t - S_1 y) \quad (18)$$

where $S_1 = \sqrt{\frac{\xi Pr}{2}}$.

Substitute $C(y,t) = e^{i\xi t}G(y)$ in equation (11), we get

$$G''(y) - (i\xi + k_c)Sc\, G(y) = -S_0 Sc\, f''(y) \quad (19)$$

with the boundary condition

$$G(0) = 1, G(\infty) = 0$$

Hence the solution of equation (19) is

$$\mathrm{G}(y) = -S_2 e^{-y\sqrt{(i\xi+k_c)\,Sc}} + S_2 e^{-y\sqrt{i\xi Pr}} \quad (20)$$

Here $S_2 = a_1 + ib_1$, where $a_1 = \frac{-S_0 Sc\xi^2 \mathrm{Pr}(Pr-Sc)}{(k_c Sc)^2 + (\xi(Pr-Sc))^2}$ and $b_1 = \frac{S_0 Sc^2 \xi\, \mathrm{Pr}\, k_c}{(k_c Sc)^2 + (\xi(Pr-Sc))^2}$

Hence $C(y,t) = e^{i\xi t}\left[-S_2 e^{-y\sqrt{(i\xi+k_c)\,Sc}} + S_2 e^{-y\sqrt{i\xi Pr}}\right]$ (21)

Separating into real and imaginary parts of the above equation and taking the real part only we get

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$$C_r(y,t) = -e^{-yS_3}(a_1 Cos(\xi t - yS_4) - b_1 Sin(\xi t - yS_4) + e^{-yS_1}(a_1 Cos(\xi t - yS_1) - b_1 Sin(\xi t - yS_1) \quad (22)$$

where $S_3 = \sqrt{\frac{(k_c\, Sc)+\sqrt{(k_c\, Sc)^2 + (\xi\, Sc)^2}}{2}}$ and $S_4 = \sqrt{\frac{-(k_c\, Sc)+\sqrt{(k_c\, Sc)^2 + (\xi\, Sc)^2}}{2}}$

Also substitute $q(y,t) = e^{i\xi t} g(y)$ in (13), we have

$$cg''(y) - (i\xi + \beta)g(y) = -Grf(y) - Gc\, G(y) \quad (23)$$

Here $c = \left(1 + \frac{1}{\gamma}\right)$, $\beta = a_2 + ib_2$ where $a_2 = \frac{M^2}{1+m^2}$ and $b_2 = -\left(\frac{M^2 m}{1+m^2} + 2K^2\right)$

The boundary conditions are,

$$g(0) = e^{-i\xi t}, g(\infty) = 0 \quad (24)$$

The solution of (3.23) is given by

$$g(y) = \left(e^{-i\xi t} + \frac{Gr + GcS_2}{i\xi Prc - (\xi i + \beta)} - \frac{GcS_2}{c(i\xi + k_c)Sc - (\xi i + \beta)}\right) e^{\frac{-y}{\sqrt{c}}\sqrt{i\xi + \beta}} - \frac{Gr+GcS_2}{i\xi Prc-(i\xi + \beta)} e^{-y\sqrt{i\xi Pr}} + \frac{GcS_2}{c(i\xi+k_c)Sc-(\xi i+\beta)} e^{-y\sqrt{(i\xi+k_c)\, Sc}} \quad (25)$$

Hence $q(y,t) = e^{i\xi t}\left[\left(e^{-i\xi t} + \frac{Gr+GcS_2}{i\xi Prc-(\xi i+\beta)} - \frac{GcS_2}{c(i\xi+k_c)Sc-(\xi i+\beta)}\right) e^{-y\sqrt{i\xi + \beta}} - \frac{Gr+GcS_2}{i\xi Prc-(i\xi + \beta)} e^{-y\sqrt{i\xi Pr}} + \frac{GcS_2}{c(i\xi+k_c)Sc-(\xi i+\beta)} e^{-y\sqrt{(i\xi+k_c)\, Sc}}\right]$

Separating the above equation into real and imaginary parts, we get

$$u(y,t) = e^{-yS_5}[L_5\, Cos(\xi t - yS_6) - L_6\, Sin(\xi t - yS_6)] - e^{-yS_1}\,[L_7\, Cos(\xi t - yS_1) - L_8\, Sin(\xi t - yS_1)] + e^{-yS_3}\,[L_9\, Cos(\xi t - yS_4) - L_{10}\, Sin(\xi t - yS_4)] \quad (26)$$

$$w(y,t) = e^{-yS_5}[L_5\, Sin(\xi t - yS_6) + L_6\, Cos(\xi t - yS_6)] - e^{-yS_1}\,[L_7\, Sin(\xi t - yS_1) + L_8\, Cos(\xi t - yS_1)\,] + e^{-yS_3}\,[L_9\, Sin(\xi t - yS_4) + L_{10}\, Cos(\xi t - yS_4)\,] \quad (27)$$

where

$$S_5 = \frac{1}{\sqrt{c}}\sqrt{\frac{a_2+\sqrt{(a_2)^2 + (\xi+b_2)^2}}{2}};\ S_6 = \frac{1}{\sqrt{c}}\sqrt{\frac{-a_2+\sqrt{(a_2)^2 + (\xi+b_2)^2}}{2}};\ L_1 = \frac{-a_2}{(\xi\, Prc-\xi-b_2)^2+{a_2}^2}$$

$$L_2 = \frac{-(\xi\, Prc-\xi-b_2)}{(\xi\, Prc-\xi-b_2)^2+{a_2}^2};\ L_3 = \frac{ck_c\, Sc-a_2}{(c\, k_c\, Sc-a_2)^2+(c\,\xi\, Sc-\xi-b_2)^2};\ L_4 = \frac{-(c\,\xi\, Sc-\xi-b_2)}{(c\, k_c\, Sc-a_2)^2+(c\,\xi\, Sc-\xi-b_2)^2}$$

$$L_5 = Cos\xi t + GrL_1 + GcL_1\, a_1 - Gc\, L_2\, b_1 - Gc\, L_3\, a_1 + GcL_4\, b_1$$

$$L_6 = -Sin\xi t - GcL_1\, b_1 + Gc\, L_2\, a_1 - Gc\, L_3\, b_1 + Gr\, L_2 - GcL_4\, a_1$$

$$L_7 = Gr\, L_1 + GcL_1 a_1 - GcL_2\, b_1;\ L_8 = Gr\, L_2 + GcL_1 b_1 + GcL_2\, a_1$$

$$L_9 = Gc\, L_3\, a_1 - Gc\, L_4\, b_1;\ L_{10} = Gc\ L_3\, b_1 - GcL_4\, a_1$$

Results and Discussion

The aim of this study is to analyze the effect of various parameters influencing Casson fluid velocity. From the graph the following results are obtained. From figures 2 and 3, it is observed that the velocity increases gradually near the plate and decrease slowly away from the plate. This is due to first the heat energy gained by the molecules which in turn increases their kinetic energy then away from the plate, there is the cooling effect that results in reduction in kinetic energy that causes a decline in the velocity. An increase in Hall parameter causes a significant decrease in both primary and secondary velocity. This is due to the established fact that the application of a magnetic field to an electrically conducting fluid gives rise to a force, known as Lorentz force, which tends to resist the fluid motion.

Prandtl number being the ratio of momentum diffusivity to the thermal diffusivity induces the temperature. The effect of Prandtl number on primary velocity and secondary velocity components are depicted in the graphs 4 and 5. Here it is observed that the primary

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velocity accelerates and the secondary velocity decreases with increasing values of Prandtl number.

Soret number is the ratio of coefficient of thermal diffusion and coefficient of ordinary diffusion. When the Soret number is increased the primary velocity component retards and secondary velocity component accelerates which are clearly depicted in figures 6 and 7. The effect of modified Grashof number in primary and secondary velocity components are depicted in the graph 8 and 9. It clearly show that the primary velocity decreases and secondary velocity component increases with increasing values of modified Grashof number.

Figures 10 and 11 depict that an increase in Schmidt number reduces the velocity of the fluid flow. Schmidt number is the ratio between the momentum diffusivity and the species diffusivity. As the Schmidt number increases, the diffusivity also increase and hence the fluid flow is decreased which is clearly represented in figures. The effect of Chemical reaction parameter on the primary velocity and secondary velocity are indicated through the figures 12 and 13. When the value of chemical reaction parameter is increased, the primary velocity profile gradually increases near the plate and attains a free stream velocity away from the plate. Whereas the secondary velocity profile retard in its motion for increasing values of the chemical reaction parameter.

Figures 14 and 15 examine the influence of Casson fluid parameter on Primary and secondary velocity components. It is observed that the momentum decreases with increase of Casson fluid parameter. Casson parameter effects inversely on the yield stress. It is evident that the boundary layer thickness becomes thin with increase of the Casson parameter. It may conclude that the boundary layer thickness is more for the Newtonian fluid than the Casson fluid as the higher value of Casson parameter involves with the non-Newtonian property of fluid.

Conclusion

From the above results and discussion the conclusions are made.

- The velocity components decrease for increase in Hall parameter, Casson fluid parameter and Schmidt number.
- With the increase of Prandtl number and chemical reaction parameter, the primary velocity increases and the secondary velocity decreases.
- The Primary velocity component increases and secondary velocity component deccreases for increase in Modified Grashof number and Soret number.

Reference

1. Ericken, L.E., Fan, L.T. and Fox, V.G., 1966. Heat and mass transfer on a moving continuous flat plate with suction or injection, Int. Eng. Chem. Fundam, 5: 19-25.
2. Grover, G.M., Cotter, T.P. and Erickson, G.F., 1964. Journal of Applied Physics, 35(6):1990.
3. Eckert. E.R.G. and Drake, R.M., 1972. Analysis of Heat and Mass Transfer, McGraw-Hill, Newyork.
4. Charles Soret, 1879. Arch. Sci. Phys. Nat., 2: 48-61
5. Elbashbeshy, E.M.A., 1997. Heat transfer over a stretching surface with variable surface heat flux, J. Phys. D. Appl. Phys, 31: 1951-1954.

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6. Chein- Hsin-Chen, 2004. Combined heat and mass transfer in MHD Free Convective from a vertical surface with Ohmic heating and viscous dissipation, Int. J. Eg. Science. 42: 699-713.
7. Makinde. O.D, 2010. Similarity solution of hydromagnetic heat and mass transfer over a vertical plate with a convective surface boundary condition, International Journal of the physical sciences, 5(6): 700-710.
8. Joaquin Zucco, Luis. M. Lopez Ochoa, Pablo Eguia and Joaquin Collazo, 2011. Combined heat and mass transfer by Natural convection from a semi-infinite plate submitted to a magnetic field with Hall currents, Engineering applications of computational Fluid mechanics, 5(2): 188-200.
9. Hayat, T. Shehzad, S.A. and Alsaedi, A, 2012. Soret and Dufour effects on magnetohydrodynamic flow of Casson fluid, Appl. Math. Mech. – Engl Ed, 33(10): 1301-1312.
10. Mukhopadhyay, S., Mondal, I.C. and Hayat, T. 2014. MHD boundary layer flow of Casson fluid passing through an exponentially stretching permeable surface with thermal radiation, Chin. Phys. B, 23: 1056-1074.
11. Casson, N, 1959. A flow equation for Pigment –oil suspension of the priniting ink type. In: Mill, C.C., Ed., Rheology of disperse systems, Pergamon press, Oxford, 84-104.

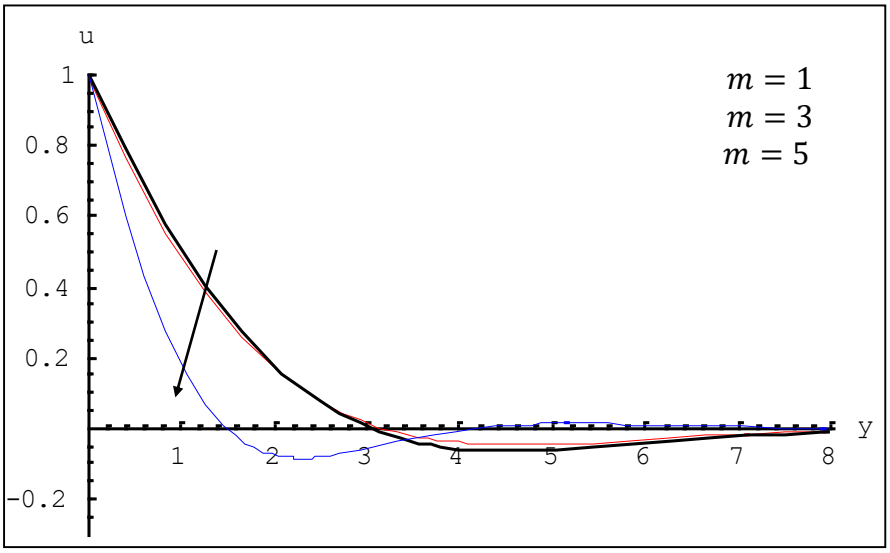

Figure 2 : Effect of Hall parameter (m) on primary velocity profile when $\gamma = 0.2$; $t = 1$; $Pr = 0.71$; $M^2 = 1$; $K^2 = 2$; $Gr = 1$; $Gc = 1$; $S_0 = 1$; $Sc = 1$; $\xi = 1$; $k_c = 1$

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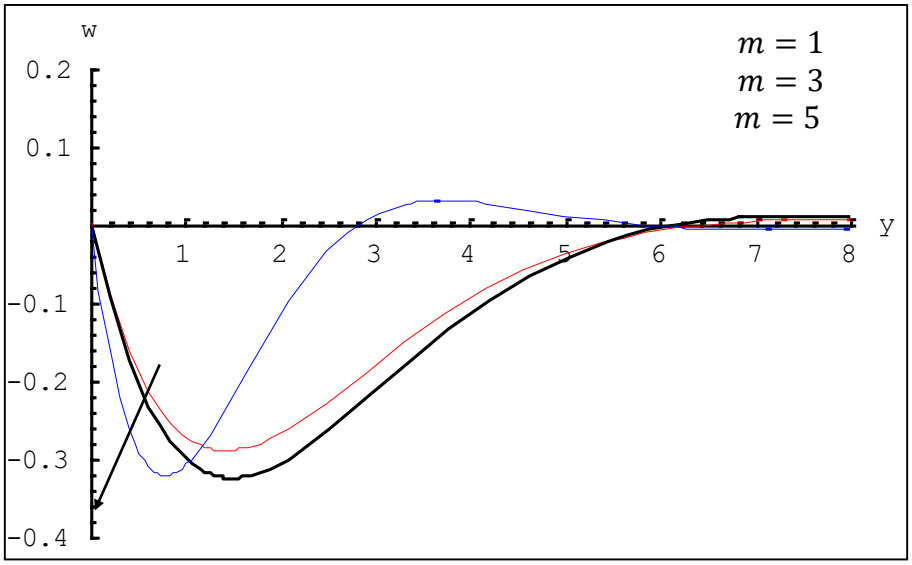

Figure 3 : Effect of Hall parameter (m) on secondary velocity profile when $\gamma = 0.2$; $t = 1$; $Pr = 0.71$; $M^2 = 1$; $K^2 = 2$; $Gr = 1$; $Gc = 1$; $S_0 = 1$; $Sc = 1$; $\xi = 1$; $k_c = 1$

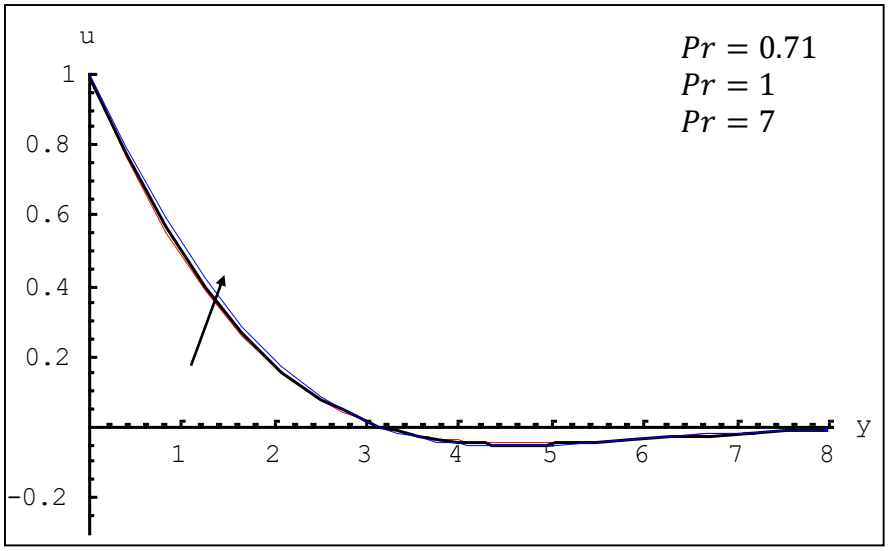

Figure 4 : Effect of Prandtl number (Pr) on primary velocity profile when $\gamma = 0.2$; $t = 1$; $m = 1$; $M^2 = 1$; $K^2 = 2$; $Gr = 1$; $Gc = 1$; $S_0 = 1$; $Sc = 2$; $\xi = 1$; $k_c = 1$

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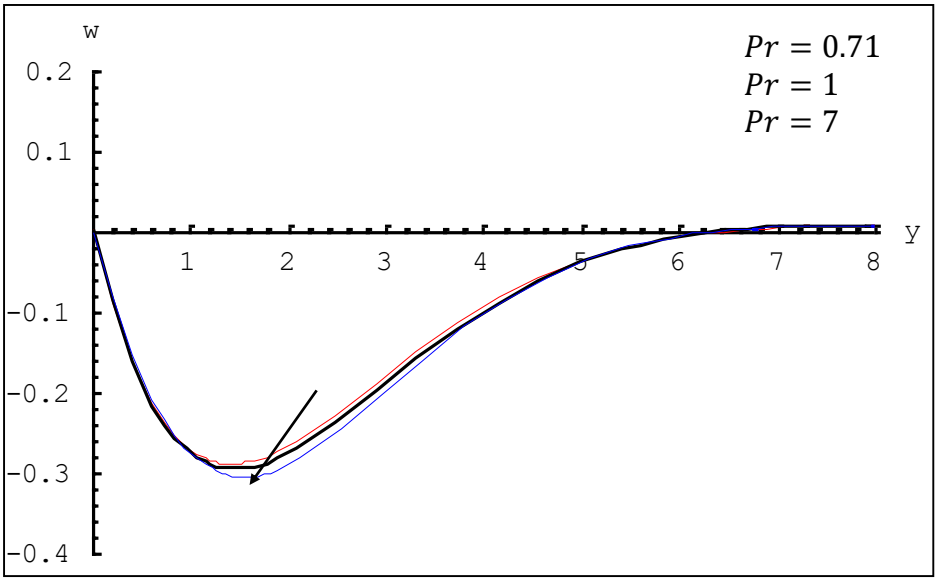

Figure 5 : Effect of Prandtl number (Pr) on secondary velocity profile when $\gamma = 0.2; t = 1;\ m = 1;\ M^2 = 1;\ K^2 = 2;\ Gr = 1; Gc = 1; S_0 = 1; Sc = 2;\ \xi = 1;\ k_c = 1$

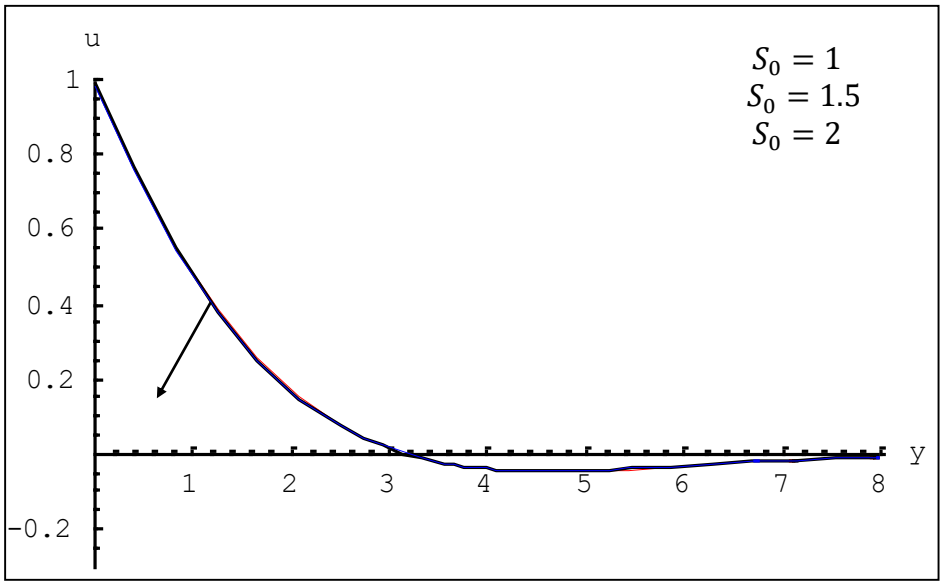

Figure 6 : Effect of Soret number (S_0) on primary velocity profile when $\gamma = 0.2; t = 1;\ m = 1;\ M^2 = 1;\ K^2 = 2;\ Pr = 1; Gr = 1; Gc = 1; Sc = 2;\ \xi = 1;\ k_c = 1$

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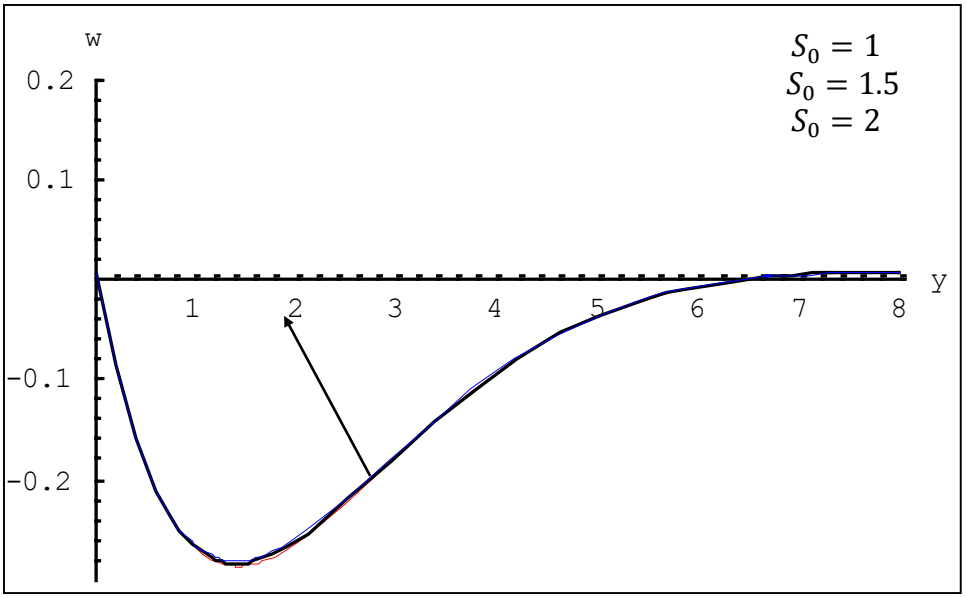

Figure 7 : Effect of Soret number (S_0) on secondary velocity profile when $\gamma = 0.2$; $t = 1$; $m = 1$; $M^2 = 1$; $K^2 = 2$; $Pr = 1$; $Gr = 1$; $Gc = 1$; $Sc = 2$; $\xi = 1$; $k_c = 1$

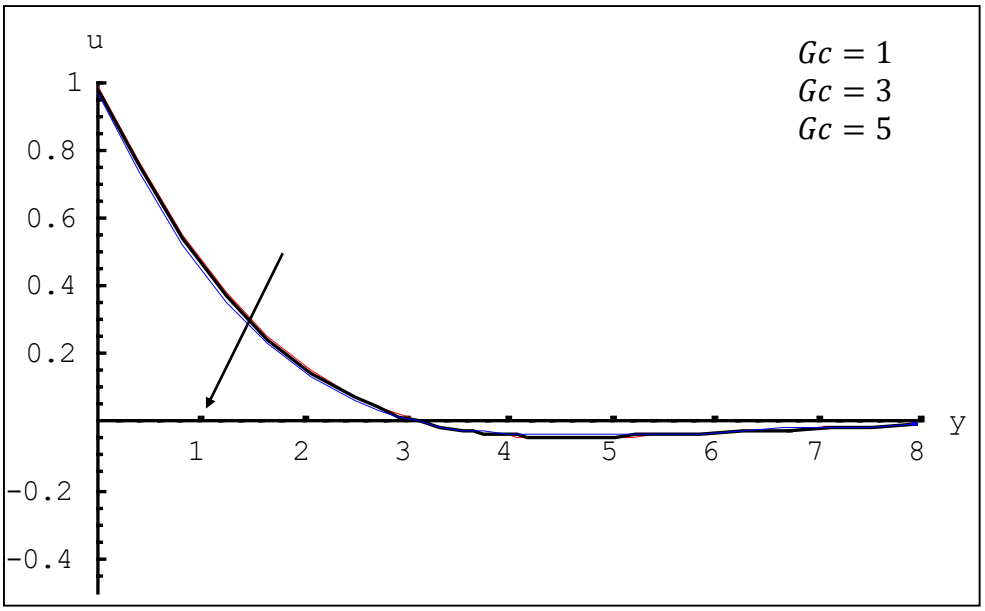

Figure 8 : Effect of modified Grashof number (Gc) on primary velocity profile when $\gamma = 0.2$; $t = 1$; $m = 1$; $M^2 = 1$; $K^2 = 2$; $Pr = 1$; $Gr = 1$; $S_0 = 1$; $Sc = 2$; $\xi = 1$; $k_c = 1$

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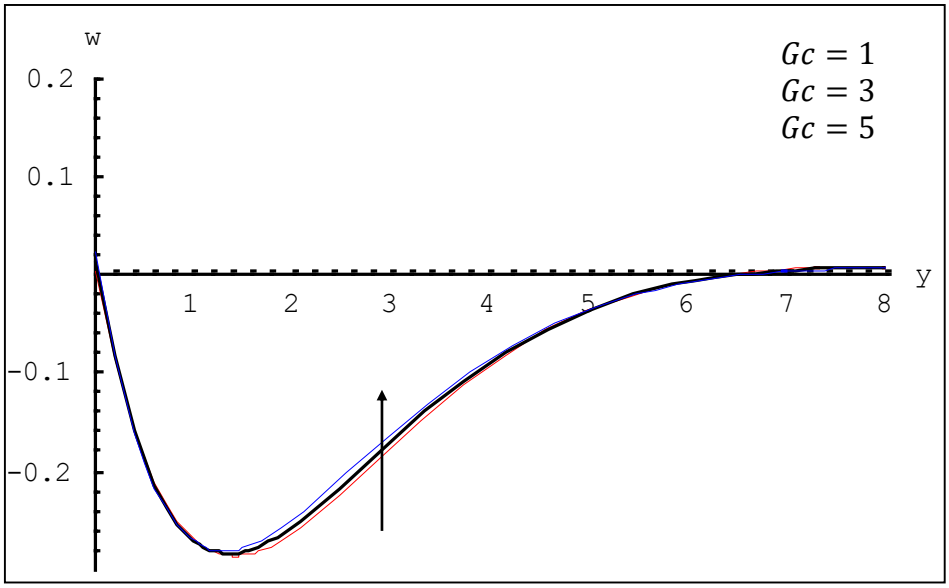

Figure 9 : Effect of modified Grashof number (Gc) on secondary velocity profile when $\gamma = 0.2; t = 1;\ m = 1;\ M^2 = 1;\ K^2 = 2;\ Pr = 1; Gr = 1; S_0 = 1; Sc = 2;$ $\xi = 1;\ k_c = 1$

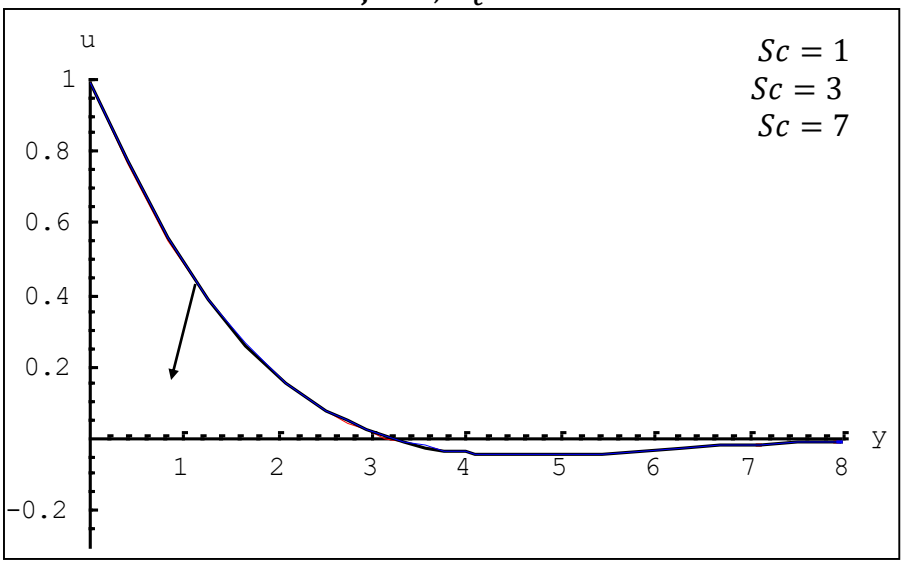

Figure 10 : Effect of Schmidt number (Sc) on primary velocity profile when $\gamma = 0.2; t = 1;\ m = 1;\ M^2 = 1;\ K^2 = 2;\ Pr = 1; Gr = 1; Gc = 1; S_0 = 1;$ $\xi = 1;\ k_c = 1$

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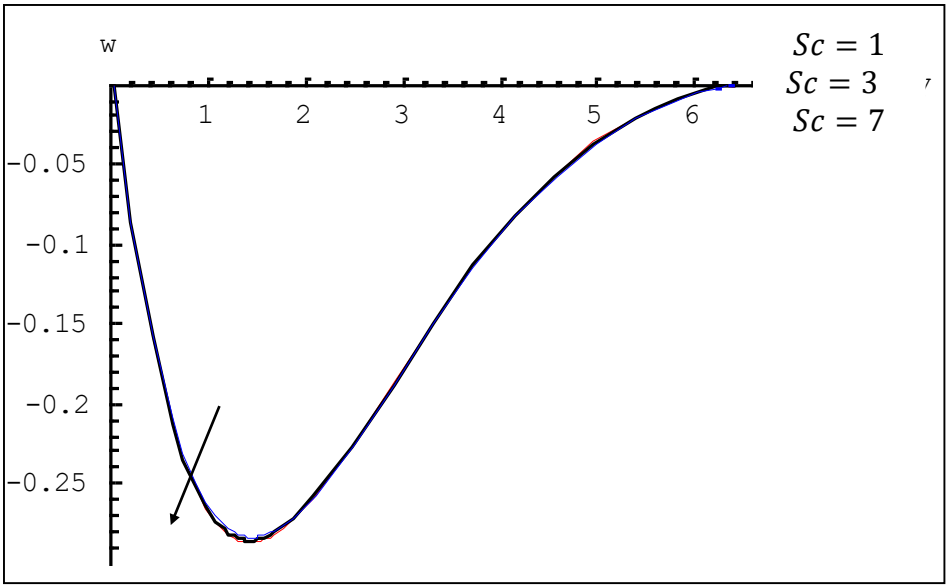

Figure 11 : Effect of Schmidt number (Gr) on secondary velocity profile when $\gamma = 0.2; t = 1;\ m = 1;\ M^2 = 1;\ K^2 = 2;\ Pr = 1; Gr = 1; Gc = 1; S_0 = 1;\ \xi = 1;\ k_c = 1$

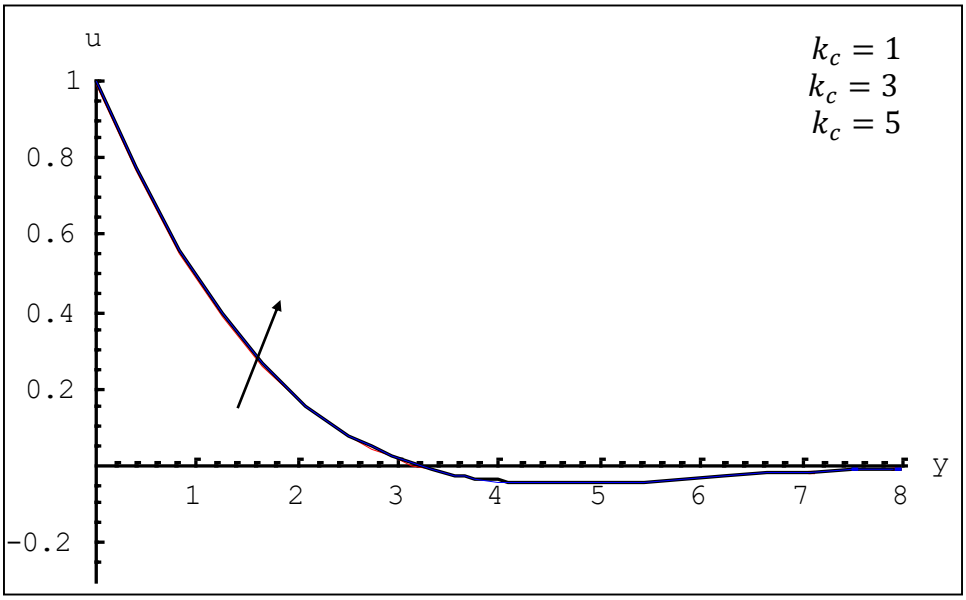

Figure 12 : Effect of chemical reaction (k_c) on primary velocity profile when $\gamma = 0.2; t = 1;\ m = 1;\ M^2 = 1;\ K^2 = 2;\ Pr = 1; Gr = 1; Gc = 1; S_0 = 1;\ \xi = 1;\ Sc = 1$

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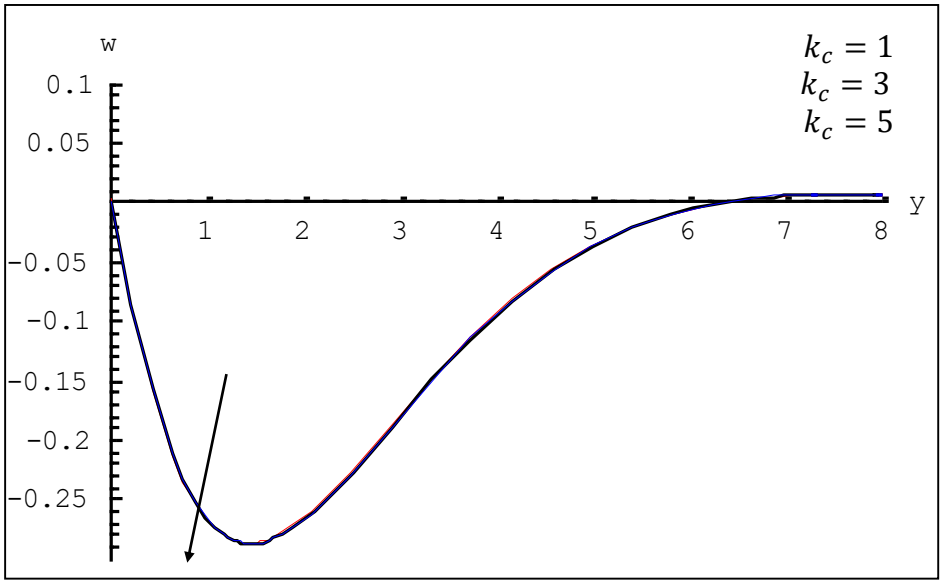

Figure 13 : Effect of chemical reaction (k_c) on secondary velocity profile when $\gamma = 0.2; t = 1;\ m = 1;\ M^2 = 1;\ K^2 = 2;\ Pr = 1; Gr = 1; Gc = 1; S_0 = 1;$ $\xi = 1;\ Sc = 1$

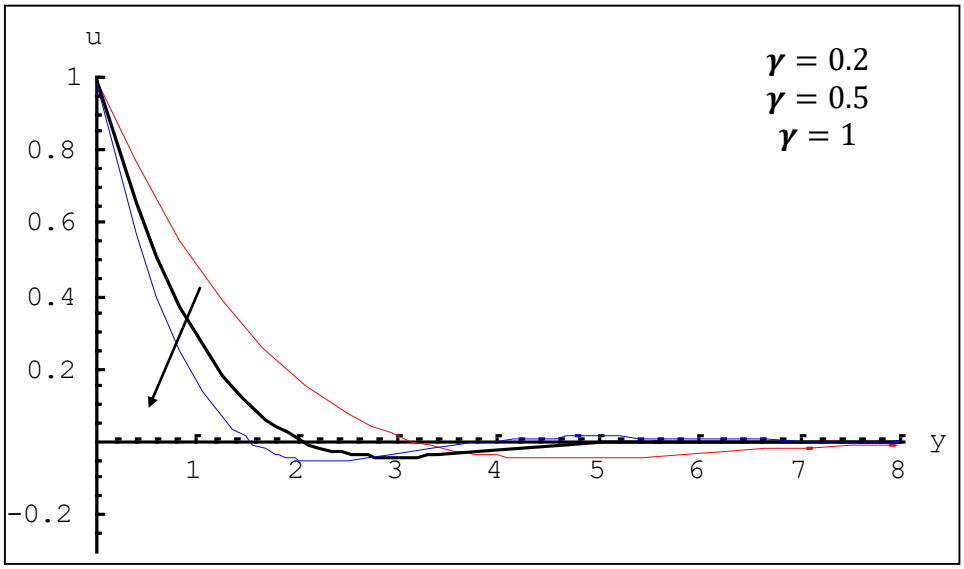

Figure 14 : Effect of time (t) on primary velocity profile when $t = 1$; $k_c = 1;\ m = 1;\ M^2 = 1;\ K^2 = 2;\ Pr = 1; Gr = 1; Gc = 1; S_0 = 1;\ \xi = 1;\ Sc = 2$

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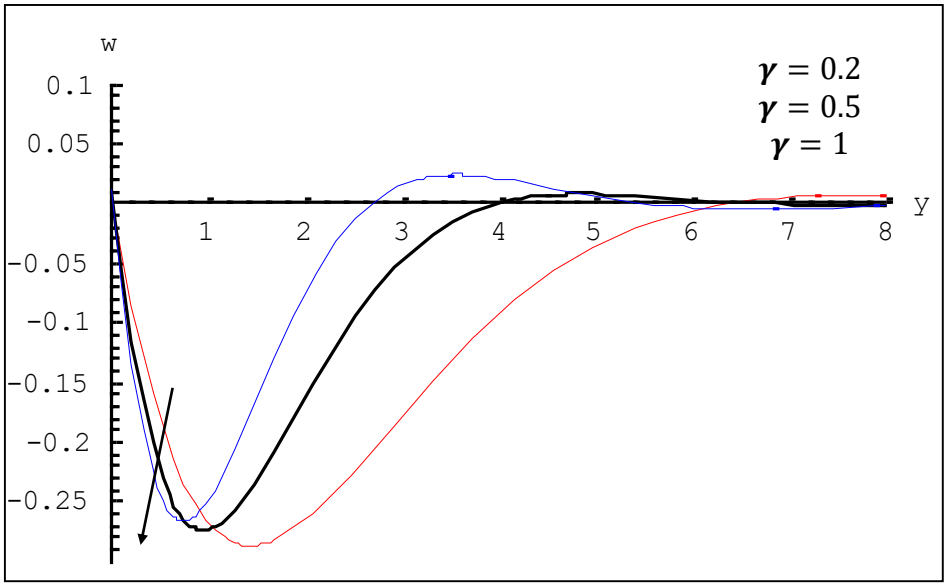

Figure 15 : Effect of time (t) on secondary velocity profile when $t = 1$; $k_c = 1$; $m = 1$; $M^2 = 1$; $K^2 = 2$; $Pr = 1$; $Gr = 1$; $Gc = 1$; $S_0 = 1$; $\xi = 1$; $Sc = 2$

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AN APPLICATION OF ROUGH MATRIX IN GULLY CRICKET

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Abstract

In this paper, Rough matrix concept is applied based on the rough set. The theory of rough matrix is discussed. In addition, another effective arrangement strategy has been produced to decision making which may contain more than one criteria in the game of gully cricket.

Keywords: rough set, rough matrix, rough approximation, multiple criteria decision making.

Introduction

Pawlak [1] introduced the concept of RST-Rough Set Theory (1982). RST s[1]is a mathematical tool deals with the data analysis of distinctness or vagueness. Yiyu Yao [3] defined the notes on rough approximations If boundary region is nonempty then set is rough set. Vijayabalaji and balaji [7] introduced rough matrix theory and its decision making based on rough set .Vijayabalaji and balaji [8] is a MCDM method in cricket by rough matrix theory. In this paper, Rough sets, theory of rough matrix and application of rough matrix in gully cricket and results of best players are given.

Basic rough sets

Definition [1]

Let U $\neq \phi$ be a universe of discourse and Y be a subset of U. An equivalence relation *R*, classifies U into a set of subsets $U\backslash R = \{Y_1, Y_2, \ldots, Y_n\}$ in which the following conditions are satisfied:

1. $Y_i \subseteq U$, $Y_i \neq \phi$ For any i.

2. $Y_i \cap Y_j \neq \phi$ For any i, j.

3. $\bigcup_{R \in P} IND(R)$

Definition [1] Any subset *Yi*, which called a category, class or granule, represents an equivalence class of R. A category in *R* containing an object $Y \in U$ is denoted by $[Y]_R$. To a family of equivalence relations $P \subseteq R$, an indiscernibility relation over P is denoted by *IND(P)*, and is defined by the equation $IND(P) = \bigcup_{R \in P} IND(R)$.

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Definition[1]The set Y can be divided according to the basic sets of *R*, namely a lower approximation set and upper approximation set. Approximation is used to represent the roughness of the knowledge. Suppose a set $Y \subseteq U$ represents a vague concept, the *R*-lower and *R*-upper approximations of Y are defined by the equations *R*-lower approximation of y: $\underline{R}(y) = \bigcup_{R \in P} \{R(y) : R(y) \subseteq y\}$

R-upper approximation of y: $\overline{R}(y) = \bigcup_{R \in P} \{ R(y): R(y) \cap y \neq \phi \}$

R-boundary region of y: $RN_R(y) = \overline{R}(y) - \overline{R}(y)$ Also we can define

R-positive of *y*: $POS_R(y) = \underline{R}(y)$

R-negative of *y*: $NEG_R(y) = U - \overline{R}(y)$

Definition [1] Rough Membership function defined by Pawlak [3] is given by $Y : U \rightarrow < 0, 1 >$, where $\mu_y^R(y) = |X \cap R(y)| \frac{|y \cap R(y)|}{|R(y)|}$, $|Y|$ denotes cardinality of *Y*. The rough membership function can be used to define approximations and the boundary region of a set, as shown below $\underline{R}(y) = \{y \in u : \mu_y^R(y) = 1\}$,

$\overline{R}(y) = \{y \in u : 0 < \mu_y^R(y) > 0$ *and* $RN_R(y) = \{y \in u : 0 < \mu_y^R(y)\ \mu_y^R(y) < 1\}$

1. Set Y is rough with respect to R if $\underline{R}(y) \neq \overline{R}(y)$.

2. Set Y is rough with respect to R if for some y, $0 < \mu_y^R(y) < 1$.

The Theory of Rough Matrix

Definition[7]: We can define rough membership function $\mu_X^R(x) = \begin{Bmatrix} 1 \ if \mu_X^R(x) = 1 \\ 0 \ if \mu_X^R(x) = 0 \end{Bmatrix}$ *where* $\mu_X^R : U \rightarrow \langle 0,1 \rangle$

Definition[7]: We can formulate the matrix from ranking $R_M(PR;CR) = \begin{Bmatrix} 1 \ if \ PR \leq CR \\ 0 \ otherwise \end{Bmatrix}$

Here PR represents present ranking. i.e., Ranking after evaluation.CR represents conditional ranking (Given).

Definition[7]: We can define a Rough Matrix $R_M = [rij]$ of order m × n as follows.

$R_M = [r_{ij}] = \begin{pmatrix} r11 & r12 & \dots & r1n \\ r21 & r22 & \dots & r2n \\ rm1 & rm2 & \dots & rmn \end{pmatrix}$ where each $r_{ij} \in \mu_y^R(y)$

Rough Matrix in Gully Cricket:

Basic rules in gully cricket

1. The match consisting of 20 overs per side.
2. Based upon the toss, toss winning captain should decide batting or bowling.
3. Batting Average = Total runs/number of times got out.
4. Bowling Economy = total runs conceded/total overs bowled.

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5. Fielding performance = Total catches taken / total innings played.
6. A bowler can bowl non-consecutive and maximum 4overs per match.

Problem statement:

Gully cricket management committee say 6 batsmen, 5 bowlers, 2 wicket keepers form pool of 13 players for all the3 venues namely Nehru street , Wooden apple street, Ruby street, respectively and also they given data based upon their performance. Further for winning best players awards management ask us a best players related proposal by our rough matrix theory results. The data table is given below here N,W,R represents Nehru street, Wooden apple, Ruby street, respectively.

Table1: Batsmen and Bowlers information data table

Identity		Bowlers/economy		
1	Ram	31.02	32.23	21.12
2	Raj	27.32	26.12	28.30
3	Amar	25.01	24.41	27.32
4	Thiga	24.20	29.04	30.42
5	Ragu	20.05	19.4	21.32

Identity		Batsmen/average		
	Batsmen	**N**	**W**	**R**
1	Bala	20.07	22.37	16.37
2	Bharathi	21.42	20.23	19.02
3	Siva	18.27	19.31	20.03
4	Thiru	18.04	16.11	17.29
5	Deva	15.07	22.17	21.02
6	Jaga	20.21	17.18	21.12

Table2: Wicket Keeper information data table

Wicket Keeper		N	W	R
1	Anand	0.433	1.112	0.512
2	Karthik	0.233	0.453	0.253

Assign the ranking based upon their performance in their field in every match. Based upon ranking construct decision attribute which widely useful to decide who the best player in every match.

Let RS ={U,X,Y}
U={ Batsmen(1….6), Bowler(1…5),wicket Keeper(1,2)}
X={Nehru Street, Wooden apple street, Ruby Street}
Y={Yes, No}

Table3: Decision table for batsmen

Identity		Batting ranking			Decision		
	Batsmen	**N**	**W**	**R**	**N**	**W**	**R**
1	Bala	3	1	6	Yes	Yes	No
2	Bharathi	1	2	4	Yes	Yes	Yes
3	Siva	4	4	3	Yes	Yes	Yes
4	Thiru	5	6	5	No	No	No
5	Deva	6	2	2	No	Yes	Yes
6	Jaga	2	5	1	Yes	No	Yes

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Table4: Decision table for Bowlers

Identity		Bowling Ranking			Decision		
		N	**W**	**R**	**N**	**W**	**R**
1	Ram	1	1	5	Yes	Yes	No
2	Raj	2	3	3	Yes	Yes	Yes
3	Amar	3	4	2	Yes	No	Yes
4	Thiga	4	2	1	No	Yes	Yes
5	Ragu	5	5	4	No	No	No

Table5: Decision table for wicket keeper

Identity		Wicket keeper			Decision		
		N	**W**	**R**	**N**	**W**	**R**
1	Anand	1	2	1	Yes	No	Yes
2	Karthik	2	1	2	No	Yes	No

Construct the matrix table by using rough matrix:

$$R_M^T = \begin{matrix} N \\ W \\ R \end{matrix} \begin{pmatrix} 1\,1\,1\,0\,0\,1\,1\,1\,1\,0\,0\,1\,0 \\ 1\,1\,1\,0\,1\,0\,1\,1\,0\,1\,0\,0\,1 \\ 0\,1\,1\,0\,1\,1\,0\,1\,1\,1\,0\,1\,0 \end{pmatrix}$$

From the rough matrix transpose table conclude that the players Bharathi, Siva, and Raj are the best players

Conclusion:

In this paper, Rough set, theory of rough matrix is discussed. Finally our approach on rough matrix application in gully cricket and the result of best players are Mr.Bharathi, MR. Siva, Mr Raj are the eligible candidate to winning the best players awards.

References:

[1]Z.Pawlak,Roughsets,Int.J.Inf.Comput.Sci.11(5)(1982)341356.http://dx.doi.org/10.1007/bf01001956
[2] Li Xiang-wei, Qi Yian-fang, A data preprocessing algorithm for classification model based on rough sets. Int. Jl.of guangzi teachers education University 2006, vol 23, no.4 pp 87-95.
[3] Yiyu yao, Notes on rough set approximations and associated measures, Journal of zhejiang ocean university(natural science), 29(5)(2010)399-410.
[4] Yiyu yao and Zhao, Attribute reduction in decision-theoretic rough set models, Information Sciences, 178(17)(2008), 3356-3373.http://dx.doi.org/10.1016/j.ins.2008.05.010
[5] Kosiantis.S, Kanellopoulos.D, P. Pintelas, Data preprocessing for supervised learning, International Journal of Computer Sciences, 1(2)(2006),111-117.
[6] Preston .I and Thomas.J, Batting strategy in limited overs cricket, Journal of the Royal Stat.Society;Series D(The Statistician), 49(1)(2006), 95 -106. http://dx.doi.org/10.1111/1467-9884.00223
[7] Vijayabalaji.S and Balaji.S, Rough matrix theory and its decision making, International JournalofPureandAppliedMathematics;87(6)(2013),845853.http://dx.doi.org/10.12732/ijpam.v87i6.13
[8] Vijayabalaji.S and Balaji.S, MCDM Method in cricket Rough matrix theory, International JournalofMathematicalAnalysis;volume.9,2015,no.18,869-875.

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STOCHASTIC MODELS FOR TIME TO RECRUITMENT IN A SINGLE GRADE MANPOWER SYSTEM WITH NON-INSTANTANEOUS EXITS AND INDEPENDENT WASTAGE OF MANPOWER WHEN THE BREAKDOWN THRESHOLD HAS TWO COMPONENTS USING UNIVARIATE CUM POLICY OF RECRUITMENT

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Abstract
In this paper, the problem of time to recruitment is analyzed for a single grade manpower system in which attrition takes place due to two types of policy decisions where this classification is done according to intensity of attrition. Assuming (i) policy decisions and exits occur at different epochs (ii) wastage of manpower due to exits and wastage due to frequent breaks taken by the personnel working in the manpower system separately form a sequence of independent and identically distributed exponential random variables with different means and (iii) breakdown threshold for the cumulative wastage of manpower in the system has two components which are independent exponential random variables. Three stochastic models are constructed and the variance of the time to recruitment is obtained using an univariate CUM policy of recruitment. While in Model I the inter-decision times form an ordinary renewal process, in Models II and III, they form a geometric process and order statistics respectively. Employing a different probabilistic analysis, analytical results in closed form for system characteristics are derived.

Keywords
Single grade manpower system, non-instantaneous exits, intensity of attrition, independent wastage of manpower, ordinary renewal process, geometric process, order statistics, breakdown threshold with two components, univariate CUM policy of recruitment and variance of time to recruitment.

2010 Mathematics Subject Classification– 90B70, 91B40, 91D35, 60H30.

Introduction

Wastage of personnel due to retirement, death and resignation is a common phenomenon in administrative as well as production oriented organizations. There are certain special problems associated with the organization engaged in sales and marketing. Frequent exits and recruitments are very common in such organizations. Whenever the organization announces revised polices regarding sales target, revision of wages, incentives and perquisites the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover of the organization. Frequent recruitments may also be expensive due to the cost of recruitments and training. As the wastage of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this wastage. The univariate recruitment policy, usually known as CUM policy of recruitment in the literature, is based on the replacement policy associated with the shock model approach in reliability theory and is stated as follows: *Recruitment is made whenever the cumulative wastage of man hours exceeds its breakdown threshold.* Several researchers have studied the problem of time to recruitment for a single grade manpower system using shock model approach. In [1] and [2] the authors have discussed some manpower planning models for a single and multi-grade manpower system using Markovian and renewal theoretic approach. In [12] the authors have analyzed the problem of time to recruitment for the single grade manpower system which is subject to attrition with **instantaneous exits**, using CUM policy of recruitment when the wastage of manpower process and inter-decision time process are independent. In [11], the author has studied the problem by associating geometric process and order

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statistics for inter-decision times. In [8], the authors have analyzed the work in [11] with (i) exponential breakdown threshold and (ii) extended exponential threshold having shape parameter 2 using a bivariate CUM policy of recruitment when the inter-decision times form a geometric process. In [9], the authors have studied the problem of time recruitment by assuming that the attrition is generated by a geometric process of inter-decision times.Variance of the time to recruitment for a single grade manpower system with optional and mandatory thresholds is obtained in [15] when inter-decision times form an order statistics. In [16], the authors have obtained variance of time to recruitment when the breakdown threshold level for the cumulative wastage of manpower is the sum of the exponential breakdown threshold levels of wastage and backup resource for manpower using Laplace transform technique. Recently, in [14], the authors have studied the work in [16] when the inter-decision times form a geometric process using a different method employed in [9]. In [3] the authors have considered the single grade manpower system with **non-instantaneous exits** and obtained variance of the time to recruitment when the wastage of manpower, inter-decision times and exit times are independent and identically distributed continuous random variables according as the mandatory breakdown threshold is an exponential random variable or extended exponential random variable with shape parameter 2 or a continuous random variable with SCBZ property. In [6] and [7] the authors have extended the research work in [3] when the inter-decision times form (i) a geometric process and (ii) an order statistics respectively. In [4], [10] and [5] the authors have studied the work in [3], [6] and [7] using the method of [9].The present paper extends the research work in [10] and [5] when the breakdown threshold level for the cumulative wastage in the manpower system is the sum of two components namely an exponential threshold for cumulative wastages due to exits and an exponential threshold for cumulative wastage due to frequent breaks of existing workers.

MODEL DESCRIPTION AND ANALYSIS FOR MODEL-I

Consider an organization taking policy decisions at random epochs in $(0,\infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated wastage of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let X_i be independent and identically distributed exponential random variables representing the amount of depletion of manpower (wastage of man hours) at the i^{th} exit epoch with probability distribution function M(.), and mean $\frac{1}{\alpha}$ ($\alpha>0$). Let S_i be the cumulative wastage of manpower in the first i exit epochs and m_i be its probability density function. The policy decisions which produce depletion of manpower are classified into two types depending upon the intensity of attrition. It is assumed that the first type of policy decisions has high attrition rate λ_1($\lambda_1>0$) and the second type has low attrition rate λ_2 ($\lambda_2>0$). Let a_1 and $(1-a_1)$ be the proportion of decisions with high and low attrition rate respectively. Let A_n be the time between $(n-1)^{th}$ and n^{th} policy decisions, forming a sequence of independent random variables. Let B_i be the time between $(i-1)^{th}$ and i^{th} exit epochs, forming a sequence of independent and identically distributed random variables with probability distribution function G(.) and density function g(.). Let D_{i+1} be the waiting time upto $(i+1)^{th}$ exit epoch. Let Y be the breakdown threshold for the cumulative wastage of manpower in the organization with probability density function h(.). It is assumed that Y is the sum of the exponential breakdown threshold level Y_1 for cumulative wastage due to exits with mean $\frac{1}{\theta_1}$ $(\theta_1>0)$ and exponential threshold Y_2 for cumulative wastage due to frequent breaks of existing workers with mean $\frac{1}{\theta_2}$ $(\theta_2>0)$. Let q ($q\neq 0$) be the probability that every policy decision has exit of personnel. Let $\chi(I)$ be the indicator function of the event I. Let T be the random variable denoting the time to recruitment with mean E(T) and variance V(T).

As in [4] we get,

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$$T=\sum_{i=0}^{\infty} D_{i+1}\,\chi\,(S_i \le Y < S_{i+1}) \quad (1)$$

$$E(T) = \sum_{i=0}^{\infty} (i+1)\,E(B)\,P\,(S_i \le Y < S_{i+1}) \quad (2)$$

and

$$E(T^2) = \sum_{i=0}^{\infty} (i+1)\,[V(B)+ (i+1)E^2(B)]\,P\,(S_i \le Y < S_{i+1}) \quad (3)$$

where $P\,(S_i \le Y < S_{i+1}) = \int_0^{\infty}\int_0^{t} \tilde{M}(t-x)m_i(x)h(t)dxdt$ (4)

It can be shown from the hypothesis on Y that

$$h(t) = \frac{\theta_1\theta_2}{\theta_1-\theta_2}\left(e^{-\theta_2 t} - e^{-\theta_1 t}\right), \theta_1 > 0, \theta_2 > 0. \quad (5)$$

Proceeding as in [4], from (2), (3), (4) and (5) and on simplification it can be shown that

$$E(T) = \left[\frac{a_1(\lambda_2-\lambda_1)+\lambda_1}{\lambda_1\lambda_2 q}\right]\left[\frac{(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)}{\theta_1\theta_2(\theta_1-\theta_2)}\right] \quad (6)$$

and

$$V(T) = \frac{\begin{array}{c}2q(\theta_1\theta_2)\left[a_1\left(\lambda_2^2-\lambda_1^2\right)+\lambda_1^2\right]\left[(\theta_1)^3\theta_2(\alpha+\theta_2)-\theta_1(\theta_2)^3(\alpha+\theta_1)\right]+\left[a_1(\lambda_2-\lambda_1)+\lambda_1\right]^2 \\ \left\{2(\theta_1)^3(\alpha+\theta_2)(\theta_1-\theta_2)[\alpha+\theta_2(1-q)]-2(\theta_2)^3(\alpha+\theta_1)(\theta_1-\theta_2)[\alpha+\theta_1(1-q)]\right\}\end{array}}{-\left[a_1(\lambda_2-\lambda_1)+\lambda_1\right]^2\left[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)\right]^2} \quad (7)$$
$$\lambda_1^2\lambda_2^2 q^2(\theta_2-\theta_1)(\theta_1\theta_2)^2$$

(6) and (7) give the mean and variance of the time to recruitment for the present model.

MODEL DESCRIPTION AND ANALYSIS FOR MODEL-II

In this model, the problem of time to recruitment for a single grade manpower system is analyzed by assuming that the inter-policy decision times form a geometric process with rate c, c>0. It is assumed that distribution of A_1 is hyper-exponential with parameters a_1, λ_1 and λ_2.

In this case from (2), (3), (4) and on simplification we get

$$E(T) = \frac{c[a_1(\lambda_2-\lambda_1)+\lambda_1]}{\lambda_1\lambda_2(c-1+q)}\left[\frac{(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)}{\theta_1\theta_2(\theta_1-\theta_2)}\right] \quad (8)$$

and

$$V(T) = \frac{\begin{array}{c}c^2[c-1+q]^2(\theta_1\theta_2)(\theta_1-\theta_2)\left[2\left(a_1\lambda_2^2+a_2\lambda_1^2\right)-(a_1\lambda_2+a_2\lambda_1)^2\right]\left[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)\right] \\ +c^2q\overline{q}(\theta_1\theta_2)(\theta_1-\theta_2)[a_1(\lambda_2-\lambda_1)+\lambda_1]^2\left[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)\right]+c^2\left[c^2-1+q\right](\theta_1-\theta_2) \\ [a_1(\lambda_2-\lambda_1)+\lambda_1]^2\left[(\theta_1)^3(\alpha+\theta_2)(2\alpha+\theta_2)-(\theta_2)^3(\alpha+\theta_1)(2\alpha+\theta_1)\right]\end{array}}{-c^2\left[c^2-1+q\right][a_1(\lambda_2-\lambda_1)+\lambda_1]^2\left[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)\right]^{2^2}} \quad (9)$$
$$[c^2-1+q](\lambda_1\lambda_2)^2\theta_1\theta_2(\theta_1-\theta_2)^2$$

(8) and (9) give the mean and variance of the time to recruitment for the present Model.

MODEL DESCRIPTION AND ANALYSIS FOR MODEL-III

Description of this model is similar to that of Model-I except the assumption on inter-decision times. In this model it is assumed that the inter-policy decision times form an order statistics. More specifically, let $\{U_{(l)}\}^{r}_{l=1}$ be a set of order statistics associated with a sample of size r taken from the population $\{U_n\}^{\infty}_{n=1}$ of independent and identically distributed hyper –exponential random variables with the population mean $E(U_n)= E(U)= \frac{a_1}{\lambda_1}+\frac{(1-a_1)}{\lambda_2}$, n=1,2,…. In this case, the cumulative distribution function S(t) of the population and the probability density function $s_{(l)}(t)$ of the l^{th} order statistics are given by

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$S(t)= a_1(1-e^{-\lambda_1 t})+(1- a_1)(1-e^{-\lambda_2 t})$, $a_1 > 0$, $\lambda_1 > 0$ and $\lambda_2 > 0$.
From the theory of order statistics [13], it is known that

$$s_{(l)}(t) = l\binom{r}{l}(S(t))^{l-1}s(t)\,[1-S(t)]^{r-l},\ l=1,2,...,r.$$

In this Model the inter - policy decision times A_n, n=1,2,…. form a sequence of independent and identically distributed random variable with cumulative distribution F(t) and probability density function f(t). By the assumption that the inter-policy decision times form an order statistics we mean that the probability density function f(t) is identified with that of the order statistics $\{U_{(l)}\}^{r}_{l=1}$. Thus f(t) can be any one of the $s_{(l)}(t)$, l =1,2,….,r.
We now determine E(T) and V(T) under two cases.

Case(i): f(t) = $s_{(1)}(t)$.

The probability density function of the first order statistics is

$$s_{(1)}(t) = r[1-S(t)]^{r-1}s(t)$$

From (2),(3),(4) and on simplification it is found that

$$E(T) = \frac{4r(\lambda_1+\lambda_2)^2[a_1(\lambda_2-\lambda_1)+\lambda_1][(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]-r(1-r)[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]}{\{\lambda_1^2\lambda_2[3a_1^2+6a_1-2]-a_1^2\lambda_2^2-\lambda_1^2\lambda_2^2[2a_1^2-5a_1+1]+4a_1\lambda_2^3+\lambda_1^3(a_1-1)\}} \quad (10)$$
$$4\lambda_1\lambda_2 q(\lambda_1+\lambda_2)^2\theta_1\theta_2(\theta_1-\theta_2)$$

and

$$V(T) = \frac{(\theta_1\theta_2)[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]\{q(k_3+k_4)-r^2[k_4+k_1^2+k_5^2(1-2r+r^2)]\}}{+(1-q)\{rk_1+r(1-r)k_2\}^2+[rk_1+r(1-r)k_2]^2[(\theta_1)^3(\alpha+\theta_2)(2\alpha+\theta_2)-(\theta_2)^3(\alpha+\theta_1)(2\alpha+\theta_1)]} -$$
$$(\theta_1-\theta_2)(\theta_1\theta_2)^2q^2$$
$$\frac{\{4r(\lambda_1+\lambda_2)^2[a_1(\lambda_2-\lambda_1)+\lambda_1][(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]-r(1-r)[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]}{\{\lambda_1^2\lambda_2[3a_1^2+6a_1-2]-a_1^2\lambda_2^2-\lambda_1^2\lambda_2^2[2a_1^2-5a_1+1]+4a_1\lambda_2^3+\lambda_1^3(a_1-1)\}\}^2} \quad (11)$$
$$16\lambda_1^2\lambda_2^2q^2(\lambda_1+\lambda_2)^4\theta_1^2\theta_2^2(\theta_1-\theta_2)^2$$

where $K_1=\left[\frac{a_1(\lambda_2-\lambda_1)+\lambda_1}{\lambda_1\lambda_2}\right]$ and $K_2=\left[\frac{\lambda_1^2\lambda_2(3a_1^2+6a_1-2)-a_1^2\lambda_2^2-\lambda_1\lambda_2^2(2a_1^2-5a_1+1)}{+4a_1\lambda_2^3+\lambda_1^3(a_1-1)}{4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2}\right]$, $K_3=\frac{2a_1(\lambda_2^{\,2}-\lambda_1^{\,2})+\lambda_1^{\,2}}{(\lambda_1\lambda_2)^2}$,

$K_4=\frac{8a_1^{\,2}\lambda_1^{\,3}\lambda_2^{\,2}+[8a_1\lambda_2^{\,2}-a_1^{\,2}\lambda_2^{\,2}+a_1\lambda_1^{\,2}-\lambda_1^{\,2}](\lambda_1+\lambda_2)^3-8a_1\lambda_1^{\,2}\lambda_2^{\,2}(\lambda_1+\lambda_2)(1-\lambda_2)}{4(\lambda_1\lambda_2)^2(\lambda_1+\lambda_2)^3}$

and $K_5=\frac{\lambda_1^{\,2}\lambda_2[3a_1^{\,2}+4a_1+2(a_1-1)]-\lambda_1\lambda_2^{\,2}[1+2a_1^{\,2}-5a_1]+a_1\lambda_2^{\,2}[4\lambda_2-a_1]+\lambda_1^{\,3}(a_1-1)}{4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2}$ (12)

(10) and (11) give the mean and variance of the time to recruitment for case (i) of the present Model.

Case (ii): $f(t) = s_{(r)}(t)$

The probability density function of the r[th] order statistics is

$$s_{(r)}(t) = r[S(t)]^{r-1}s(t)$$

From (2), (3),(4) and on simplification it is found that

$$E(T) = \frac{4r(\lambda_1+\lambda_2)^2[a_1(\lambda_2-\lambda_1)+\lambda_1][(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]+r(1-r)}{[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]4a_1\lambda_1\lambda_2(1+a_1)-a_1\lambda_2(\lambda_1+\lambda_2)^2-(1+3a_1)\lambda_1(\lambda_1+\lambda_2)^2} \quad (13)$$
$$4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2q\theta_1\theta_2(\theta_1-\theta_2)$$

and

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$$V(T)=\frac{\theta_1\theta_2[(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]}{[q\{r(2k_6+k_7)-r^2[k_7-(k_1^2)-(k_6^2)(1-r)^2]\}(1-q)[rk_1+r(1-r)k_6]^2]}\frac{[rk_1+r(1-r)k_6]^2[(\theta_1)^3(\alpha+\theta_2)(2\alpha+\theta_2)-(\theta_2)^3(\alpha+\theta_1)(2\alpha+\theta_1)]}{q^2(\theta_1-\theta_2)(\theta_1\theta_2)^2}-\left[\frac{4r(\lambda_1+\lambda_2)^2[a_1(\lambda_2-\lambda_1)+\lambda_1][(\theta_1)^2(\alpha+\theta_2)-(\theta_2)^2(\alpha+\theta_1)]+r(1-r)}{4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2q\theta_1\theta_2(\theta_1-\theta_2)}\right]^2 \quad (14)$$

where K_1,K_2, K_3, K_4, K_5 gives (12) and $k_6=\left[\frac{4\lambda_1\lambda_2(a_1+a_1^2)-a_1\lambda_2(\lambda_1+\lambda_2)^2-\lambda_1(\lambda_1+\lambda_2)^2[1+3a_1]}{4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2}\right]$

$$K_7=\frac{16a_1(1+a_1)\lambda_1^{\,3}\lambda_2^{\,2}-4a_1\lambda_2^{\,2}(\lambda_1+\lambda_2)^3-4\lambda_1^{\,2}(\lambda_1+\lambda_2)^3(a_1+1)}{(\lambda_1+\lambda_2)^3(2\lambda_1)^2(2\lambda_2)^2} \quad (15)$$

(13) and (14) give the mean and variance of the time to recruitment for case (ii) of the present Model.

CONCLUSION

The models discussed in this paper improve the earlier relevant research work in the context of admitting the realistic assumption of non-instantaneous exits in the system and taking into account wastage due to frequent breaks for this system. They will be useful in the process of planning recruitment when the system has the above cited provision. The suitability of distributions assumed in the present work can be tested by data analysis.

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REFERENCES

1. Bartholomew, D.J., *"Statistical Model for Social Processes"*, New York, John Wiley and Sons, 1973.
2. Bartholomew, D.J., and Andrew Forbes, F., *"Statistical Techniques for Manpower Planning"*, New York, John Wiley and Sons, 1979.
3. Devi, A., and Srinivasan, A., *"Variance of Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits."* International Journal of Research in Mathematics & Computation, Vol.2, No.1, 2014, pp.23-27.
4. Devi, A., and Srinivasan, A., *"Probabilistic Analysis on Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits."* International Journal of Revolution in Science and Humanity, Vol.4, No.2, 2014, pp.59-64.
5. Devi, A., and Srinivasan, A., *"Expected Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits Having Inter-Decision Times as an Order Statistics."* Mathematical Science- International Research Journal, Vol.3, No.2, 2014, pp.887-890.
6. Devi, A., and Srinivasan, A., *"A Stochastic Model for Estimation of Variance of Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits."* International Journal of Applied Mathematics and Statistical Sciences, Vol.4, No.3, 2015, pp.1-8.
7. Devi, A., and Srinivasan, A., *"Expected Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits with Inter-Decision Times as Order Statistics."* International Journal of Innovative Research in Science, Engineering and Technology, Vol.4, No.8, 2015, pp.7517-7520.

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8. Ishwarya, G., Shivaranjani, N., and Srinivasan, A., *"Expected Time to Recruitment in a Single Graded Manpower System with Inter-Decision Times as a Geometric Process."* Bonfring International Journal of ManMachine Interface, Vol.2, No.1, 2012, pp.7-10.
9. Lalitha.R, Devi.A and Srinivasan. A, *"A Stochastic Model on the Time to Recruitment for a Single Grade Manpower System with Attrition Generated by a Geometric Process of Inter-Decision Times Using Univariate Policy of Recruitment",* Journal of Engineering Computers &Applied Sciences, Vol.3, No.7, 2014, pp.12-15.
10. Mariappan, P., Devi, A., and Srinivasan, A., *"A Stochastic Model for Time to Recruitment in a Single Grade Manpower System with Different Epochs for Decisions and Exits Having Geometric Inter-Decision Times."* Second International Conference on Business Analytics and Intelligence, Bangalore, India, 2014.
11. Muthaiyan, A., *"A Study on Stochastic Models in Manpower Planning"*, Ph.D Thesis, Bharathidasan University, Tiruchirappalli, 2010.
12. Sathyamoorthi, R., and Elangovan, R., *"Shock Model Approach to Determine the Expected Time to Recruitment."* Journal of Decision and Mathematika Sciences, Vol.3, 1998, pp.67-78.
13. Sheldon Ross, *"A First Course in Probability"*, Sixth Edition, Published by Pearson Ltd, Indian branch, Patparganj, Delhi, India, 2004.
14. Srinivasan, A., and Jayalakshmi, G., *"Probabilistic Analysis on Time to Recruitment for a Single Grade Manpower System when the Breakdown Threshold has two Components."* International Journal of Scientific Engineering and Applied Science, Vol.1, No.4, 2015, pp.180-183.
15. Sridharan, J., Saivarajan, A., and Srinivasan, A., *"Expected Time to Recruitment in a Single Graded Manpower System with Two Thresholds"*, Journal of Advances in Mathematics, Vol.7, 2014, pp.1147-1157.
16. Vijayasankar, N., Elangovan, R., and Sathiyamoorthi, R., *"Determination of Expected Time to Recruitment when Backup Resources of Manpower Exits."* Ultra Scientist, Vol.25, No.1, 2013, pp.61-68.

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VARIANCE OF TIME TO RECRUITMENT IN A SINGLE GRADE MANPOWER SYSTEM WITH NON-INSTANTANEOUS EXITS AND INDEPENDENT WASTAGE OF MANPOWER WHEN THE BREAKDOWN THRESHOLD HAS TWO COMPONENTS USING UNIVARIATE MAX POLICY OF RECRUITMENT

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Abstract
In this paper, the problem of time to recruitment is analyzed for a single grade manpower system subject to classified policy decisions with non-instantaneous exits. It is assumed that exit time process is an ordinary renewal process, wastage of manpower due to exits and wastage due to frequent breaks by the personnel working in system form separately a sequence of independent and identically distributed exponential random variables with different means and the breakdown threshold for the maximum wastage of manpower has two components which are independent exponential random variables. Three stochastic models are constructed and the variance of the time to recruitment is obtained using an univariate MAX policy of recruitment. While in Model I, the inter-decision times form an ordinary renewal process, in Models II and III, they form a geometric process and order statistics respectively. Analytical results in closed form for system characteristics are derived.

Keywords
Single grade manpower system, non-instantaneous exits, wastage of manpower, ordinary renewal process, geometric process, order statistics, breakdown threshold with two components, univariate MAX policy of recruitment and variance of time to recruitment.

2010 Mathematics Subject Classification– 90B70, 91B40, 91D35, 60H30.

Introduction

Study of manpower planning for a system with one or more grades is initiated by the authors in [1] and [2] using Markovian and renewal theoretic approach. Wastage due to exit of personnel as a consequence of policy decisions is common in any marketing organization. Wastage of manpower is also observed when personnel working in the system take frequent breaks. Several recruitment policies are suggested in the literature as the factors leading to wastage are unpredictable. In [9], [11], [12] and [16] the authors have obtained variance of time to recruitment for a single grade manpower system with instantaneous exits under different conditions on inter-decision times, threshold using univariate Cum policy of recruitment. In [3], [4], [5], [6] and [7] the just mentioned research work is studied when the manpower system has non-instantaneous exits. In [13], [14] and [15] the authors have studied the problem of time to recruitment for a single grade manpower system with instantaneous exits and obtained the variance of the time to recruitment using univariate Max policy of recruitment which states that *recruitment is done whenever the maximum wastage of manpower exceeds a control limit, known as breakdown threshold*. In [8], the authors have obtained variance of the time to recruitment for a single grade manpower system with non-instantaneous exits when the breakdown threshold has one component. The present paper extends the research work in [8] when the breakdown threshold level for the cumulative wastage in the manpower system is the sum of two components namely an exponential threshold for cumulative wastages due to exits and an exponential threshold for cumulative wastage due to frequent breaks of existing workers.

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MODEL DESCRIPTION AND ANALYSIS FOR MODEL-I

Consider an organization taking policy decisions at random epochs in (0,∞) and at every decision making epoch a random number of persons quit the organization. There is an associated wastage of manpower, if a person quits. It is assumed that the loss of manpower is linear and cumulative. For i=1,2,3…, let $\overline{X_i}$ be continuous random variable representing the maximum exits take place in the first i exit epoch with mean $\frac{1}{\alpha}$ (α>0). The policy decisions which produce depletion of manpower are classified into two types depending upon the intensity of attrition. It is assumed that the first type of policy decisions has high attrition rate λ_1 (λ_1>0) and the second type has low attrition rate λ_2 (λ_2 >0). Let a_1 and (1-a_1) be the proportion of decisions with high and low attrition rate respectively. Let A_n be the time between (n-1)th and nth policy decisions, forming a sequence of independent random variables. Let B_i be the time between (i-1)th and ith exit epochs, forming a sequence of independent and identically distributed random variables with probability distribution function G(.) and density function g(.). Let D_{i+1} be the waiting time upto (i+1)th exit epoch. Let Y be the breakdown threshold for the cumulative wastage of manpower in the organization with probability density function h(.). It is assumed that Y is the sum of the exponential breakdown threshold level Y_1 for cumulative wastage due to exits with mean $\frac{1}{\theta_1}$ $(\theta_1 > 0)$ and exponential threshold Y_2 for cumulative wastage due to frequent breaks of existing workers with mean $\frac{1}{\theta_2}$ $(\theta_2 > 0)$. Let q $(q \neq 0)$ be the probability that every policy decision has exit of personnel. Let $\chi(I)$ be the indicator function of the event I. Let T be the random variable denoting the time to recruitment with mean E(T) and variance V(T).
As in [4] we get,

$$T = \sum_{i=0}^{\infty} D_{i+1}\chi(\overline{X_i} \leq Y < \overline{X_{i+1}}) \quad (1)$$

$$E(T) = \sum_{i=0}^{\infty} (i+1)E(B)P(\overline{X_i} \leq Y < \overline{X_{i+1}}) \quad (2)$$

where $P(\overline{X_i} \leq Y < \overline{X_{i+1}}) = \int_0^{\infty}[M(y)]^i h(y)dy \times \int_0^{\infty} \tilde{M}(y)\, h(y)dy$ (3)

$$E(T) = E(B)\int_0^{\infty} \tilde{M}(y)\, h(y)dy \left[\int_0^{\infty} \left(\tilde{M}(y)\right)^{-2} h(y)dy\right] = E(B)J_1J_2 \quad (4)$$

where $J_1 = \frac{\theta_1}{\alpha+\theta_1}, J_2 = \frac{\theta_1}{\theta_1-2\alpha}$ and $E(B) = \frac{a_1(\lambda_2-\lambda_1)+\lambda_1}{\lambda_1\lambda_2 q}$ (5)

It can be shown from the hypothesis on Y that

$$h(t) = \frac{\theta_1\theta_2}{\theta_1-\theta_2}\left(e^{-\theta_2 t} - e^{-\theta_1 t}\right), \theta_1 > 0, \theta_2 > 0. \quad (6)$$

Proceeding as in [4], from (2), (3), (4), (5) and (6) and on simplification it can be shown that

$$E(T) = \frac{[a_1(\lambda_2-\lambda_1)+\lambda_1](\theta_1\theta_2)^2}{\lambda_1\lambda_2 q(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)} \quad (7)$$

and

$$V(T) = \frac{\begin{matrix}2q(\theta_1\theta_2)^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)(\theta_1-3\alpha)(\theta_2-3\alpha)\left[\left[a_1(\lambda_2^2-\lambda_1^2)+\lambda_1^2\right]-[a_1(\lambda_2-\lambda_1)+\lambda_1]^2\right] \\ +(\theta_1\theta_2)^2[a_1(\lambda_2-\lambda_1)+\lambda_1]^2[2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)^2(\theta_2-2\alpha)^2-(\theta_1\theta_2)^2(\theta_1-3\alpha)(\theta_2-3\alpha)]\end{matrix}}{(\lambda_1\lambda_2 q)^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)^2(\theta_2-2\alpha)^2(\theta_1-3\alpha)(\theta_2-3\alpha)} \quad (8)$$

(7) and (8) give the mean and variance of the time to recruitment for the present Model.

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MODEL DESCRIPTION AND ANALYSIS FOR MODEL-II

In this model, the problem of time to recruitment for a single grade manpower system is analyzed by assuming that the inter-policy decision times form a geometric process with rate c, c>0. It is assumed that distribution of A_1 is hyper-exponential with parameters a_1, λ_1 and λ_2.
In this case from (2), (3), (4) and on simplification we get

$$E(T) = \frac{(\theta_1\theta_2)^2 c[a_1(\lambda_2-\lambda_1)+\lambda_1]}{(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)\lambda_1\lambda_2(c-1+q)} \quad (9)$$

and

$$V(T) = \left\{\frac{c^2[c-1+q]^2[2(a_1\lambda_2^2+a_2\lambda_1^2)-(a_1\lambda_2+a_2\lambda_1)^2]+c^2q(1-q)[a_1(\lambda_2-\lambda_1)+\lambda_1]^2}{[c-1+q]^2[c^2-1+q](\lambda_1\lambda_2)^2}\right\}$$
$$\left[\frac{(\theta_1\theta_2)^2}{(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)}\right] + \left[\frac{c[a_1(\lambda_2-\lambda_1)+\lambda_1]}{\lambda_1\lambda_2[c-1+q]}\right]^2$$
$$\left[\frac{2(\theta_1\theta_2)^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)^2(\theta_2-2\alpha)^2-(\theta_1\theta_2)^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)(\theta_1-3\alpha)(\theta_2-3\alpha)}{(\alpha+\theta_1)^2(\alpha+\theta_2)^2(\theta_1-2\alpha)^2(\theta_2-2\alpha)^2(\theta_1-3\alpha)(\theta_2-3\alpha)} + \frac{(\theta_1\theta_2)^4(\theta_1-3\alpha)(\theta_2-3\alpha)}{}\right] \quad (10)$$

(9) and (10) give the mean and variance of the time to recruitment for the present Model.

MODEL DESCRIPTION AND ANALYSIS FOR MODEL-III

Description of this model is similar to that of Model-I except the assumption on inter-decision times. In this model it is assumed that the inter-policy decision times form an order statistics. More specifically, let $\{U_{(l)}\}^{r}_{l=1}$ be a set of order statistics associated with a sample of size r taken from the population $\{U_n\}^{\infty}_{n=1}$ of independent and identically distributed hyper –exponential random variables with the population mean $E(U_n)= E(U)= \frac{a_1}{\lambda_1}+\frac{(1-a_1)}{\lambda_2}$, n=1,2,…. In this case, the cumulative distribution function S(t) of the population and the probability density function $s_{(l)}(t)$ of the l^{th} order statistics are given by
$S(t)= a_1(1-e^{-\lambda_1 t})+(1- a_1)(1-e^{-\lambda_2 t})$, $a_1 > 0$, $\lambda_1 > 0$ and $\lambda_2 > 0$.
From the theory of order statistics [10], it is known that

$$s_{(l)}(t) = l\binom{r}{l}(S(t))^{l-1}s(t)[1-S(t)]^{r-l}, \; l = 1,2,...,r.$$

In this Model the inter - policy decision times A_n, n=1,2,…. form a sequence of independent and identically distributed random variable with cumulative distribution F(t) and probability density function f(t). By the assumption that the inter-policy decision times form an order statistics we mean that the probability density function f(t) is identified with that of the order statistics $\{U_{(l)}\}^{r}_{l=1}$. Thus f(t) can be any one of the $s_{(l)}(t)$, l =1,2,….,r.
We now determine E(T) and V(T) under two cases.

Case(i): f(t) = $s_{(1)}(t)$.

The probability density function of the first order statistics is

$$s_{(1)}(t) = r[1-S(t)]^{r-1}s(t)$$

From (2),(3),(4) and on simplification it is found that

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$$E(T) = \frac{(\theta_1\theta_2)^2[rk_1+r(1-r)k_2]}{q(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)} \quad (11)$$

where $k_1 = \frac{a((\lambda_2-\lambda_1)+\lambda_1)}{q\lambda_1\lambda_2}$ and $k_2 = \frac{\lambda_1^2\lambda_2(3a_1^2+6a_1-2)-a_1^2\lambda_2^2-\lambda_1\lambda_2^2(2a_1^2-5a_1+1)+4a_1\lambda_2^3+\lambda_1^3(a_1-1)}{4\lambda_1\lambda_2 q(\lambda_1+\lambda_2)^2}$ (12)

and

$$V(T) = \frac{\begin{matrix}rq(\theta_1\theta_2)^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)(\theta_1-3\alpha)(\theta_2-3\alpha)\{(k_3+k_4)-r[k_4+k_1^2+k_5^2(1-2r+r^2)]\}\\+(\theta_1\theta_2)^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)[rk_1+r(1-r)k_2]^2\end{matrix}}{\begin{matrix}[(1-q)(\theta_1-3\alpha)(\theta_2-3\alpha)+[\theta_2(\theta_1-\alpha)-\alpha(\theta_1+\alpha)]]-(\theta_1\theta_2)^4[rk_1+r(1-r)k_2]^2\\q^2(\alpha+\theta_1)^2(\alpha+\theta_2)^2(\theta_1-2\alpha)^2(\theta_2-2\alpha)^2(\theta_1-3\alpha)(\theta_2-3\alpha)\end{matrix}} \quad (13)$$

where K_1 and K_2 are give in (11), $K_3 = \frac{2a_1(\lambda_2^{\,2}-\lambda_1^{\,2})+\lambda_1^{\,2}}{(\lambda_1\lambda_2)^2}$,

$K_4 = \frac{8a_1^{\,2}\lambda_1^{\,3}\lambda_2^{\,2}+[8a_1\lambda_2^{\,2}-a_1^{\,2}\lambda_2^{\,2}+a_1\lambda_1^{\,2}-\lambda_1^{\,2}](\lambda_1+\lambda_2)^3-8a_1\lambda_1^{\,2}\lambda_2^{\,2}(\lambda_1+\lambda_2)(1-\lambda_2)}{4(\lambda_1\lambda_2)^2(\lambda_1+\lambda_2)^3}$

and $K_5 = \frac{\lambda_1^{\,2}\lambda_2[3a_1^{\,2}+4a_1+2(a_1-1)]-\lambda_1\lambda_2^{\,2}[1+2a_1^{\,2}-5a_1]+a_1\lambda_2^{\,2}[4\lambda_2-a_1]+\lambda_1^{\,3}(a_1-1)}{4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2}$ (14)

(11) and (13) give the mean and variance of the time to recruitment for case (i) of the present model.

Case (ii): $f(t) = s_{(r)}(t)$

The probability density function of the r[th] order statistics is

$$s_{(r)}(t) = r[S(t)]^{r-1}s(t)$$

From (2), (3),(4) and on simplification it is found that

$$E(T) = r(\theta_1\theta_2)^2\left[\frac{a_1(\lambda_2-\lambda_1)+\lambda_1}{\lambda_1\lambda_2 q(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)}\right] + +r(1-r)(\theta_1\theta_2)^2\left[\frac{4a_1\lambda_1\lambda_2(1+a_1)-a_1\lambda_2(\lambda_1+\lambda_2)^2-(1+3a_1)\lambda_1(\lambda_1+\lambda_2)^2}{4\lambda_1\lambda_2 q(\lambda_1+\lambda_2)^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)}\right] \quad (15)$$

and

$$V(T) = \frac{\begin{matrix}rq(\theta_1\theta_2)^2(\theta_1-3\alpha)(\theta_2-3\alpha)\{(2k_6+k_7)-r[k_7+k_8^2+k_5^2(1-r)^2]\}\\+(\theta_1\theta_2)^2[rk_1+r(1-r)k_6]^2\{(1-q)(\theta_1-3\alpha)(\theta_2-3\alpha)+[\theta_2(\theta_1-\alpha)-\alpha(\theta_1+\alpha)]\}\end{matrix}}{q^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)(\theta_1-3\alpha)(\theta_2-3\alpha)} -$$
$$\left[\frac{4r(\lambda_1+\lambda_2)^2(\theta_1\theta_2)^2[a_1(\lambda_2-\lambda_1)]-r(1-r)(\theta_1\theta_2)^2[4a_1\lambda_1\lambda_2(1+a_1)-a_1\lambda_2(\lambda_1+\lambda_2)^2]}{4\lambda_1\lambda_2 q(\lambda_1+\lambda_2)^2(\alpha+\theta_1)(\alpha+\theta_2)(\theta_1-2\alpha)(\theta_2-2\alpha)}\right]^2 \quad (16)$$

where K_1, K_2, K_3, K_4, K_5 give (14) and $k_6 = \left[\frac{4\lambda_1\lambda_2(a_1+a_1^2)-a_1\lambda_2(\lambda_1+\lambda_2)^2-\lambda_1(\lambda_1+\lambda_2)^2[1+3a_1]}{4\lambda_1\lambda_2(\lambda_1+\lambda_2)^2}\right]$

$K_7 = \frac{16a_1(1+a_1)\lambda_1^{\,3}\lambda_2^{\,2}-4a_1\lambda_2^{\,2}(\lambda_1+\lambda_2)^3-4\lambda_1^{\,2}(\lambda_1+\lambda_2)^3(a_1+1)}{(\lambda_1+\lambda_2)^3(2\lambda_1)^2(2\lambda_2)^2}$ (17)

(15) and (16) give the mean and variance of the time to recruitment for case (ii) of the present Model.

CONCLUSION

The problem of time to recruitment is studied in a more general setting on policy decisions and breakdown threshold. Consequently, the process of planning of recruitment is more enlightened. Data analysis can be used to test the suitability of distributions assumed in the present work.

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REFERENCES

1. Bartholomew, D.J., *"Statistical Model for Social Processes"*, New York, John Wiley and Sons, 1973.
2. Bartholomew, D.J., and Andrew Forbes, F., *"Statistical Techniques for Manpower Planning"*, New York, John Wiley and Sons, 1979.
3. Devi, A., and Srinivasan, A., *"Variance of Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits."* International Journal of Research in Mathematics & Computation, Vol.2, No.1, 2014, pp.23-27.
4. Devi, A., and Srinivasan, A., *"Probabilistic Analysis on Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits."* International Journal of Revolution in Science and Humanity, Vol.4, No.2, 2014, pp.59-64.
5. Devi, A., and Srinivasan, A., *"Expected Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits Having Inter-Decision Times as an Order Statistics."* Mathematical Science- International Research Journal, Vol.3, No.2, 2014, pp.887-890.
6. Devi, A., and Srinivasan, A., *"A Stochastic Model for Estimation of Variance of Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits."* International Journal of Applied Mathematics and Statistical Sciences, Vol.4, No.3, 2015, pp.1-8.
7. Devi, A., and Srinivasan, A., *"Expected Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits with Inter-Decision Times as Order Statistics."* International Journal of Innovative Research in Science, Engineering and Technology, Vol.4, No.8, 2015, pp.7517-7520.
8. Devi, A., and Srinivasan, A., *"Probabilistic Analysis on Time to Recruitment for a Single Grade Manpower System with Different Epochs for Decisions and Exits having two types of Policy Decisions "* International Journal of Pure and Applied Mathematics, Vol.118, No.6, 2018, pp.455-463.
9. Muthaiyan, A., *"A Study on Stochastic Models in Manpower Planning"*, Ph.D Thesis, Bharathidasan University, Tiruchirappalli, 2010.
10. Sheldon Ross, *"A First Course in Probability"*, Sixth Edition, Published by Pearson Ltd, Indian branch, Patparganj, Delhi, India, 2004.
11. Srinivasan, A., and Jayalakshmi, G., *"Probabilistic Analysis on Time to Recruitment for a Single Grade Manpower System when the Breakdown Threshold has two Components."* International Journal of Scientific Engineering and Applied Science, Vol.1, No.4, 2015, pp.180-183.
12. Sridharan, J., Saivarajan, A., and Srinivasan, A., *"Expected Time to Recruitment in a Single Graded Manpower System with Two Thresholds"*, Journal of Advances in Mathematics, Vol.7, 2014, pp.1147-1157.
13. B.Venkatesh and A.Srinivasan, Mean and variance of time to recruitment in a single graded manpower system using an univariate policy of recruitment involving a constant threshold, *Acta Ciencia Indica*, 33 M (4) (2007) 1671-1674.
14. B.Venkatesh and A.Srinivasan, Mean and variance of time to recruitment in a single graded manpower system using an univariate policy of recruitment when the threshold distribution has SCBZ property, *Acta Ciencia Indica*, 33 M(4) (2008) 1679-1682.
15. B.Venkatesh and A.Srinivasan, Mean and variance of time to recruitment in a single graded manpower system using an univariate policy of recruitment when the threshold distribution is continuous, *ActaCienciaIndica*, 33 M (4) (2007) 1675-1678.
16. Vijayasankar, N., Elangovan, R., and Sathiyamoorthi, R., *"Determination of Expected Time to Recruitment when Backup Resources of Manpower Exits."* Ultra Scientist, Vol.25, No.1, 2013, pp.61-68.

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Some results on Fixed points for generalized (α,ϕ,ψ) contractive multifunctions in symmetric spaces

Abstract: Recently Abdelbasset Felhi introduced a class of generalized (α,ϕ,ψ) proximal contraction non-self maps in semi-metric spaces and gave some results on best proximity points and fixed points. In this paper, we used this generalized (α,ϕ,ψ) contraction for multivalued maps in symmetric spaces for the existence of fixed points and some related results for self maps. We provide sufficient conditions for the existence and uniqueness of fixed points by using the concept of α- admissible.

Key words: Symmetric spaces, multivalued maps, Hansdorff metric, fixed point,α- admissible .

I. Introduction

Semi-metric spaces were considered by many authors like Frechet[7],Menger[12]and Wilson[19] as a generalization of metric spaces.After that some fixed point results for semi-metric spaces have been investigated in [1],[3]-[15].

The contraction is one of the important tool to prove the existence and uniqueness of a fixed point. Banach contraction principle is one of the most fascinating and classical result of the last century in the field of non linear analysis. Following Banach contraction mapping Nadler [16] introduced the concept of multivalued contraction mapping and established that a multivalued contraction mapping possesses a fixed point in a complete metric space. There are so many fixed point theorems for multivalued mappings in metric spaces satisfying contractive type conditions.

On the other hand, Hicks [10], and Hicks and Rhoades [11] started the study of existence of fixed points in symmetric spaces.

Samet, Vetro and Vetro [17] introduced the notion of α-ψ-contractive type mappings and established some fixed point theorems in complete metric spaces. Mohammadi, Rezapour, Shahzad gave some new results for α-ψ-ciric generalized multifunctions.

Abdelbasset Felhi [2] introduced generalized (α,ϕ,ψ) contractions for non-se3lf maps in semi-metric spaces for the existence and uniqueness of best proximity points. The aim of this paper is to establish some fixed points theorems for multivalued mappings using the generalized (α,ϕ,ψ) contractions which was introduced by Abdelbasset Felhi[2].

Our results generalize and improve various known results from fixed point theory.

II. Preliminaries

1. *Definition* [11]: A symmetric o a set X is a non negative real valued function d on X × X such that (i) d(x,y) = 0 iff x=y (ii) d(x,y)= d(y,x)

   Let d be a symmetric on a set X and for r>0 and any x∈X, let B(x,r) = {y∈ X: d(x,y)< r}. A topology t(d) on X is given by U ∈ t(d) if and only if for each x∈U, B(x,r)⊂U for some r > 0.
2. *Definition [11]:* A symmetric d is said to be semi-metric if for each x∈X and each r > 0, B(x, r) is a neighbourhood of x in the topology t(d) . Note that $\lim_{n\to\infty} d(x_n, x) = 0$ if and only if $x_n \to x$ in the topology t(d) or τ_d

   We need the following two axioms (W.3) and (W.4) given by Wilson[19] in a symmetric space (X,d).

   (W.3) Given $\{x_n\}$, x and y in X , $\lim_{n\to\infty} d(x_n, x) = 0$ and $\lim_{n\to\infty} d(x_n, y) = 0$ imply x=y.

   (W.4) Given $\{x_n\}$, $\{y_n\}$ and x in X, $\lim_{n\to\infty} d(x_n, x) = 0$ and $\lim_{n\to\infty} d(x_n, y_n) = 0$ imply that $\lim_{n\to\infty} d(y_n, x) = 0$
3. *Proposition [11]:* Let (x, d) be a symmetric space. Then (x, d) is semi-metric space if and only if the following conditions hold:

   (1) (x, τ_d) is first countable

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(2) For any sequence $\{x_n\}$ in X, $d(x_n, x) \to 0$ is equivalent to $x_n \to x$ in the topology τ_d.

4. *Definition[9,11]:* Let (X,d) be a symmetric space and $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ is d-cauchy sequence if and only if $\lim_{n,m\to\infty} d(x_n, x_m) = 0$. Furthermore, (X, d) is said to be d-cauchy complete if every d- Cauchy sequence converges to some $x \in X$ in τ_d.

It is easy to see that for a semi-metric d, if τ_d is Hausdroff, then (W.3) holds. Let (X, d) be a symmetric space. CB(X) denotes the collection of all closed bounded subsets of X. For any $x \in X$ and A is a non-empty subset of X. $d(x, A) = \inf\{d(x, y) : y \in A)$ and $H(A, B) = \max\left\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\right\}$.

H is known as the Hausdroff metric introduced by d on CB(x)[16].Further, if (x,d) is complete (CB(X),H) is also complete.

5. *Definition:* Let (X,d) be a symmetric space and A is a non empty subset of X.
   1. We say that A is d-closed if and only if $\overline{A}^d = A$ where $\overline{A}^d = \{x \in X : d(x, A) = 0\}$ and $d(x, A) = \inf\{d(x, y) : y \in A\}$
   2. We say that A is d-bounded if and only if $\delta_d(A) = \sup\{d(x, y) : x, y \in A\}$ .For main theorem we need the following lemma.
6. *Lemma[6]*: Let (X,d) be a d-bounded symmetric space. Let $A,B \in CB(X)$ and q>1 . For each $x\in A$, there exists $y\in B$ such that $d(x,y)\leq qH(A,B)$.
7. *Definition*: Let (X, d) be a symmetric space. We say that (X,d) satisfies the property (WC) if for all sequences $\{x_n\}$, $\{y_n\}$ in X and all $x,y\in X$ such that $\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(y_n, y) = 0$, one has $d(x, y) \leq \liminf_{n\to\infty} d(x_n, y_n)$.
8. *Remark:*
   1. If (X,d) be a symmetric space satisfying the property (WC), then it is also satisfying the Fatou property.
   2. Each metric space satisfies the property (WC).
9. *Definition [17]*: Let (X,d) be a symmetric space. Let $T : X \to 2^X$ be a multivalued function then T is said to be α-admissible whenever $\alpha(x, y) \geq 1$ implies $\alpha(Tx, Ty) \geq 1$ for all $x,y\in X$.
10. *Definition [17]*: Let (X,d) be a symmetric space and $\alpha : X\times X \to [0,\infty)$. A mapping $T : X \to 2^X$ is said to be triangular α-admissible if (T1) T is α-admissible, (T2) $\alpha(x, y) \geq 1$ and $\alpha(y, z) \geq 1 \Rightarrow \alpha(x, z) \geq 1$, for all x,y,z in X .

*Definition[18]:*Denote by ψ the set of functions $\psi : [0,\infty) \to [0,\infty)$ satisfying (ψ1) ψ is non-decreasing; (ψ2) $\lim_{n\to\infty} \psi^n(t) = 0$ for each t>0, where ψ^n is the n-th iterate of ψ.Also denote by ϕ the set of functions $\phi : [0,\infty) \to [0,\infty)$ satisfying (ϕ1) ϕ is non-decreasing; (ϕ2) $\phi^{-1}(\{0\}) = \{0\}$ and $\lim_{n\to 0^+} \phi(x) = 0$

11. *Lemma [18]*: If $\psi\in\psi$, then $\psi(t)<t$ for all t > 0, ψ is continuous at 0 and $\psi(0)=0$.
12. *Lemma [2]*: Let (X,d) be a symmetric space and $\phi\in\Phi$. Consider the function $\phi\circ d : X\times X \to [0,\infty)$ defines as follows $\phi\circ d(x, y) = \phi(d(x, y))$ for all $x,y\in X$. Then (X, $\phi\circ d$) is also a symmetric space.

Using the definiton of (α,ϕ,ψ) contraction given in [2],we extend it to the multivaued maps.

13. *Definition*: Let (X,d) be a symmetric space, let $T : X \to CB(X)$, $\phi\in\Phi$, $\psi\in\Psi$ and $\alpha : X\times X \to [0,\infty)$ .Then T is generalized (α,ϕ,ψ) contractive multifunction if $\alpha(x,y)\geq 1$ $\Rightarrow \phi(H(Tx, Ty)) \leq \psi(\phi(d(x, y)))$ for $x,y \in X$.

Using the above definiton ,we extend and generalize the following map.

14. *Definition*: Let (X,d) be a symmetric space, let $T : X \to CB(X)$, $\phi\in\Phi$, $\psi\in\Psi$ and $\alpha : X\times X \to [0,\infty)$ .Then T is generalized (α,ϕ,ψ) contractive multifunction if $\alpha(x,y)\geq 1$

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$$\Rightarrow \phi(H(Tx,Ty)) \le \psi\left(\max\left\{ \begin{matrix} \phi\circ d(x,y), \phi\circ d(x,Tx), \phi\circ d(y,Ty), \\ \frac{1}{2}\left[\phi\circ d(x,Ty) + \phi\circ d(y,Tx)\right] \end{matrix} \right\} \right) \quad \text{for } x,y\in X.$$

Now we state and prove our main results. In the following result, we use the argument similar to that in the corollary 3.5[2].The main results consist of the existence and uniqueness of fixed point for generalized (α,ϕ,ψ) contraction multivalued maps in semi-metric spaces.

III. Main results

Theorem A:

Let (X, d) be d-bounded and S-complete semi-metric space, $\alpha : X\times X \to [0,\infty)$ be a function , $\phi \in \Phi$ and $\psi \in \Psi$ and $T : X \to CB(X)$ be a closed-valued multifunction,triangular α-admissible and (α,ϕ, ψ) contractive multifunction on X such that $\phi(H(Tx,Ty)) \le \psi(\phi(d(x,y)))$ for all $x, y \in X$. Suppose that there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \ge 1$ Assume that if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \ge 1$ for all n and $x_n \to x$, then, $\alpha(x_n, x) \ge 1$ for all n. Then T has a fixed point.

Proof:

Let $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \ge 1 \Rightarrow \alpha(Tx_0, Tx_1) \ge 1$

Define the sequence $\{x_n\}$ in x by $x_{n+1} = Tx_n$ for all n≥0. So $\alpha(x_n, x_{n+1}) \ge 1$

Since T is triangular α-admissible then $\alpha(x_n, x_{n+1}) \ge 1$ and $\alpha(x_{n+1}, x_{n+2}) \ge 1$

$\Rightarrow \alpha(x_n, x_{n+2}) \ge 1$

$\Rightarrow \alpha(Tx_n, Tx_{n+2}) \ge 1$

Then by induction, we get $\alpha(x_n, x_m) \ge 1$ for all m>n≥0.So $\alpha(Tx_n, Tx_m) \ge 1$

For all n=0, 1, 2… we denote $\delta_n = \sup_{j,k\in N} \phi(d(x_{n+j}, x_{n+k}))$

Since X is d-bounded and the fact that ϕ is non-decreasing function, we have $\delta_n < \infty$, for all n=0,1,2,…

By continuing this process, $d(x_{n+j}, Tx_{n+j-1}) = 0$, $d(x_{n+k}, Tx_{n+k-1}) = 0$ for all n,j,k∈N.It follows ,

$$\begin{aligned} d(x_{n+j}, x_{n+k}) = d(Tx_{n+j-1}, Tx_{n+k-1}) &\le \phi(H(Tx_{n+j-1}, Tx_{n+k-1})) \\ &\le \psi(\phi\circ d(x_{n+j-1}, x_{n+k-1})) \\ &= \psi(\delta_{n-1}) \end{aligned}$$

$d(x_{n+j}, x_{n+k}) \le \phi\circ H(Tx_{n+j-1}, Tx_{n+k-1}) \le \psi(\delta_{n-1})$ for all j<k.

Since ψ is non-decreasing function then $d(x_{n+j}, x_{n+k}) \le \phi\circ H(Tx_{n+j-1}, Tx_{n+k-1}) \le \psi(\delta_{n-1})$ for all j<k.

By symmetry of d, we get $d(x_{n+j}, x_{n+k}) \le \phi\circ H(Tx_{n+j-1}, Tx_{n+k-1}) \le \psi(\delta_{n-1})$ for all j>k.

Also for j = k, we have $d(x_{n+j}, x_{n+k}) \le \phi\circ H(Tx_{n+j-1}, Tx_{n+k-1}) = \phi(0) = 0 \le \psi(\delta_{n-1})$ Thus $d(x_{n+j}, x_{n+k}) \le \phi\circ H(Tx_{n+j-1}, Tx_{n+k-1}) \le \psi(\delta_{n-1})$ for all j,k∈N.

So we have $\delta_n = \psi^n(\delta_0)$ for all n∈N .

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Now we have $d(x_n, x_m) \le \phi(H(Tx_{n-1}, Tx_{m-1}) \le \delta_{n-1} \le \psi^{n-1}(\delta_0)$ for all n,m≥1.

This implies that $\lim_{n\to\infty} d(x_n, x_m) = 0$ which shows that $\{x_n\}$ is a d- Cauchy sequence in X Since X is S-complete, $x_n \to x^*$ for some $x^* \in X$. Since $\alpha(x_n, x^*) \ge 1$ for all n which implies that $\alpha(Tx_n, Tx^*) \ge 1$ for all n, thus $d(x_{n+1}, Tx^*) = d(Tx_n, Tx^*) \le \phi(H(Tx_n, Tx^*)) \le \psi(\phi(d(x_n, x^*)))$ Letting $n\to\infty$, we get $d(x^*, Tx^*) = 0$. So T has a fixed point.

Theorem B:

Let (X, d) be d-bounded and S-complete symmetric space, $\alpha : X \times X \to [0,\infty)$ be a function , $\phi \in \Phi$ and $\psi \in \Psi$ and $T : X \to CB(X)$ be a closed-valued multifunction,triangular α-admissible and (α,ϕ, ψ) contractive multifunction on X such that $\phi(H(Tx, Ty)) \le \psi(\phi(d(x, y)))$ for all $x, y \in X$. Suppose that there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \ge 1$. If T is τ_d - continuous and (X, d) satisfies the property (WC), then T has a fixed point.

Proof:

Following the proof of theorem A, there exists a d- Cauchy sequence $\{x_n\}$ in X and since X is S-complete, there exists $x^* \in X$ as $n\to\infty$ in the topology τ_d .

Since T is τ_d -continuous, then $Tx_n = Tx^*$ in τ_d and so $\lim_{n\to\infty} d(Tx_n, Tx^*) = 0$. Since (X, d) satisfies the property (WC), we have $d(x^*, Tx^*) \le \liminf_{n\to\infty} d(x_{n+1}, Tx_n) = 0$ which implies that $d(x^*, Tx^*) = 0$.

∴ T has a fixed point in X.

Theorem C:

Let (X, d) be d-bounded and S-complete symmetric space satisfying (W.4), $\alpha : X \times X \to [0,\infty)$ be a function , $\phi \in \Phi$ and $\psi \in \Psi$ and $T : X \to CB(X)$ be a closed-valued multifunction,triangular α-admissible and (α,ϕ, ψ) contractive multifunction on X such that $\phi(H(Tx, Ty)) \le \psi(\phi(d(x, y)))$ for all $x, y \in X$. Suppose that there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \ge 1$. Then T has a fixed point.

Proof:

Following the proof of theorem A, there exists a d- Cauchy sequence $\{x_n\}$ in X and since X is S-complete, there exists $x^* \in X$ as $n\to\infty$ in the topology τ_d .

Let $\varepsilon > 1$.From Lemma 6 , for each $n \in \{1,2,\ldots\}$ there exists $y_n \in Tx^*$ such that

$$d(x_{n+1}, y_n) \le \varepsilon H(Tx_n, Tx^*)$$

$$\le \varepsilon\psi(\phi(d(x_n, x^*))), n = 1, 2, \ldots$$

Which implies that $\lim_{n\to\infty} d(x_{n+1}, y_n) = 0$. In view of (W.4), we have $\lim_{n\to\infty} d(y_n, x^*) = 0$ and therefore $x^* \in \overline{Tx^*}^d = Tx^*$. So $x^* = Tx^*$. Therefore T has a fixed point.

Example:

Let $X = [0,\infty)$, $d(x, y) = (x - y)^2$ and $\delta \in (0,1)$ be a fixed numbers. Define $T : X \to 2^X$ by $Tx = [0, \delta x]$ for all $x \in X$ and $\alpha : X \times X \to [0,\infty)$ by α(x, y)=1 whenever $x, y \in [0,1]$ and α(x, y)=0 whenever $x \notin [0, 1]$ or $y \notin [0, 1]$. Now, we show that T is α- admissible. If α(x, y)≥1, then x,

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$y \in [0, 1]$ and so Tx and Ty are subsets of [0, 1]. Thus $a, b \in [0, 1]$ for all $a \in Tx$ and $b \in Ty$. This implies that $\alpha(Tx, Ty) = \inf\{\alpha(a, b) : a \in Tx, b \in Ty\} = 1$. Therefore, T is α - admissible. Now we show that T is an (α, ϕ, ψ) contractive multifunction, where $\psi(t)=\delta t$ for all $t \geq 0$ and $\phi(t) = \sqrt{t}$ for all $t \geq 0$. If $x \notin \left[0, \frac{1}{\delta}\right]$ or $y \notin \left[0, \frac{1}{\delta}\right]$, then it is easy to show that $\alpha(Tx, Ty) = 0$. If $0 \leq x, y \leq \frac{1}{\delta,}$ then $\alpha(Tx, Ty) = 1$ By using the definition of the Hansdorff metric, it is easy to see that $H(Tx, Ty) \leq \delta d(x, y)$ for $x, y \in \left[0, \frac{1}{\delta}\right]$. Thus, $\phi(H(Tx, Ty)) \leq \psi(\phi(d(x, y)))$ for all $x, y \in X$. Therefore, T is an (α, ϕ, ψ) contractive multifunction.

Theorem D:

Let (X, d) be a d-bounded and S-complete symmetric space, $\alpha : X \times X \to [0, \infty)$ be a function, $\phi \in \Phi$ and $\psi \in \Psi$, $T : X \to CB(X)$, be a closed-valued multifunction,triangular α-admissible and (α, ϕ, ψ) contractive multifunction on X such that

$$\phi(H(Tx, Ty)) \leq \psi\left(\max\left\{\begin{matrix}\phi \circ d(x, y), \phi \circ d(x, Tx), \phi \circ d(y, Ty), \\ \frac{1}{2}[\phi \circ d(x, Ty) + \phi \circ d(y, Tx)]\end{matrix}\right\}\right)$$

for all $x, y \in X$ satisfying $\alpha(x, y) \geq 1$. Suppose the following conditions hold:

1. T is triangular α-admissible.
2. There exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$.
3. Assume that if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \geq 0$ and $\lim_{n \to \infty} d(x_n, x) = 0$, then $\alpha(x_n, x) \geq 1$ for all $n \geq 0$.

Then T has fixed point in X..

Proof:

Let $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1 \Rightarrow \alpha(Tx_0, Tx_1) \geq 1$

Define the sequence $\{x_n\}$ in x by $x_{n+1} = Tx_n$ for all $n \geq 0$. So $\alpha(x_n, x_{n+1}) \geq 1$

Since T is triangular α-admissible then $\alpha(x_n, x_{n+1}) \geq 1$ and $\alpha(x_{n+1}+, x_{n+2}) \geq 1$

$\Rightarrow \alpha(x_n, x_{n+2}) \geq 1$

$\Rightarrow \alpha(Tx_n, Tx_{n+2}) \geq 1$

Then by induction, we get $\alpha(x_n, x_m) \geq 1$ for all $m > n \geq 0$.So $\alpha(Tx_n, Tx_m) \geq 1$

For all n=0, 1, 2… we denote $\delta_n = \sup_{j,k \in N} \phi(d(x_{n+j}, x_{n+k}))$

Since X is d-bounded and the fact that ϕ is non-decreasing function, we have $\delta_n < \infty$, for all n=0,1,2,…

By continuing this process, $d(x_{n+j}, Tx_{n+j-1}) = 0$, $d(x_{n+k}, Tx_{n+k-1}) = 0$ for all $n, j, k \in N$.It follows ,

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$$d(x_{n+j},x_{n+k}) = d(Tx_{n+j-1},Tx_{n+k-1}) \le \phi(H(Tx_{n+j-1},Tx_{n+k-1}))$$
$$\le \psi\left(\max\left\{\begin{matrix}\phi\circ d(x_{n+j-1},x_{n+k-1}),\phi\circ d(x_{n+j-1},Tx_{n+j-1}),\phi\circ d(x_{n+k-1},x_{n+k-1}),\\ \frac{1}{2}\left[\phi\circ d(x_{n+j-1},Tx_{n+k-1})+\phi\circ d(x_{n+k-1},Tx_{n+j-1})\right]\end{matrix}\right\}\right)$$
$$= \psi\left(\max\left\{\begin{matrix}\phi\circ d(x_{n+j-1},x_{n+k-1}),\phi\circ d(x_{n+j-1},x_{n+j}),\phi\circ d(x_{n+k-1},x_{n+k}),\\ \frac{1}{2}\left[\phi\circ d(x_{n+j-1},x_{n+k})+\phi\circ d(x_{n+k-1},x_{n+j})\right]\end{matrix}\right\}\right)$$

$d(x_{n+j},x_{n+k}) \le \phi\circ H(Tx_{n+j-1},Tx_{n+k-1}) \le \psi(\delta_{n-1})$ for all j<k.

Since ψ is non-decreasing function then $d(x_{n+j},x_{n+k}) \le \phi\circ H(Tx_{n+j-1},Tx_{n+k-1}) \le \psi(\delta_{n-1})$ for all j<k.

By symmetry of d, we get $d(x_{n+j},x_{n+k}) \le \phi\circ H(Tx_{n+j-1},Tx_{n+k-1}) \le \psi(\delta_{n-1})$ for all j>k.

Also for j = k, we have $d(x_{n+j},x_{n+k}) \le \phi\circ H(Tx_{n+j-1},Tx_{n+k-1}) = \phi(0) = 0 \le \psi(\delta_{n-1})$ Thus $d(x_{n+j},x_{n+k}) \le \phi\circ H(Tx_{n+j-1},Tx_{n+k-1}) \le \psi(\delta_{n-1})$ for all j,k∈N.

So we have $\delta_n = \psi^n(\delta_0)$ for all n∈N .

Now we have $d(x_n,x_m) \le \phi(H(Tx_{n-1},Tx_{m-1}) \le \delta_{n-1} \le \psi^{n-1}(\delta_0)$ for all n,m≥1.

This implies that $\lim_{n\to\infty} d(x_n,x_m) = 0$ which shows that $\{x_n\}$ is a d- Cauchy sequence in X.Since X is S-complete, $x_n \to x^*$ for some $x^* \in X$. Since $\alpha(x_n,x^*) \ge 1$ for all n which implies that $\alpha(Tx_n,Tx^*) \ge 1$ for all n, thus $d(x_{n+1},Tx^*) = d(Tx_n,Tx^*) \le \phi(H(Tx_n,Tx^*)) \le \psi(\phi(d(x_n,x^*)))$ Letting $n\to\infty$, we get $d(x^*,Tx^*) = 0$. So T has a fixed point.

Theorem E:

Let (X,d) be a d-bounded and S-complete symmetric space and $\phi \in \Phi, \psi \in \Psi$ and $\alpha : X\times X \to [0,\infty)$, $T : X \to CB(X)$ such that

$$\phi\circ H(Tx,Ty) \le \psi\left(\max\left\{\begin{matrix}\phi\circ d(x,y),\phi\circ d(x,Tx),\phi\circ d(y,Ty),\\ \frac{1}{2}[\phi\circ d(x,Ty)+\phi\circ d(y,Tx)]\end{matrix}\right\}\right)$$ for all x,y∈X satisfying α(x,y)≥1. Suppose the following conditions hold:

1. T is triangular α-admissible.
2. There exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \ge 1$.
3. T is τ_d continuous.
4. (X, d) satisfies the property (WC).

Then T has fixed point in X.

Proof:

Following the proof of theorem C, there exists a d- Cauchy sequence $\{x_n\}$ in X . Since X is S-complete, there exists $x^* \in X$ as $n\to\infty$ in the topology τ_d .

Since T is τ_d -continuous, then $Tx_n = Tx^*$ in τ_d and so $\lim_{n\to\infty} d(Tx_n,Tx^*) = 0$. Since (X, d) satisfies the property (WC), we have $d(x^*,Tx^*) \le \liminf_{n\to\infty} d(x_{n+1},Tx_n) = 0$ which implies that $d(x^*,Tx^*) = 0$.

∴ T has a fixed point in X.

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Now we prove the uniqueness of the fixed pont in the above theorem. For this, we denote the set of fixed points of T by Fix(T).

Theorem F:

Assume that all the hypothesis of Theorem A,B,C,D and E hold. Also suppose $\forall x, y \in \mathrm{Fix}(T)$, there exists $z \in X$ such that $\alpha(x,z) \geq 1, \alpha(y,z) \geq 1$ holds, then the fixed point of T is unique.

Proof:

Suppose there exists u, $w \in X$ such that $d(u., Tu) = d(w, Tw) = 0$. Now by the assumption, we have $\alpha(u,w) \geq 1$, it follows $\alpha(Tu, Tw) \geq 1$.Then

$$\phi(H(Tu,Tw)) = \phi(H(u,w))$$
$$\leq \psi\left(\max\left\{\begin{matrix}\phi\circ d(u,w), \phi\circ d(u,u), \phi\circ d(w,w),\\ \frac{1}{2}\left[\phi\circ d(u,w) + \phi\circ d(w,u)\right]\end{matrix}\right\}\right)$$
$$= \psi\left(\max\left\{\phi\circ d(u,w), \phi(0)\right\}\right) = \psi\left(\phi\circ d(u,w)\right)$$

Which implies that $\phi\circ d(u,w) = 0$ and so u=w.

Corollary G:

Let (X,d) be a d-bounded and S-complete symmetric space satisfying (W4) and $T: X \to CB(X)$, $\phi \in \Phi$, $\psi \in \Psi$ and $\alpha: X \times X \to [0,\infty)$ such that $\phi(H(Tx,Ty)) \leq \psi\left(\max\left\{\begin{matrix}\phi\circ d(x,y), \phi\circ d(x,Tx), \phi\circ d(y,Ty),\\ \frac{1}{2}\left[\phi\circ d(x,Ty) + \phi\circ d(y,Tx)\right]\end{matrix}\right\}\right)$ for all x,y∈X.Suppose that there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$ Then T has fixed point in X..

Corollary H:

Let (X,d) be a d-bounded and S-complete symmetric space and $\alpha: X \times X \to [0,\infty)$, $T: X \to CB(X)$, $\psi \in \Psi$ such that $H(Tx,Ty) \leq \psi\left(\max\left\{\begin{matrix}d(x,y), d(x,Tx), d(y,Ty),\\ \frac{1}{2}\left[d(x,Ty) + d(y,Tx)\right]\end{matrix}\right\}\right)$ for all x,y∈X. Suppose that there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$. If T is τ_d continuous then T has fixed point of X.

Corollary I:

Let (X,d) be a d-bounded and S-complete symmetric space satisfying (W4) and $\phi \in \Phi$, $\psi \in \Psi$ and $T: X \to CB(X)$, such that $\phi(H(Tx,Ty)) \leq \psi\left(\max\left\{\begin{matrix}\phi(d(x,y)), \phi(d(x,Tx)), \phi(d(y,Ty)),\\ \frac{1}{2}\left[\phi(d(x,Ty)) + \phi(d(y,Tx))\right]\end{matrix}\right\}\right)$ for all x,y∈X. Then T has fixed point in X.

Proof: It suffices to take $\alpha(x,y) = 1$ in Theorem D.The uniqueness of z holds since the condition in Theorem F is satisfied.

Corollary J:

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Let (X,d) be a d-bounded and S-complete symmetric space satisfying (W4) and $T: X \to X$, $\phi \in \Phi$, $\psi \in \Psi$ and $\alpha: X \times X \to [0,\infty)$ such that $\phi(d(Tx,Ty)) \le \psi\left(\max\left\{\phi\circ d(x,y), \phi\circ d(x,Tx), \phi\circ d(y,Ty), \frac{1}{2}[\phi\circ d(x,Ty)+\phi\circ d(y,Tx)]\right\}\right)$ for all x,y∈X.Suppose that there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0,x_1) \ge 1$ and T is triangular α-admissible . Then T has fixed point in X.

Corollary K :

Let (X,d) be a d-bounded and S-complete symmetric space and $T: X \to X$, $\psi \in \Psi$ such that $d(Tx,Ty) \le \psi\left(\max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{1}{2}[d(x,Ty)+d(y,Tx)]\right\}\right)$ for all x,y∈X. If T is τ_d continuous then T has fixed point of X.

IV. Conclusion:

Receently many results appeared in the literature giving the problems related to the fixed point for multivalued maps.In this paper we obtained the results for existence of the fixed points of multivalued maps that satisfying a generalized contractive conditions.As a consequence we obtained some fixed point for multivalued contraction. We present an example and some corollaries.

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V. References

[1] Aamri, M and Mountawakil, D.El. *Common fixed points under contractive conditions in symmetric space*, Appl.Math.E-notes,3, 156-162,(2003).
[2] Abdelbasset Felhi, *Best proximity point for generalized (α, ϕ, ψ)- proximal contractions on Semi-metric spaces*, Facta universities (NIS), ser.Math.Inform. Vol-32, N05, 687-702,2017.
[3] Aliouche, A., *A common fixed pont theorem for weakly compatible mappings in symmetric spaces satisfying a contractive condition of integral type*, Journal of Mathematical Analysis and Applications, Vol.322, n0.2, pp.796-802,(2006),.
[4] Cho,Y.J, Sharma, B.K and Sahu, D.R., (1995),*Semi-compatibility, and fixed points*, Math Japonica ,42, 91-98,(1995).
[5] Cicchese, M.,*Questioni di comptetezza e contrazioni in spazimetrici generalization*, Boll. Un. Math. Italy., 13-A 5, 175-179,(1976).
[6] Driss El Moutawakil, *A fixed point theorem for multivalued maps in symmetric spaces*, Applied Mathematics E-notes, 26-32, 4(2004).
[7] Frechet, M., *Sur quelques points du calculfunctionnel*, Rend. Circ. Mat. Palermo 22 1-74,(1906).
[8] Imdad, M., Ali, J., and Khan, l., *Coincidence and fixed points in symmetric spaces under strict contractions*, Jouranal of Mathematical Analysis and Applications, Vol.320, no.1, pp.352-360,(2006).Corrections J.Math.Anal.Appl.329,752,2007.
[9] .J. Jachymski, J .Matkowski, T. Swiatkowski, *Nonlinear contractions on semi metric spaces*. J. Appl. Anal. 125-134, 1m 1995.
[10] T.L.Hicks, *Fixed point theorems for multivalued mappings* II, Indian J.Pure Appl.Math, 133-137, 29(2), 1998
[11] T.L.Hicks,B.E.Rhoades,*Fixed point theory in symmetric spaces with application to probablistic spaces*,Non linear Analysis,36(1999),331-344,(1999)
[12] K.Menger:Untersuchungen ber allgemeiine.Math.Annalen 100,75-163,1928.
[13] B.E.Rhoades, *Proving fixed point theorems using general principles*, Indian J. Pure Appl. Math, 21(8), 741-770, 1996.
[14] R.P.Pant,V.Pant,*Common fixed points under strict contractive conditions*,Journal of Mathematical Analysis and Applications,248(1),327-332,(2000).
[15] R.P.Pant,V.Pant,*Fixed points in fuzzy metric space for noncompatible maps*,Soochow J.Maths,33(4),647-655,(2007).
[16] S.B Nadler Jr., *Multivalued contraction mappings*, Pacific J.Math, 639-640,30,1969.

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[17] B. Samet, C. Vetro, P. Vetro, *Fixed point theorems for α-ψ-contractive type mappings*, Non linear Anal. 75, 2154-2165, 2012.
[18] I. A. Rus, A. Petrusel, G. Petrusel, Fixed point theory, Cluj University Press, Cluj-Napoca, 1.3, 1, 2008.
[19] W.A. Wilson, *On semi metric spaces*, Amer.J.Math., 53, 361-363, 1931.

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Fuzzy Inventory Model without Shortage Using Octagonal Fuzzy Numbers

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Abstract:
This paper proposed an inventory model with the concept of without shortages along with the fuzzy concept and using the new octagonal fuzzy numbers. The main objective of this paper is to determine the fuzzy optimal order quantity and fuzzy minimum total cost for the proposed an inventory model. The octagonal fuzzy numbers are introduced to achieve this objective. The carrying cost and ordering cost are computed in terms of octagonal fuzzy numbers. A signed distance method was introduced for defuzzifying the fuzzy total cost and fuzzy optimal order quantity. Mathematical model and some numerical examples are given to illustrate for both the crisp and fuzzy model.

Keywords : octagonal fuzzy number, α – cut of an octagonal fuzzy number, signed distance method

Introduction

An inventory consists of raw materials, work-in-progress, or finished goods. The inventory system is taking an important part of cost controlling in business. Effective inventory control is essential for manufacturing organizations for many reasons. Thus, inventory control is very important field for both real world applications and research purpose.

Fuzzy mathematical programming has been applicable to several fields like project network, reliability, optimization, transportation, media selection for advertising, air pollution etc., The first quantitative treatment of inventory was the simple EOQ model. In 1915, the first inventory model was developed by Harris [4]. The most widely used inventory model is the Economic order quantity (EOQ) model, in which the successive operations are classified as supply and demand. This model was developed by Harris [4], Wilson [9] interest in the EOQ model in academics and industries. Many applications of fuzzy set theory can be found in Zimmerman [8].

Later in 1965, first time the concept of fuzzy sets was introduced by Zadeh [10]. Fuzzy set theory is an extension of classical set theory where elements have degrees of membership. The theory set attracted the attention of many researches. In 1987, Park [6] proposed the model on fuzzy set theoretical interpretation of economic order quantity inventory problem. In 2012, Dutta and Pavankumar [1] presented a fuzzy inventory model without shortage using trapezoidal fuzzy numbers with sensitivity analysis. In 2014, Stephen and Rajesh [7] have developed an optimal total cost and optimal order quantity for fuzzy inventory model without shortage using Hexagonal fuzzy numbers. In 2015, Stephen and Rajesh [13] introduced inventory model with allowable shortage using LR – type Hexagonal fuzzy numbers.

The model developed by S. Gajalakshmi et.al [3] that they introduced Octagonal fuzzy numbers for inventory and solved using signed defuzzification. The EOQ model developed by A.Farithaasma, C.Manjula [2],that they optimizing Octagonal fuzzy number using nearest interval approximation method.

2. Preliminaries

Definition 2.1 :

A fuzzy number $\widetilde{A}$ is said to be linear octagonal fuzzy number or simply an octagonal fuzzy number whose membership function $\mu_{\widetilde{A}}$ is given by

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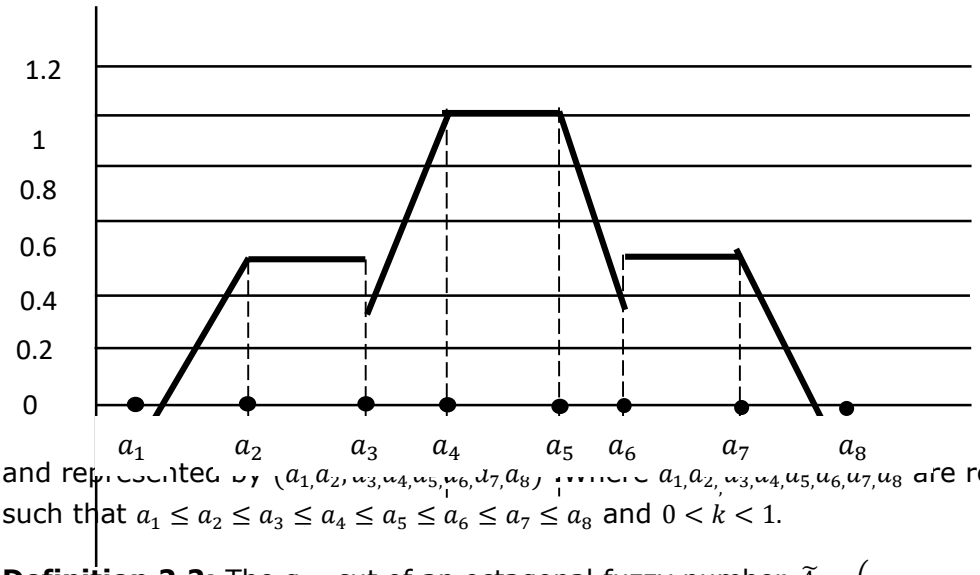
$$\mu_{\tilde{A}}(x) = \begin{cases} k\left(\frac{x-a_1}{a_2-a_1}\right) & a_1 \le x \le a_2 \\ k & a_2 \le x \le a_3 \\ k+(1-k)\left(\frac{x-a_3}{a_4-a_3}\right) & a_3 \le x \le a_4 \\ 1 & a_4 \le x \le a_5 \\ k+(1-k)\left(\frac{a_6-x}{a_6-a_5}\right) & a_5 \le x \le a_6 \\ k & a_6 \le x \le a_7 \\ k\left(\frac{a_8-x}{a_8-a_7}\right) & a_7 \le x \le a_8 \\ 0 & otherwise \end{cases}$$



and represented by $(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8)$ where $a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8$ are real numbers such that $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ and $0 < k < 1$.

Definition 2.2: The α – cut of an octagonal fuzzy number $\tilde{A} = \left(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8\right)$ is defined as

$$[\tilde{A}] = [A_\alpha^L, A_\alpha^R] = \begin{cases} \left[\left(A_\alpha{}^L\right)_1,\ \left(A_\alpha{}^R\right)_1\right] & \alpha \in [0,\mathrm{k}] \\ \left[\left(A_\alpha{}^L\right)_2,\ \left(A_\alpha{}^R\right)_2\right] & \alpha \in [\mathrm{k},1] \end{cases} \qquad for \alpha \in [0,\mathrm{k}]$$

Where $(A_\alpha^L)_1 = a_1 + \frac{\alpha}{k}(a_2 - a_1)$

$(A_\alpha^L)_2 = a_3 + \frac{\alpha-k}{1-k}(a_4 - a_3)$

$(A_\alpha^R)_1 = a_8 - \frac{\alpha}{k}(a_8 - a_7)$

$(A_\alpha^R)_2 = a_6 - \frac{\alpha-k}{1-k}(a_6 - a_5)$

Definition 2.3 : Defuzzification of $\tilde{A}$ can be found by signed distance method. If $\tilde{A}$ is an octagonal fuzzy number ,then the signed distance from $\tilde{A}$ to 0 is defined as $d(\tilde{A},0) = \frac{1}{2}\int_0^1([A_L(\alpha), A_R(\alpha)], 0)\, d\alpha$.

Where, $A_\alpha = [A_L(\alpha), A_R(\alpha)]$

$A_\alpha = [a + (b-a)\alpha\ ,\ \mathrm{d} - (\mathrm{d}-\mathrm{c})\alpha] \quad \alpha\epsilon[0,1]$.

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2.4 : Arithmetic operations on an octagonal fuzzy numbers

Let $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ and $\widetilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ are two octagonal fuzzy numbers. Then

- The addition of $\widetilde{A}$ and $\widetilde{B}$ is

$$\widetilde{A} + \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8).$$

- The subtraction of $\widetilde{A}$ and $\widetilde{B}$ is

$$\tilde{A} - \tilde{B} = \begin{pmatrix} \max(0,(a_1 - b_8)), \max(0,(a_2 - b_7)), \max(0,(a_3 - b_6)), \\ \max(0,(a_4 - b_5)), \max(0,(a_5 - b_4)), \max(0,(a_6 - b_3)), \\ \max(0,(a_7 - b_2)), \max(0,(a_8 - b_1)). \end{pmatrix}$$

- The multiplication of $\widetilde{A}$ and $\widetilde{B}$ is When $\widetilde{A}$ and $\widetilde{B}$ are two positive octagonal fuzzy numbers, then $\widetilde{A} \otimes \widetilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7, a_8b_8)$.
- The division of $\widetilde{A}$ and $\widetilde{B}$ is $\frac{\widetilde{A}}{\widetilde{B}} = \left(\frac{a_1}{b_8}, \frac{a_2}{b_7}, \frac{a_3}{b_6}, \frac{a_4}{b_5}, \frac{a_5}{b_4}, \frac{a_6}{b_3}, \frac{a_7}{b_2}, \frac{a_8}{b_1}\right)$.

3.Notations and Assumptions :

Notations :

We define the following symbols:

D – Total demand over the planning time period [0,T]
o – Ordering cost per order.
s – Carrying cost per unit quantity per unit time.
L – Length of the plan.
Q – Order quantity per cycle.
TC – Total cost for the period [0.T].
Q^* – Optimal order quantity.
TC^* – Minimum total cost for [0,T] .
$\widetilde{Q}^*$ – Fuzzy optimal order quantity.
$F(\widetilde{Q}^*)$ – Fuzzy minimum total cost for [0,T] .

Assumptions :

In inventory management , economic order quantity (EOQ) is the order quantity that minimizes the total holding cost and ordering cost. The following assumptions are considered in Fuzzy Inventory Model Without Shortage Using Octagonal Fuzzy Numbers

- Total demand is constant.
- Length of the plan is constant .
- Shortages are not allowed .
- Only carrying cost and ordering cost are in fuzzy nature.

3.1. Proposed inventory model in crisp sense : First , we handle an inventory model without shortage in crisp environment. The economic lot size is obtained by the following model equation,

$$q = \sqrt{\frac{2Do}{sL}} \quad -----(1)$$

The total cost for the period [0,T] is ,

Total cost = Carrying cost + Ordering cost .

$$TC = \frac{QsL}{2} + \frac{Do}{Q} \quad -----(2)$$

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The optimum q^* and TC^* can be obtained by equating the first order partial derivatives of TC to zero and solving the resulting equations.

3.1.1. To find an optimal order quantity :

Differentiate partially TC with respect to order quantity per cycle (Q) , $\frac{\partial TC}{\partial Q} = \frac{sL}{2} - \frac{Do}{Q^2}$, Now $\frac{\partial TC}{\partial Q} = 0$

$$Q^* = \sqrt{\frac{2Do}{sL}}$$

Therefore , the optimal order quantity, $q^* = \sqrt{\frac{2Do}{sL}}$.

3.1.2. To find the minimum total cost :

$$TC = \frac{QsL}{2} + \frac{Do}{Q} = \frac{sL\sqrt{\frac{2Do}{sL}}}{2} + \frac{Do}{\sqrt{\frac{2Do}{sL}}} = \sqrt{2DosL} \quad -----(4)$$

Therefore , the minimum total cost , $TC^* = \sqrt{2DosL}$.

3.2. Proposed inventory model in fuzzy sense:

Now, we handle an inventory model without shortage in fuzzy environment . Since the carrying cost (s) and ordering cost (o) are fuzzy nature . We represent them by octagonal fuzzy numbers .

Let

$\tilde{s}$ - Carrying cost per unit quantity .

$\tilde{o}$ - Ordering cost per cycle .

The length of plan (L) and the total demand (D) are considered as constant .

Now we fuzzify total cost given in (2) , the fuzzy total cost is given by $\widetilde{TC} = \frac{\widetilde{Q}\,\tilde{s}\,L}{2} + \frac{D\tilde{o}}{\widetilde{Q}}$

Our aim to apply signed distance method to defuzzify the fuzzy total cost and obtain the fuzzy optimal order quantityby using the simple calculus technique .

Suppose,$\tilde{s} = (s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8)$, $\tilde{o} = (o_1, o_2, o_3, o_4, o_5, o_6, o_7, o_8)$ are octagonal fuzzy numbers.

We have $\widetilde{TC} = \widetilde{TC}(\tilde{s}, \tilde{o}) = \left(\tilde{s} \otimes \frac{\widetilde{Q}\,L}{2}\right) \oplus \left(\tilde{o} \otimes \frac{D}{\widetilde{Q}}\right)$

$$\widetilde{TC} = \begin{pmatrix} s_1\frac{\widetilde{Q}\,L}{2} + o_1\frac{D}{\widetilde{Q}}, s_2\frac{\widetilde{Q}\,L}{2} + o_2\frac{D}{\widetilde{Q}}, s_3\frac{\widetilde{Q}\,L}{2} + o_3\frac{D}{\widetilde{Q}}, s_4\frac{\widetilde{Q}\,L}{2} + o_4\frac{D}{\widetilde{Q}}, \\ s_5\frac{\widetilde{Q}\,L}{2} + o_5\frac{D}{\widetilde{Q}}, s_6\frac{\widetilde{Q}\,L}{2} + o_6\frac{D}{\widetilde{Q}}, s_7\frac{\widetilde{Q}\,L}{2} + o_7\frac{D}{\widetilde{Q}}, s_8\frac{\widetilde{Q}\,L}{2} + o_8\frac{D}{\widetilde{Q}} \end{pmatrix} \quad ----(5)$$

Defuzzification $\widetilde{TC}$ in (5) by using signed distance method , we have

$d\left(\widetilde{TC}(\tilde{s}, \tilde{o}), 0\right) = \frac{1}{2}\int_0^1 ([A_L(\alpha), A_R(\alpha)], 0)\, d\alpha.$

To find left α-cut $A_L(\alpha)$:

$A_L(\alpha) = [(A_\alpha^L)_1 , (A_\alpha^R)_1]$ Where $(A_\alpha^L)_1 = a_1 + \frac{\alpha}{k}(a_2 - a_1)$;

$(A_\alpha^R)_1 = a_8 - \frac{\alpha}{k}(a_8 - a_7).$

To find $(A_\alpha^L)_1$:

$$(A_\alpha^L)_1 = a_1 + \frac{\alpha}{k}(a_2 - a_1) = \left(s_1\frac{\widetilde{Q}\,L}{2} + o_1\frac{D}{\widetilde{Q}}\right) + \frac{\alpha}{k}\left[\left(s_2\frac{\widetilde{Q}\,L}{2} + o_2\frac{D}{\widetilde{Q}}\right) - \left(s_1\frac{\widetilde{Q}\,L}{2} + o_1\frac{D}{\widetilde{Q}}\right)\right]$$

$$(A_\alpha^L)_1 = \frac{\widetilde{Q}\,L}{2}\left[s_1 + \frac{\alpha}{k}(s_2 - s_1)\right] + \frac{D}{\widetilde{Q}}\left[o_1 + \frac{\alpha}{k}(o_2 - o_1)\right]$$

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To find $(A_\alpha^R)_1$:

$$(A_\alpha^R)_1 = a_8 - \frac{\alpha}{k}(a_8 - a_7) = \left(s_8\frac{\widetilde{Q}L}{2} + o_8\frac{D}{\widetilde{Q}}\right) - \frac{\alpha}{k}\left[\left(s_8\frac{\widetilde{Q}L}{2} + o_8\frac{D}{\widetilde{Q}}\right) - \left(s_7\frac{\widetilde{Q}L}{2} + o_7\frac{D}{\widetilde{Q}}\right)\right]$$

$$= \frac{\widetilde{Q}L}{2}\left[s_8 - \frac{\alpha}{k}s_8 + \frac{\alpha}{k}s_7\right] + \frac{D}{\widetilde{Q}}\left[o_8 - \frac{\alpha}{k}o_8 + \frac{\alpha}{k}o_7\right] = \frac{\widetilde{Q}L}{2}\left[s_8 - \frac{\alpha}{k}(s_8 - s_7)\right] + \frac{D}{\widetilde{Q}}\left[o_8 - \frac{\alpha}{k}(o_8 - o_7)\right]$$

$since\ A_L(\alpha) = [(A_\alpha^L)_1\ , (A_\alpha^R)_1]$

$$A_L(\alpha) = \left[\left(\frac{\widetilde{Q}L}{2}\left[s_1 + \frac{\alpha}{k}(s_2 - s_1)\right] + \frac{D}{\widetilde{Q}}\left[o_1 + \frac{\alpha}{k}(o_2 - o_1)\right]\right), \left(\frac{\widetilde{Q}L}{2}\left[s_8 - \frac{\alpha}{k}(s_8 - s_7)\right] + \frac{D}{\widetilde{Q}}\left[o_8 - \frac{\alpha}{k}(o_8 - o_7)\right]\right)\right]$$

To find right α-cut $A_R(\alpha)$:

$A_R(\alpha) = [(A_\alpha^L)_2\ ,\ (A_\alpha^R)_2]$ where $(A_\alpha^L)_2 = a_3 + \frac{\alpha-k}{1-k}(a_4 - a_3)$;

$$(A_\alpha^R)_2 = a_6 - \frac{\alpha-k}{1-k}(a_6 - a_5)$$

To find $(A_\alpha^L)_2$: $(A_\alpha^L)_2 = a_3 + \frac{\alpha-k}{1-k}(a_4 - a_3)$

$$= \left(s_3\frac{\widetilde{Q}L}{2} + o_3\frac{D}{\widetilde{Q}}\right) + \frac{\alpha-k}{1-k}\left[\left(s_4\frac{\widetilde{Q}L}{2} + o_4\frac{D}{\widetilde{Q}}\right) - \left(s_3\frac{\widetilde{Q}L}{2} + o_3\frac{D}{\widetilde{Q}}\right)\right]$$

$$(A_\alpha^L)_2 = \frac{\widetilde{Q}L}{2}\left[s_3 + \frac{\alpha-k}{1-k}(s_4 - s_3)\right] + \frac{D}{\widetilde{Q}}\left[o_3 + \frac{\alpha-k}{1-k}(o_4 - o_3)\right]$$

To find $(A_\alpha^R)_2$: $(A_\alpha^R)_2 = a_6 - \frac{\alpha-k}{1-k}(a_6 - a_5)$

$$= \left(s_6\frac{\widetilde{Q}L}{2} + o_6\frac{D}{\widetilde{Q}}\right) - \frac{\alpha-k}{1-k}\left[\left(s_6\frac{\widetilde{Q}L}{2} + o_6\frac{D}{\widetilde{Q}}\right) - \left(s_5\frac{\widetilde{Q}L}{2} + o_5\frac{D}{\widetilde{Q}}\right)\right]$$

$$(A_\alpha^R)_2 = \frac{\widetilde{Q}L}{2}\left[s_6 - \frac{\alpha-k}{1-k}(s_6 - s_5)\right] + \frac{D}{\widetilde{Q}}\left[o_6 - \frac{\alpha-k}{1-k}(o_6 - o_5)\right]$$

$since\ A_R(\alpha) = (A_\alpha^L)_2\ ,\ (A_\alpha^R)_2$

$$A_R(\alpha) = \left[\left(\frac{\widetilde{Q}L}{2}\left[s_3 + \frac{\alpha-k}{1-k}(s_4 - s_3)\right] + \frac{D}{\widetilde{Q}}\left[o_3 + \frac{\alpha-k}{1-k}(o_4 - o_3)\right]\right), \left(\frac{\widetilde{Q}L}{2}\left[s_6 - \frac{\alpha-k}{1-k}(s_6 - s_5)\right] + \frac{D}{\widetilde{Q}}\left[o_6 - \frac{\alpha-k}{1-k}(o_6 - o_5)\right]\right)\right]$$

Since $d(\widetilde{TC}\ (\tilde{s}, \tilde{o}), 0) = \frac{1}{2}\int_0^1 ([A_L(\alpha)\ , A_R(\alpha)]\ , 0)\ d\alpha$ $d(\widetilde{TC}(\tilde{s}, \tilde{o}), 0) =$

$$\frac{1}{2}\int_0^1 \left[\begin{Bmatrix}\left(\frac{\widetilde{Q}L}{2}\left[s_1 + \frac{\alpha}{k}(s_2 - s_1)\right] + \frac{D}{\widetilde{Q}}\left[o_1 + \frac{\alpha}{k}(o_2 - o_1)\right]\right), \\ \left(\frac{\widetilde{Q}L}{2}\left[s_8 - \frac{\alpha}{k}(s_8 - s_7)\right] + \frac{D}{\widetilde{Q}}\left[o_8 - \frac{\alpha}{k}(o_8 - o_7)\right]\right)\end{Bmatrix}, \begin{Bmatrix}\left(\frac{\widetilde{Q}L}{2}\left[s_3 + \frac{\alpha-k}{1-k}(s_4 - s_3)\right] + \frac{D}{\widetilde{Q}}\left[o_3 + \frac{\alpha-k}{1-k}(o_4 - o_3)\right]\right), \\ \left(\frac{\widetilde{Q}L}{2}\left[s_6 - \frac{\alpha-k}{1-k}(s_6 - s_5)\right] + \frac{D}{\widetilde{Q}}\left[o_6 - \frac{\alpha-k}{1-k}(o_6 - o_5)\right]\right)\end{Bmatrix}\right] d\alpha$$

$$d(\widetilde{TC}\ (\tilde{s}, \tilde{o}), 0) = \frac{\widetilde{Q}L}{4}\left(\begin{bmatrix}s_1 + \frac{1}{2k}(s_2 - s_1) + s_3 + \frac{1-2k}{2(1-k)}(s_4 - s_3)\end{bmatrix}, \begin{bmatrix}s_8 - \frac{1}{2k}(s_8 - s_7) + s_6 - \frac{1-2k}{2(1-k)}(s_6 - s_5)\end{bmatrix}\right)$$

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$$+\frac{D}{2\widetilde{Q}}\left(\begin{bmatrix} o_1+\frac{1}{2k}(o_2-o_1)+o_3+\frac{1-2k}{2(1-k)}(o_4-o_3)\end{bmatrix}, \\ \begin{bmatrix} o_8-\frac{1}{2k}(o_8-o_7)+o_6-\frac{1-2k}{2(1-k)}(o_6-o_5)\end{bmatrix}\right)---(6)$$

$d(\widetilde{TC}\ (\tilde{s},\tilde{o}),0)=F(\widetilde{Q}\ )(say).$

Case 1: k = 0.5 ,Then equation (6) becomes,

$$F(\widetilde{Q}\ )=\frac{\widetilde{Q}\ L}{4}\left(\begin{bmatrix} s_1+\left(\frac{1}{1}\right)(s_2-s_1)+s_3+\left(\frac{1-1}{2(0.5)}\right)(s_4-s_3)\end{bmatrix}, \\ \begin{bmatrix} s_8-\left(\frac{1}{1}\right)(s_8-s_7)+s_6-\left(\frac{1-1}{2(0.5)}\right)(s_6-s_5)\end{bmatrix}\right)$$

$$+\frac{D}{2\widetilde{Q}}\left(\begin{bmatrix} o_1+\left(\frac{1}{1}\right)(o_2-o_1)+o_3+\left(\frac{1-1}{2(0.5)}\right)(o_4-o_3)\end{bmatrix}, \\ \begin{bmatrix} o_8-\left(\frac{1}{1}\right)(o_8-o_7)+o_6-\left(\frac{1-1}{2(0.5)}\right)(o_6-o_5)\end{bmatrix}\right)$$

$$F(\widetilde{Q}\ )\quad=\frac{\widetilde{Q}\ L}{4}[(s_2+s_3),(s_6+s_7)]+\frac{D}{2\widetilde{Q}}[(o_2+o_3),(o_6+o_7)]$$

$F(\widetilde{Q}\ )$is minimum , when $\frac{dF(\widetilde{Q})}{d\widetilde{Q}}=0$ and $\frac{d^2F(\widetilde{Q})}{d\widetilde{Q}^2}>0$. Now differentiate $F(\widetilde{Q}\ )$ with respect to $\widetilde{Q}$ and then equate into zero. (i.e), $\frac{dF(\widetilde{Q})}{d\widetilde{Q}}=0$.

$\frac{dF(\widetilde{Q})}{d\widetilde{Q}}=\frac{L}{4}[(s_2+s_3),(s_6+s_7)]-\frac{D}{2\widetilde{Q}^2}[(o_2+o_3),(o_6+o_7)]$.

Since $\frac{dF(\widetilde{Q})}{d\widetilde{Q}}=0$.

$\widetilde{Q}^{\,2}=\frac{2D}{L}\frac{[(o_2+o_3),(o_6+o_7)]}{[(s_2+s_3),(s_6+s_7)]}$.

$\widetilde{Q}^{\,*}=\sqrt{\frac{2D}{L}\frac{[(o_2+o_3),(o_6+o_7)]}{[(s_2+s_3),(s_6+s_7)]}}$.

This is the fuzzy optimal order quantity . Also $\widetilde{Q}=\widetilde{Q}^{\,*}$, we have $\frac{d^2F(\widetilde{Q})}{d\widetilde{Q}^2}>0$

This shows that $F(\widetilde{Q}\ )$ is minimum at $\widetilde{Q}=\widetilde{Q}^{\,*}$ and from (6)

$F(\widetilde{Q}^{\,*})=\frac{\widetilde{Q}^{\,*}L}{4}[(s_2+s_3),(s_6+s_7)]+\frac{D}{2\widetilde{Q}^{\,*}}[(o_2+o_3),(o_6+o_7)]$.

This is the minimum fuzzy total cost for [0 ,T] .

Case 2: k = 0.8, Then equation (6) becomes,

$$F(\tilde{Q})\quad=\frac{\tilde{Q}L}{4}\left(\begin{bmatrix} s_1+\left(\frac{1}{1.6}\right)(s_2-s_1)+s_3+\left(\frac{1-1.6}{2(0.2)}\right)(s_4-s_3)\end{bmatrix}, \\ \begin{bmatrix} s_8-\left(\frac{1}{1.6}\right)(s_8-s_7)+s_6-\left(\frac{1-1.6}{2(0.2)}\right)(s_6-s_5)\end{bmatrix}\right)+$$

$$\frac{D}{2\tilde{Q}}\left(\begin{bmatrix} o_1+\left(\frac{1}{1.6}\right)(o_2-o_1)+o_3+\left(\frac{1-1.6}{2(0.2)}\right)(o_4-o_3)\end{bmatrix}, \\ \begin{bmatrix} o_8-\left(\frac{1}{1.6}\right)(o_8-o_7)+o_6-\left(\frac{1-1.6}{2(0.2)}\right)(o_6-o_5)\end{bmatrix}\right).$$

$$F(\tilde{Q})=\frac{\tilde{Q}L}{4}\begin{bmatrix}(0.375s_1+0.625s_2+2.5s_3-1.5s_4),\\(0.375s_8+0.625s_7+2.5s_6-1.5s_5)\end{bmatrix}+$$

$$\frac{D}{2\tilde{Q}}\begin{bmatrix}(0.375o_1+0.625o_2+2.5o_3-1.5o_4),\\(0.375o_8+0.625o_7+2.5o_6-1.5o_5)\end{bmatrix}.$$

$F(\tilde{Q})$is minimum , when $\frac{dF(\tilde{Q})}{d\tilde{Q}}=0$ and $\frac{d^2F(\tilde{Q})}{d\tilde{Q}^2}>0$.

Now differentiate $F(\tilde{Q})$ with respect to $\tilde{Q}$ and then $\frac{dF(\tilde{Q})}{d\tilde{Q}}=0$.

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$$\tilde{Q}^* = \sqrt{\frac{2D}{L}\frac{\left[\begin{matrix}(0.375o_1 + 0.625o_2 + 2.5o_3 - 1.5o_4)\,, \\ (0.375o_8 + 0.625o_7 + 2.5o_6 - 1.5o_5)\end{matrix}\right]}{\left[\begin{matrix}(0.375s_1 + 0.625s_2 + 2.5s_3 - 1.5s_4)\,, \\ (0.375s_8 + 0.625s_7 + 2.5s_6 - 1.5s_5)\end{matrix}\right]}}$$

This is the fuzzy optimal order quantity . Also $\tilde{Q} = \tilde{Q}^*$, we have $\frac{d^2F(\tilde{Q})}{d\tilde{Q}^2} > 0$.
This shows that $F(\tilde{Q})$ is minimum at $\tilde{Q} = \tilde{Q}^*$ and from (6)

$$F(\tilde{Q}^*) = \frac{\tilde{Q}^*L}{4}\left[\begin{matrix}(0.375s_1 + 0.625s_2 + 2.5s_3 - 1.5s_4)\,, \\ (0.375s_8 + 0.625s_7 + 2.5s_6 - 1.5s_5)\end{matrix}\right] + \frac{D}{2\tilde{Q}^*}\left[\begin{matrix}(0.375o_1 + 0.625o_2 + 2.5o_3 - 1.5o_4)\,, \\ (0.375o_8 + 0.625o_7 + 2.5o_6 - 1.5o_5)\end{matrix}\right].$$

This is the minimum fuzzy total cost for [0 ,T] .

3.3. Algorithm :

Step 1 : First, calculate an optimal order quantity (Q^*) and minimum total cost (TC^*) for the crisp model.

Step 2 : Determine fuzzy total cost which can be obtained by fuzzifying the total cost, i.e. taking carrying cost $(\tilde{s})$ and ordering cost $(\tilde{o})$ as octagonal fuzzy numbers.

Step 3 : Calculate $F(\tilde{Q})$ by using signed distance method for defuzzification of the fuzzy total cost $\widetilde{TC}$. Then put the values of k as we required.

Step 4 : Find fuzzy optimal order quantity $\tilde{Q}^*$ which can be obtained by putting the first total derivative of $F(\tilde{Q})$ with respect to $\tilde{Q}$ and then equate to zero and second derivate of $F(\tilde{Q})$ is positive at $\tilde{Q} = \tilde{Q}^*$.

Step 5 : Calculate minimum fuzzy total cost $F(\tilde{Q}^*)$ by substituting fuzzy optimal order quantity $(\tilde{Q}^*)$in $F(\tilde{Q})$.

4. Numerical Example

Crisp Model : Let o = Rs.21/- per unit , s = Rs.12/- , D = 550 unit , L = 10 days . Then the economic order quantity and total cost are respectively. q^* = 24.90 units TC^*= Rs. 1583 /-

Fuzzy Model : D = 5550 units , L = 10 days

Let $\tilde{o}$ =(15,17,19,21,23,24,25,27) , $\tilde{s}$ =(8,10,11,12,13,14,15,16)

Then we find that, For k = 0.5 ,$\tilde{Q}^*$= 24.65 units, F($\tilde{Q}^*$) = Rs. 1579.49/-

For k=0.8 ,$\tilde{Q}^*$= 24.83 units , F($\tilde{Q}^*$) = Rs. 1577.40/-

Demand	k=0.5		k=0.8	
	$\tilde{Q}^*$	F($\tilde{Q}^*$)	$\tilde{Q}^*$	F($\tilde{Q}^*$)
450	22.54	1428.65	22.75	1348.25
500	27.72	1523.63	27.89	1447.81
550	**24.65**	**1579.49**	**24.83**	**1577.40**
600	25.81	1649.64	25.94	1630.44
650	27.01	1717.10	27.20	1701.01

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5. CONCLUSION

In this paper, Octagonal fuzzy number is utilized in developing the notion of Inventory model. The main aim of the authors is to applied signed distance method to defuzzify the fuzzy total cost and obtain the fuzzy optimal order quantity by using the simple calculus technique. The optimal order quantity and minimum total cost for the proposed inventory model derived both crisp and fuzzy sense. A new arithmetic operation of octagonal fuzzy numbers is proposed to get the expected result. A numerical example is added to illustrate the process of obtaining the fuzzy optimal order quantity and the fuzzy minimal total cost.

References

[1] Dutta, D and Pavankumar*, Fuzzy inventory model without shortages using Trapezoidal fuzzy number with sensitivity analysis*.

[2] FarithaAsma, A., *Fuzzy optimization technique in EOQ model using nearest interval approximation method*, vol.4, issue 10, October 2015.

[3] Gajalakshmi, S and Parvathi, P ,*Solving an EOQ model in an Inventory problem by using Octagonal Fuzzy Numbers*.

[4] Harris, F., *Operations and cost* , AW Shaw co.chicago, (1915).

[5] Malini, S.U and FelbinC. Kennedy, *An approach for solving fuzzy transportation problem using Octagonal Fuzzy Numbers*.

[6] Park, K.S., *Fuzzy set Theoretical Interpretation of economic order quantity*, IEEE Trans.systemMan.cybernet SMC, 17,1987,1082-1084.

[7] Stephen Dinagar, D and Rajesh Kannan, J., *"On Optimal Total cost And Optimal Order Quantity For Fuzzy Invventory Model Without Shortage Using HexagonalFuzzy Numbers"*, International Journal of Fuzzy Mathematics and system, 4(2)(2014), 193-201,ISSN 2248-9940.

[8]Zimmerman, H. J., Using fuzzy sets in operational research, European journal of operational research

[9] Wilson, R., *A scientific routine for stock control*. Harvard Business Review, 13, 1934, 116-128.

[10] Zadeh, L.A., Fuzzy sets, *Information control*, 8, 338-353, 1965.

[11] *Fuzzy sets and its applications* by George J.Klir / Boyaun (1995).

[12] KantiSwarp, P.K.Gupta, Manmohann (1997) *Operations Research* 8th edition, Sultan Chand and sons New Delhi.

[13] D. Stephen Dinagar and Rajesh Kannan, J., *"On Invventory Model With Allowable Shortage Using LR – Hexagonal Fuzzy Numbers"*, International Journal of Fuzzy sets and Artificial intelligence 5(2015) 75–90, ISSN 2241–1240.

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PATTERN CLASSIFICATION OF CONTINUED FRACTIONS WITH CENTERED POLYGONAL NUMBERS AS BASE

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Abstract:

A study of number sequence is an enthusiastic field in number theory. Among these sequence are centered polygonal numbers that yields a unique copiousness in its suitability. Here the study focuses on the ratios of centered polygonal numbers as continued fractions in two dimensional approach. In this article different kinds of properties and characteristics of continued fractions which represent ratios of centered polygonal numbers of the same rank are explicated. As any polygonal can be sub divided into like smaller triangles, squares, pentagons and the corresponding figurate numbers taken as base. Here the study depends on the essentiality and periodicity of the general sequence.

Keywords: *continued fraction, centered polygonal numbers, triangular number, centered triangular number, centered square number*.

Notations:

1. $\langle x_0, x_1, x_2, x_3, \ldots, x_n \rangle$ – continued fraction expansion
2. $C_{l,n}$ - centered polygonal number of order $'l'$ and rank $'n'$.
3. T_n - Triangular number

AMS Classification: 11A55

1. Introduction

The theory of numbers is that area of mathematics whose aim is to uncover the many deep and subtle relationships between different sorts of numbers. The pythagoreans also linked numbers with geometry. They introduced the idea os triangular numbers, square numbers, pentagonal numbers, etc,..The reason for this geometrical nomenclature is clear when the numbers are represented by dots arranged in the form of triangles, squares, pentagons etc ,...

Next to integers, the study of real numbers, in general, has got great excitement. One of the ways of representing a real number is called continued fraction expression. The Golden ratio which is an irrational number has continued fraction expression. Also it can be represented diagrammatically. The usefulness of continued fractions arises from the fact that successive convergents can be calculated by simple recurring formulae and that the sequence of convergents have very valuable properties which are helpful in finding rational approximations to real numbers. A continued fraction is symmetric if the partial quotients equidistant from the beginning and the end are equal. This motivated me to represent the ratios of centered polygonal numbers which are rational numbers as Continued fraction expressions. Here the continued fraction expansion of centered polygonal numbers of consecutive ranks are studied. In this paper, an attempt has been made to analyse some characterizations of ratios of centered polygonal numbers with reference the centered triangular numbers which is the building block of all centered polygonal numbers.

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The centered polygonal numbers are a family of sequences of two dimensional figurate numbers, each formed by a central dot (for $n = 0$) surrounded by polygonal layers with constant number of rank k_0 of 0- dimensional elements (or vertices y) , thus a constant number k_1 equal to k_0 of 1-dimensional elements (edge e) . Each side v of a polygonal layer contains one more dot than a side in the previous layer. So starting from the second polynomial layer each layer of a centered k_0- gonal number contains k_0 -gonal number contains centered k_0 more points than the previous layer. The n[th] centered k-gonal number can be obtained by placing k copies of the $(n-1)^{th}$ triangular around a centered point therefore, the n[th] centered k-gonal number of rank $'n'$ can be represented as,

$$C_{k,n} = \frac{kn}{2}(n+1) + 1$$

1.1 Continued Fraction:

An expression of the form

$$\frac{a}{b} = x_0 + \frac{y_0}{x_1 + \frac{y}{x_2 + \frac{y_2}{x_3 + \frac{y_3}{\ddots}}}}$$

where x_i, y_i are real or complex numbers is called a continued fraction.

1.2 The Continued Fraction algorithm:

This algorithm can be used to find the continued fraction expansion of any real number.

Let q be any real number. Choose $q_0 = q$ and set $x_0 = q_0$,

Define $q_i = \frac{1}{q_0 - [q_0]}$ and set $x_1 = [q_1]$

$q_2 = \frac{1}{q_1 - [q_1]}$, which implies that $x_2 = [q_2]$,....., $q_k = \frac{1}{q_{k-1} - [q_{k-1}]}$, which implies that $x_k = [q_k]$,.....,

This process is continued infinitely or to some finite stage till an $q_i \in N$, exists such that $x_i = [q_i]$.

1.3 Theorem:

The continued fraction expression of ratios of centered polygonal numbers having multiple of three as its order to centered polygonal numbers of whose ranks form an arithmatic progression can be classified as follows:

$$i)\ \frac{C_{3l,n}}{C_{l(3k+1),n}} = \langle 0, k, 3, \frac{1}{3k-2}\left(lT_n - (k-1)\right)\rangle$$

$$ii)\ \frac{C_{3l,n}}{C_{l(3k+2),n}} = \langle 0, k, 1,1,1, \frac{1}{3k-1}\left(lT_n - (2k-1)\right)\rangle$$

$$iii)\ \frac{C_{3l,n}}{C_{l(3k+3),n}} = \langle 0, k, 1, \frac{1}{k}\left(3lT_n - (k-1)\right)\rangle$$

where $'l'$ is any positive integer and T_n is a triangular number of rank $'n'$.

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Proof:

i) Consider,

$$C_{3l,n} = \frac{3ln(n+1)}{2} + 1 = \frac{3ln^2 + 3ln + 2}{2}$$

$$C_{l(3k+1),n} = \frac{l(3k+1)n(n+1)}{2} + 1 = \frac{3kln^2 + 3kln + ln^2 + ln + 2}{2}$$

Now,

$$\frac{C_{3l,n}}{C_{l(3k+1),n}} = \frac{3ln^2 + 3ln + 2}{3kln^2 + 3kln + ln^2 + ln + 2}$$

$$= 0 + \frac{3kln^2 + 3kln + ln^2 + ln + 2}{3ln^2 + 3ln + 2}$$

$$= k + \frac{ln^2 + ln - 2k + 2}{3ln^2 + 3ln + 2}$$

$$= 3 + \frac{6k-4}{ln^2+ln-2k+2}$$

$$= \frac{ln(n+1)}{6k-4} - \frac{2(k-1)}{6k-4}$$

$$= \frac{1}{3k-2}\left(lT_n - (k-1)\right)$$

$$\frac{C_{3l,n}}{C_{l(3k+1),n}} = \langle 0, k, 3, \frac{1}{3k-2}\left(lT_n - (k-1)\right)\rangle$$

Illustration:

when $l = 1, k = 1$ weget $\frac{C_{3,n}}{C_{4,n}} = \langle 0,1,3,T_n\rangle$

when $l = 1, k = 2$ weget $\frac{C_{3,n}}{C_{7,n}} = \langle 0,2,3,\frac{1}{4}(T_n - 1)\rangle$

when $l = 1, k = 3$ weget $\frac{C_{3,n}}{C_{10,n}} = \langle 0,3,3,\frac{1}{7}(T_n - 2)\rangle$

The continued fraction of the ratios of centered triangular number to centered polygonal numbers of orders 4,7,10,…(AP) of the form,

$$\frac{C_{3l,n}}{C_{l(3k+1),n}} = \langle 0, k, 3, \frac{1}{3k-2}\left(lT_n - (k-1)\right)\rangle$$

ii) Consider,

$$\frac{C_{3l,n}}{C_{l(3k+2),n}} = \frac{3ln^2 + 3ln + 2}{3kln^2 + 3kln + 2ln^2 + 2ln + 2}$$

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$$= 0 + \frac{3kln^2 + 3kln + 2ln^2 + 2ln + 2}{3ln^2 + 3ln + 2}$$

$$= k + \frac{2ln^2 + 2ln - 2k + 2}{3ln^2 + 3ln + 2}$$

$$= 1 + \frac{ln^2+ln+2k}{2ln^2+2ln-2k+2}$$

$$= 1 + \frac{ln^2 + ln - 4k + 2}{ln^2 + ln + 2k}$$

$$= 1 + \frac{6k - 2}{ln^2 + ln - 4k + 2}$$

$$= \frac{ln(n+1)}{2(3k-1)} - \frac{2(2k-1)}{2(3k-1)}$$

$$= \frac{1}{3k-1}\left(lT_n - (2k-1)\right)$$

$$\frac{C_{3l,n}}{C_{l(3k+2),n}} = \langle 0, k, 1,1,1, \frac{1}{3k-1}\left(lT_n - (2k-1)\right)\rangle$$

iii) Consider,

$$\frac{C_{3l,n}}{C_{l(3k+3),n}} = \frac{3ln^2 + 3ln + 2}{3kln^2 + 3kln + 3ln^2 + 3ln + 2}$$

$$= 0 + \frac{3kln^2 + 3kln + 3ln^2 + 3ln + 2}{3ln^2 + 3ln + 2}$$

$$= k + \frac{3ln^2 + 3ln - 2k + 2}{3ln^2 + 3ln + 2}$$

$$= 1 + \frac{2k}{3ln^2 + 3ln - 2k + 2}$$

$$= \frac{3ln(n+1)}{2k} - \frac{2(k-1)}{2k}$$

$$= \frac{1}{k}\left(3lT_n - (k-1)\right)$$

$$\frac{C_{3l,n}}{C_{l(3k+3),n}} = \langle 0, k, 1, \frac{1}{k}\left(3lT_n - (k-1)\right)\rangle$$

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1.4 Theorem:

The ratios of certain centered polygonal numbers of rank $'n'$ with respect to centered triangular numbers, centered square numbers, centered pentagonal numbers etc,.. of corresponding rank can be expressed in terms of triangular numbers of rank $'n'$ as follows:

$$i)\ \frac{C_{l,n}}{C_{lk+l,n}} = \langle 0, k, 1, \frac{1}{k}\left(lT_n - (k-1)\right)\rangle$$

$$ii)\ \frac{C_{l,n}}{C_{lk+l-1,n}} = \langle 0, k, 1, l-2, 1, \frac{1}{lk-1}\left(T_n - \left((l-1)k - 1\right)\right)\rangle$$

$$iii)\ \frac{C_{l,n}}{C_{lk+1,n}} = \langle 0, k, l, \frac{1}{lk-(l-1)}\left(T_n - (k-1)\right)\rangle$$

where $'l'$ is any positive integer and T_n is a triangular number of rank $'n'$.

Proof:

$i)$ Consider,

$$\frac{C_{l,n}}{C_{lk+l,n}} = \frac{ln^2 + ln + 2}{lkn^2 + ln^2 + lkn + ln + 2}$$

$$= 0 + \frac{lkn^2 + ln^2 + lkn + ln + 2}{ln^2 + ln + 2}$$

$$= k + \frac{ln^2 + ln - 2k + 2}{ln^2 + ln + 2}$$

$$= 1 + \frac{2k}{ln^2 + ln - 2k + 2}$$

$$= \frac{ln(n+1)}{2k} - \frac{2(k-1)}{2k}$$

$$= \frac{1}{k}\left(lT_n - (k-1)\right)$$

$$\frac{C_{l,n}}{C_{lk+l,n}} = \langle 0, k, 1, \frac{1}{k}\left(lT_n - (k-1)\right)\rangle$$

$ii)$ Consider,

$$\frac{C_{l,n}}{C_{lk+l-1,n}} = \frac{ln^2 + ln + 2}{lkn^2 + ln^2 - n^2 + lkn + ln - n + 2}$$

$$= 0 + \frac{lkn^2 + ln^2 - n^2 + lkn + ln - n + 2}{ln^2 + ln + 2}$$

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$$= \quad k + \frac{ln^2 - n^2 + ln - n - 2k + 2}{ln^2 + ln + 2}$$

$$= \quad 1 + \frac{n^2 + n + 2k}{ln^2 - n^2 + ln - n - 2k + 2}$$

$$= \quad (l-2) + \frac{n^2 - 2kl + n + 2k + 2}{n^2 + n + 2k}$$

$$= \quad 1 + \frac{2kl - 2}{n^2 - 2kl + n + 2k + 2}$$

$$= \quad \frac{n(n+1)}{2(lk-1)} - \frac{2(l-1)k - 2}{2(lk-1)}$$

$$= \quad \frac{1}{lk-1}(T_n - (l-1)k - 1)$$

$$\frac{C_{l,n}}{C_{lk+l-1,n}} = \langle 0, k, 1, l-2, 1, \frac{1}{lk-1}(T_n - (l-1)k - 1)\rangle$$

iii) Consider,

$$\frac{C_{l,n}}{C_{lk+1,n}} = \frac{ln^2 + ln + 2}{lkn^2 + n^2 + lkn + n + 2}$$

$$= \quad 0 + \frac{lkn^2 + n^2 + lkn + n + 2}{ln^2 + ln + 2}$$

$$= \quad k + \frac{n^2 + n - 2k + 2}{ln^2 + ln + 2}$$

$$= \quad l + \frac{2 - 2l + 2kl}{n^2 + n - 2k + 2}$$

$$= \quad \frac{n(n+1)}{2(1+lk-l)} - \frac{2(k-1)}{2(1+lk-l)}$$

$$= \quad \frac{1}{lk-(l-1)}(T_n - (k-1))$$

$$\frac{C_{l,n}}{C_{lk+1,n}} = \langle 0, k, l, \frac{1}{lk-(l-1)}(T_n - (k-1))\rangle$$

Illustration:

when $l = 3, k = 1$ weget $\frac{C_{3,n}}{C_{4,n}} = \langle 0,1,3, T_n\rangle$
when $l = 4, k = 1$ weget $\frac{C_{4,n}}{C_{5,n}} = \langle 0,1,4, T_n\rangle$
when $l = 5, k = 1$ weget $\frac{C_{5,n}}{C_{6,n}} = \langle 0,1,5, T_n\rangle$

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Thus we attain the result.

Conclusion:

As polygonal numbers are rich in variety we can find the continued fraction expressions of ratios of polygonal numbers with respect to polygonal numbers and polygonal numbers with respect to centered polygonal numbers and so on and classify them into different patterns.

References:

[1] P.Balamurugan, A.Gnanam,"Pattern classification of continued fractions with triangular number as base",International journal of pure and applied mathematics,Vol.119,No.15,2018, 3413-3418.
[2] A.Gnanam, S.Krithika, "Ratios of Polygonal Numbers as Continued Fractions", International journal of Engineering Science, Advanced Computing and Bio-Technology,Vol.8,No.3, July-September 2017, pp 143-155.
[3] George E. Andrews, "Number Theory", Hindustan Publishing Corporation.1992 .
[4] Ivan Niven, Herbert S.Zukerman, Hugh L.Montgomery,"An introduction to theory of Numbers", 5th edition, John Wiley and Sons 1991.
[5] "An introductory course in Elementary Number theory" by Wissam Raji ,May 9,2013.
[6] "An alternative approach to analytic Number Theory" by Andrew Granville,2014.
[7] Adams, William; Shanks, Daniel, "Mathematics of computation", 39 (159), pp 255 – 300, MR 0658231.
[8] Nathanson, M.B, "Sums of polygonal numbers", In analytic Number Theory and Diophantine Problem:Proceedings of a Conference at Oklahoma State University,1984 (Ed.A.Adolphson etal).M. ABoston, Birkhauser, p. 305 – 316, 1987.
[9] Rosen. K.H. "Elementary Number theory and its Applications", Fourth edition,Addison – Wesley, Reading, Massachusets, 2000.
[10] Shailesh Shirali, "A primer on Number sequences, Mathematics student ",Vol. 45, No.2. pp. 63 – 73, 1997.
[11] Telang, S.G. "Number Theory", Tata Mcgraw Hill publishing Company, Newyork, 1996.
[12] Uspensky, J.V. and M.A. Heaslet, "Elementary Number Theory".Mcgraw Hill Book Company, 1939.
[13] Vinogradev. "An Introduction to the theory of numbers", London and Newyork, Pergamon Press, 1995.
[14] Jonathan Browein,Alfvander Poorten, Jeffrey Shallit, Wadim Zudilin, "Neverending Fractions. An Introduction to continued fractions", Cambridge University Press, 2014.
[15] Joseph H.Silverman," A friendly introduction to Number Theory", Fourth edition, 1996.
[16] Neville Robbins,"Beginning Number Theory",Second edition, Narosa PublishingHouse,2005.

International Journal of Management, IT & Engineering
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Soft semi # generalized closed sets in Soft Topological Spaces

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Abstract :

In this paper a new class of soft generalized closed sets, namely soft semi # generalized closed set has been introduced in Soft Topological Spaces. Also some basic properties and characteristics of soft semi#g – closed sets have been investigated in soft topological spaces.

Keywords: Soft g-closed sets, Soft sg-closed sets, soft semi-closed sets, soft semi*-closed sets, soft g*s-closed sets, soft s#g - closed sets.

1.Introduction:

The study of soft generalized closed sets in soft topological space was introduced by K. Kannan. Also soft semi closed sets was introduced by D.Molodtasov in 1999 and soft α-open sets & soft α-closed sets was introduced by, Metin Akdag and Alkan Ozkan in 2014. The concept of soft pre-generalised closed sets was introduced by J. Subhashinin and Dr. C. Sekar. I. Aroliarani and A. Arokia Lancy introduced the concept of soft sg-closed sets, soft gs-closed sets, soft gp-closed sets, soft αg-closed sets. Then soft g*s-closed sets, and soft s*g closed sets have been introduced by M. Suraiya Begum and M. Sheik John. soft w-closed sets, soft g#s-closed sets, and soft g*-closed sets are respectively defined by D.Molodtasov, T. Rajendrakumar & V. Kaladevi, A. Kalavathi and G. Sai Sundara Krishnan.

In this paper, a new class of soft closed sets called soft semi#g-closed set has been introduced and which is properly placed in between the class of soft gs-closed sets and the class of soft sg-closed sets. Also the characterization of soft s#g-closed sets and many fundamental properties of soft s#g-closed sets have been investigated.

2.Preliminaries

Throughout this paper (X, τ, E) represents a soft topological space on which no separation axiom is assumed unless otherwise mentioned. For a soft subset (A, E) of

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a soft topological space (X, τ, E), cl(A, E) and int(A, E) denote the closure and interior of (A, E) respectively.

Definition 2.1: [4] A subset (A,E) of soft topological space (X,τ,E) is called **soft generalized closed** if $Cl(A,E)\subseteq(H,E)$, whenever $(A,E)\subseteq(H,E)$ and (H,E) is soft open set in X.

Definition 2.2: [4] A subset (A,E) of soft topological spaces (X,τ,E) is called **soft generalized semi closed**(soft gs - closed) if $sCl(A,E)\subseteq(H,E)$ whenever $(A,E)\subseteq(H,E)$ and (H,E) is soft open set in X.

Definition 2.3: [12] Let (X, τ) be a topological space and A⊆ X. The generalized closure of A denoted by cl*(A) is defined as cl*(A) = ∩{ generalized closed sets } ⊃ A and generalized interior of A denoted by int*(A) is defined as int*(A) = ∪ { generalized open sets} ⊆ A.

Definition 2.4 : [12] Let (X, τ, E) be a soft topological space over X and (F,E) be a soft set over X. Then the **soft generalized closure** of (F, E) denoted by cl*(F,E) is the intersection of all soft generalized closed super sets of (F, E).

Let (X, τ, E) be a soft topological space over X and (F,E) be a soft set over X. Then the **soft generalized interior** of (F, E) denoted by int*(F,E) is the union of all soft generalized-open sub sets of (F, E).

Definition 2.5 : [5] A subset (A,E) of soft topological space (X,τ,E) is called **soft semi-open set** if $(A,E)\subseteq Cl(Int(A,E))$ and a **soft semi - closed** if $Int(Cl(A,E))\subseteq(A,E)$.

Definition 2.6 : [14] A subset (A,E) of soft topological space (X,τ,E) is called **soft α -open** if $(A,E)\subseteq Int(Cl(Int(A,E)))$ and a **soft α -closed** if $Cl(Int(Cl(A,E)))\subseteq(A,E)$.

Definition 2.7 : [8] A subset (A,E) of soft topological space (X,τ,E) is called **soft pre-open** if $(A,E)\subseteq Int(Cl(A,E))$ and a **soft pre-closed** if $Cl(Int(A,E))\subseteq(A,E)$.

Definition 2.8 : [10] A subset (A, E) of a soft topological space (X, τ, E) is called **soft semi*-open** if (A, E) ⊆ cl*(int(A, E)).

A subset (A, E) of a soft topological space (X, τ, E) is called **soft semi*-closed** if int*(cl(A, E) ⊆ (A, E).

Definition 2.9 : [10] Let (X, τ, E) be soft topological space and (A, E) is a soft subsets of X. Then the **soft semi-closure of (A, E)** denoted by scl(A,E) and is defined as scl(A,E) = ∩{soft semi-closed sets} ⊃ (A, E).

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Definition 2.10 : [6] A subset (A,E) of a soft topological space (X, τ,E) is called **soft semi-generalized closed set** in a soft topological space (X, τ,E), if scl(A,E) ⊆ (H,E) whenever (A,E) ⊆ (H,E) and (H,E) is soft semi-open in X.

Soft generalized semi closed if scl(A,E) ⊆ (H,E) whenever (A,E) ⊆ (H,E) and (H,E) is soft open in X.

Definition 2.11 : [6] A subset (A,E) of soft topological spaces (X,τ,E) is called **soft α - generalized closed**(soft αg - closed) if $\alpha Cl(A,E)\subseteq(H,E)$ whenever $(A,E)\subseteq(H,E)$ and (H,E) is soft open set in X .

Definition 2.12 : [7] Let (X, τ, E) be a soft topological space and (A, E) be a soft subset of X. The soft set (A, E) is said to be **soft w–closed** if cl(A, E)⊆ (H,E) whenever (A, E) ⊆ (H,E) and (H,E) is soft semi–open.

Definition 2.13 : [7] Let (X, τ, E) be a soft topological space and (A, E) be a soft subset of X. The soft set (A, E) is said to be **soft wg–closed** if cl(int(A, E))⊆ (H,E) Whenever (A, E) ⊆ (H,E) and (H,E) is soft open.

Definition 2.14 : [7] Let (X, τ, E) be a soft topological space and (A, E) be a soft subset of X. The soft set (A, E) is said to be **soft wgα –closed** if αcl(int(A, E)) ⊆ (H,E) Whenever (A, E) ⊆ (H,E) and (H,E) is soft α–open.

Definition 2.15 : [7] Let (X, τ, E) be a soft topological space and (A, E) be a soft subset of X. The soft set (A, E) is said to be **soft wα–closed** if αcl(A, E) ⊆ (H,E) Whenever (A, E) ⊆ (H,E) and (H,E) is soft wα–open.

Definition 2.16 : [15] Let (X, τ, E) be a soft topological space and (A, E) be a soft subset of X. The soft set (A, E) is said to be **soft g*s –closed** if scl(A, E)⊆ (H,E) Whenever (A, E) ⊆ (H,E) and (H,E) is soft gs –open.

Definition 2.17 : [15] Let (X, τ, E) be a soft topological space and (A, E) be a soft subset of X. The soft set (A, E) is said to be **soft s*g –closed** if s*cl(A, E)⊆ (H,E) Whenever (A, E) ⊆ (H,E) and (H,E) is soft semi* –open in X.

Definition 2.18 : [13] Let (X, τ, E) be a soft topological space and (A, E) be a soft subset of X. The soft set (A, E) is said to be **soft *g –closed** if cl(A, E)⊆ (H,E) Whenever (A, E) ⊆ (H,E) and (H,E) is soft w –open in X.

3. Soft semi#generalised closed set

Definition 3.1: A soft subset (A, E) of a soft topological space (X, τ, E) is called **soft semi# generalized closed set** (in short form **s#g-closed**) if scl(A, E) ⊆ (H, E) whenever (A, E) ⊆ (H, E) and (H, E) is soft semi*-open in (X, τ, E).

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Theorem 3.2 : Every soft-closed set is soft semi# g-closed set.

Proof : Consider a soft closed set (A, E). Let (A, E) ⊆ (H, E), (H, E) is soft semi*-open. Since (A, E) is soft closed, cl(A, E) = (A, E) ⊆ (H, E). But scl(A, E) ⊆ cl(A, E). Thus we have scl(A, E) ⊆ (H, E) whenever (A, E) ⊆ (H, E) and (H, E) is soft semi*-open. Therefore, (A, E) is a soft semi# generalized closed set.

Theorem 3.3: Every soft semi-closed set is soft s#g-closed.

Proof : Consider a soft semi closed set (A, E). Let (A, E) ⊆ (H, E), (H, E) is soft semi*-open. Since (A, E) is soft semi-closed, scl(A, E) = (A, E) ⊆ (H, E). Therefore (A, E) is a soft semi# generalized closed set.

Theorem 3.4: Every soft semi*-closed set is soft s#g – closed.

Proof : Let (A, E) be a soft semi*-closed set. Let (A, E) ⊆ (H, E), (H, E) is soft semi*-open. Since (A, E) is soft semi*-closed, int*(cl(A, E)) ⊆ (H, E). But scl(A, E) ⊆ int*(cl(A, E)) ⊆ (H, E). Thus we have scl(A, E) ⊆ (H, E) whenever (A, E) ⊆ (H, E) and (H, E) is soft semi*-open. Therefore (A, E) is soft s#g-closed.

Theorem 3.5: Every soft sg-closed set is soft s#g-closed.

Proof : Let (A, E) be a soft sg-closed set. Let (A, E) ⊆ (H, E), (H, E) is soft semi*-open. Then (H, E) is soft semi-open. Since (A, E) is soft sg-closed, scl(A, E) ⊆ (H, E) whenever (A, E) ⊆ (H, E) and (H, E) is soft semi*-open. Therefore (A, E) is soft s#g -closed.

Theorem 3.6: Every soft s#g-closed set is soft gs-closed.

Proof : Let (A, E) be a soft s#g-closed. Let (A, E) ⊆ (H, E), (H, E) is soft open. Then (H, E) is semi*-open. Since (A, E) is soft s#g- closed, scl(A, E) ⊆ (H, E). Thus we have scl(A, E) ⊆ (H, E) whenever (A, E) ⊆ (H, E) and (H, E) is soft-open. Therefore (A, E) is soft gs-closed set.

Theorem 3. 7: Every soft w-closed sets is soft s#g-closed.

Proof : Let (A, E) be a soft w-closed set. Let (A, E) ⊆ (H, E), (H, E) is soft semi*-open. Then (H, E) is soft semi-open. Since (A, E) is soft w-closed, cl(A, E) ⊆ (H, E). But scl(A, E) ⊆ cl(A, E). Thus, we have scl(A, E) ⊆ (H, E) whenever (A, E) ⊆ (H, E) and (H, E) is semi*-open. Therefore, (A, E) is soft semi#g-closed.

Theorem 3.8: Every soft g*s-clsoed sets is soft s#g-closed.

Proof : Let (A, E) be a soft g*s – closed. Let (A, E) ⊆ (H, E), (H, E) is soft semi*-open. Then (H, E) is soft gs-open. Since (A, E) is soft g*s-closed, scl(A, E) ⊆ (H, E). Thus, we have scl(A, E) ⊆ (H, E) whenever (A, E) ⊆ (H, E) and (H, E) is soft semi*-open. Therefore, (A, E) is soft s#g-closed.

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Theorem 3.9: Every soft semi*g-closed set is soft s#g-closed.

Proof : Let (A, E) be a soft semi*g-closed set. Let (A, E) $\subseteq$ (H, E), (H, E) is soft semi*-open. Since (A, E) is soft semi*g-closed, s*cl(A, E) $\subseteq$ (H, E). But scl(A, E) $\subseteq$ s*cl(A, E). Thus, we have scl(A,E) $\subseteq$ (H, E) whenever (A, E) $\subseteq$ (H, E) and (H, E) is soft semi*-open. Therefore (A, E) is a soft s#g – closed.

Conclusion :

In this paper a new soft closed set called soft semi# generalized closed set have been defined and some of its properties have been analyzed. The results shows the relationship between soft s#g – closed sets and the other existing sets.

References

1. D.Molodtasov, Soft Set Theory-First Result , An International Journal of Computer and Mathematics with Application , 37,1999,19-31

2. Muhammad Shabir and Munazza Naz, On soft topological spaces, Computer and Mathematics with Application , 61,2011,1786-1799..

3. S. Sekar and K. Mariappa, Sg-Interior and sg-Closure in Topological Spaces, International Journal of Scientific & Engineering Research, Volume 3, Issue 6, June 2012.

4. K.Kannan, Soft Generalized Closed Sets in Soft Topological Spaces,Journal of Theoretical and Applied Information Technology ,37,2012,17-21.

5. J.Mahanta, P.K.Das , On soft topological space via semi open and semi closed soft sets, arXiv:1203.4133v1[math.GN],2012,1-12.

6. I.Arokiarani and A.Arokia Lancy, Generalized soft gβ closed sets and soft gsβ closed sets in soft topological spaces, International Journal of Mathematical Archive, 4, 2013, 17-23.

7. T. Nandhini, A. Kalaichelvi, SRW Closed sets in Soft Topological spaces, International Journal of Innovative Research in Science, Engineering and Technology, ISSN: 2319-8753, Vol. 3, Issue 6, June 2014.

8. Metin Akdag and Alkan Ozkan, On soft pre open sets and soft pre separation axioms, Gazi University Journal of Science,4,2014,1077-1083.

9. J. Subhashinin and Dr. C. Sekar, Soft Pre-Generalized -Closed sets in a Soft Topological Space, International Journal of Engineering Trends and Technology, Volume 12, Number 7, Jun 2014.

10. K. Kannan and D. Rajalakshmi, Soft Semi Star Generalized Closed Sets, Malaysian Journal of Mathematical Sciences, 9(1): 77-88, 2015.

Vol. 9 Issue 1, (Special Issue HICAAMMC- January 2019)
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11. T. Rajendrakumar and V. Kaladevi, Soft g#s-closed sets in Soft Topological Spaces, Aryabhatta Journal of Mathematics and Informatics, Vol. 8, Issue 2, July – Dec. 2016.

12. S. Saranya and Dr. K. Bageerathi, Semi# generalized closed sets in Topological Spaces, International Journal of Mathematics Trends and Technology (IJMTT), Vol. 36, Number 3, August 2016.

13. A. Kalavathi and G. Sai Sundara Krishnan, Soft g* Closed and Soft g* Open sets in soft topological spaces, Journal of interdisciplinary Mathematics, 19:1, 65-82, 2016.

14. D. Sivaraj and V.E. Sasikala, A Study on Soft α-open sets, IOSR Journal of Mathematics (IOSR – JM), (Sep. – Oct. 2016), PP 70 – 74.

15. M. Suraiya Begum and M. Sheik John, Soft g*s-Closed Sets in Soft Topological Spaces, International Journal of Scientific Research and Education, (2016), 5466-5470.

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An Inventory Model For Non-instantaneous Deteriorating Items With Stock Level Demand Rate and Shortages

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Abstract

In this paper a deterministic inventory model for non-instantaneous deteriorating items with stock level demand rate has been developed. In this model shortages are permitted through the entirely backlogged. An economic order quantity (EOQ) model has been attained and the derived model is demonstrated by a numerical example.

Keywords: Inventory, EOQ model, deteriorating items, stock level demand rate.

I. Introduction

In practice, when shortages arise, some customers are willing to wait for backorder while others are annoyed to wait and therefore would turn to buy from other competitors or buy alternative item. For inventory models with stock dependent consumption rate, some authors assumed shortages to be completely backlogged while others assume the shortages to be partially backlogged. In this model shortages are allowed and the demand rate function is a power function of inventory level by proposing an EOQ model for delayed deteriorating items with linear inventory level dependent demand rate function and partial backlogging rate. We assume a fixed fraction of demand rate during the shortage period to be the backlogging rate as in Wee (1995). In the initial stage, inventory depletes down to a certain level of the inventory due to market demand only but dependent on stock level. In the second stage the inventory level gets depleted due to the effect of both market demand and deterioration but still dependent on stock until the inventory level falls to zero. In the final phase of the cycle, shortages are allowed and the unsatisfied demand is partially backlogged at a rate which is a fixed fraction of demand rate during the shortage period. Theoretical results for the optimal replenishment policy of the inventory system in order to minimize the total average system cost per unit time are established. Furthermore, some numerical examples are presented to illustrate the application of the model developed and then use the examples to study the effects of various changes in some model parameters on the decision variables.

Chang et al. (2010) modified the model of Padmanabhan and Vrat (1995) by considering a problem of determining the optimal replenishment policy for non-instantaneous deteriorating items with linearly stock dependent demand and constant unit holding cost. In the model, shortages are allowed and the backlogging rate is a variable and dependent on the waiting time for the next replenishment. Other inventory models using non-instantaneous deteriorating rate with linearly stock-dependent demand rate include those in Jain et al. (2008) and Uthayakumar and Geetha (2009) who used linear functional form to represent stock-dependent demand rate. Mahata and Goswami (2009) modified the model of Giri and

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Chaudhuri (1998) by considering non-instantaneous deteriorating items with fuzzy deteriorating rate and power-form stock-dependent demand rate. Other researchers who considered non-instantaneous deteriorating inventory for items with stock-dependent demand pattern are Chang and Lin (2010), Chang et al. (2010) and so on. Baraya and Sani (2011) developed an inventory model for delayed deteriorating items with inventory level dependent demand rate. In all of these models, the holding cost per unit of time was taken as a constant. This paper presents EOQ model for non-instantaneous deteriorating items with stock-dependent demand, allowing shortages and fixed partial backlogging rate, and it is organized as follows.

II. NOTATIONS

The following notations are used in developing the model.

Q : The Economic Order Quantity which enters into inventory at time $t = 0$

S_d : The inventory level up to which market demand rate is stock-dependent and reduction is based only on demand

θ : The rate of deterioration, constant fraction $\theta(0 < \theta < 1)$, of the on-hand inventory which deteriorates

C_o : The ordering cost per order for each cycle

t_d : The length of time for which the product has no deterioration and after this period of time, the inventory starts to reduces due to both deterioration and demand

T : The length of optimal ordering/production cycle time

t_1 : The length of time after which the shortage time begins

$B(t)$: The level of backorder at time t for which the product has shortage

$I_1(t)$: The on-hand inventory level at time t for which the product has no deterioration

$I_2(t)$: The on-hand inventory level at time t for which the product has deterioration

μ_1 : The unit inventory holding cost per unit time of item

μ_2 : The unit deterioration cost per unit time of item

M : Maximum Inventory Level

C_h : The inventory holding cost per unit item per unit time

$I(t)$: The inventory level at time t

μ_3 : The unit shortage cost for backlogged items per unit time of item

μ_4 : The unit opportunity cost due to lost sales per unit time of item

TC : The total inventory cost per unit time

ASSUMPTIONS

The inventory system is developed on the basis of following assumptions:

1. The unit cost of item and the traditional parameters of replenishment cost and holding cost are known and constant.
2. Shortages are allowed and Lead time is zero.
3. Rate of inflation is constant.
4. Replenishment is instantaneous at infinite rate.
5. The inventory system involves only one item and one stocking point in each cycle.
6. The time horizon of the inventory system is infinite. Only a typical inventory cycle length is considered, all remaining cycles are identical.

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7. A constant fraction θ representing the rate of deterioration per time period of the on-hand inventory when the item begins to deteriorate.
8. The inventory level dependent demand rate $D[I(t)]$ of the product at any instant of time t, is assumed a linearly dependent demand rate function such that $D[I(t)] = a + bI(t)$, where a is a positive constant, b is the stock dependent demand rate parameter, $0 < b < 1$, and $I(t)$ is the inventory level at time t.
9. Shortages are allowed and partially backlogged. The unsatisfied demand is backlogged, and the partial backlogging rate parameter is δ $(0 < \delta < 1)$. The extreme cases $\delta = 0$ and $\delta = 1$ represent the scenarios of no shortage allowed and complete backlogging, respectively.

III. Mathematical Model and Analysis

Using the above assumptions, a typical inventory cycle for the variation of inventory level with time where the shortages are allowed is shown in Figure 1.

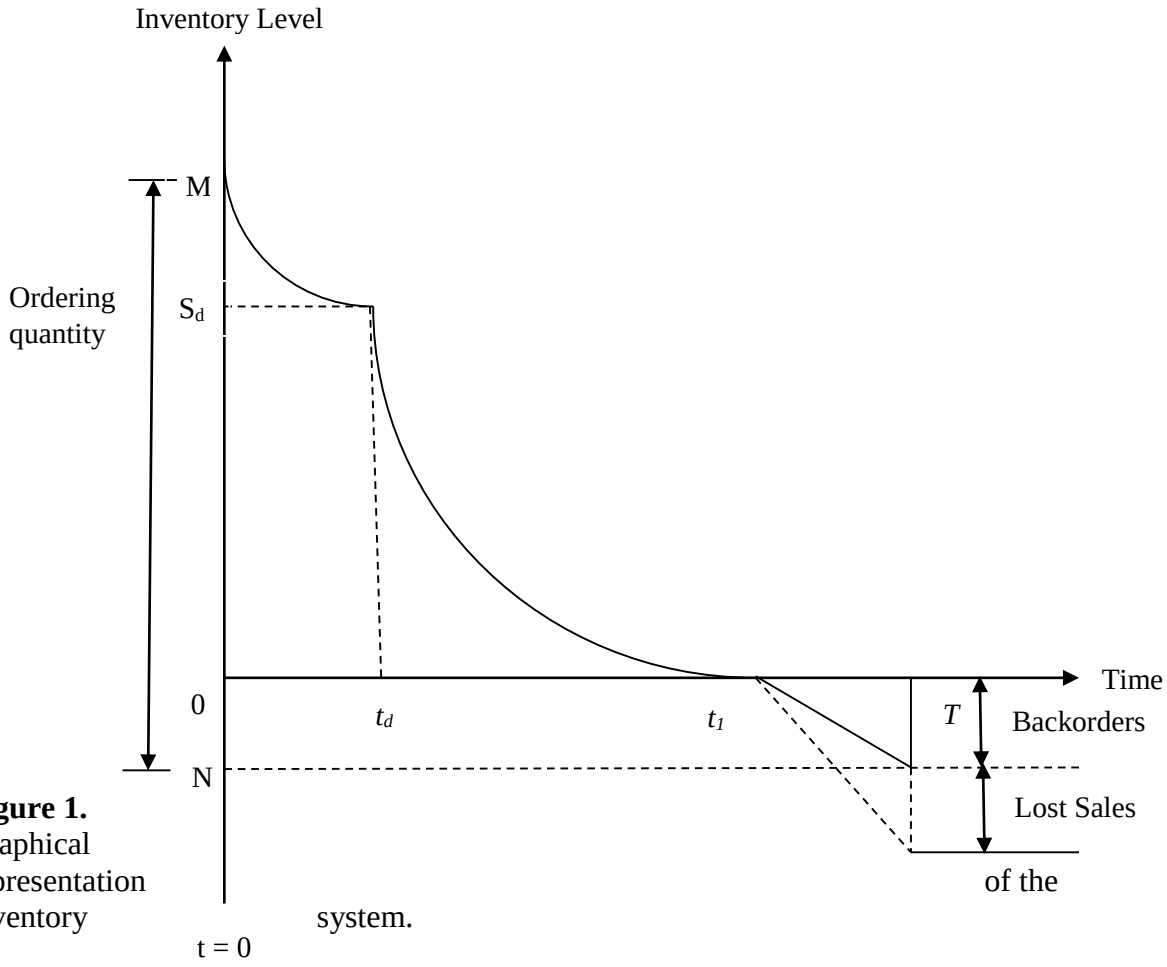

Figure 1. Graphical representation of the inventory system.

The inventory level dependent demand rate $D[I(t)]$ of the product at any instant of time t is a linear function of $I(t)$ and is given by $D[I(t)] = a + bI(t)$. An order of size M units of item is received in the inventory at the beginning of each cycle. During the time interval $[0, t_d]$, the on-hand inventory level gradually depletes from M to the level S_d at time $t = t_d$, due to market demand only. The inventory level further depletes from S_d to zero due

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to market demand and deterioration during the time interval$[t_d, t_1]$. Then during the time interval $[t_1, T]$shortages set in and unsatisfied demand is backlogged at a rate δ of the demand rate. The whole process is then repeated.

The inventory level at any time t in the interval $[0, t_d]$ can be represented by the following differential equation:

$$\frac{dI_1(t)}{dt} = -[a + bI_1(t)], \qquad 0 \le t \le t_d \qquad (1)$$

With the initial condition $I_1(0) = M$.

Solution of equation (1) is obtained as follows:

$\frac{dI_1(t)}{dt} + bI_1(t) = -a$ is a linear differential equation in $I_1(t)$ whose integrating factor is e^{bt}

So that

$$I_1(t)e^{bt} = -\frac{a}{b}e^{bt} + k_1 \ \ where \ \ k_1 \ is \ a \ constant.$$

With the initial condition $I_1(0) = M$, we have $k_1 = M + \frac{a}{b}$

and

$$I_1(t) = Me^{-bt} - \frac{a}{b}(1 - e^{-bt}) \qquad (2)$$

During the time interval $[t_d, t_1]$ the product depletes owing to the effect of both market demand and deterioration. Given that θ is a constant fraction of the on-hand inventory that gets deteriorated per unit time, the differential equation governing the instantaneous state of $I_2(t)$ over the time period in $[t_d, t_1]$ is given by

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -[a + bI_2(t)], \qquad t_d \le t \le t_1 \qquad (3)$$

with boundary condition $I_2(t_1) = 0$

Solution of equation (3) is obtained as follows:

$\frac{dI_2(t)}{dt} + (b + \theta)I_2(t) = -a$ is also a linear differential equation in $I_2(t)$ whose integrating factor is $e^{(b+\theta)t}$

So that

$I_2(t)e^{(b+\theta)t} = -\frac{a}{(b+\theta)}e^{(b+\theta)t} + k_2$, where k_2 is a constant of integration.

With the boundary condition $I_2(t_1) = 0$, we have $k_2 = \frac{a}{(b+\theta)}e^{(b+\theta)t_1}$

and

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$$I_2(t) = \frac{a}{(b+\theta)}\left[e^{(b+\theta)(t_1-t)} - 1\right] \quad (4)$$

During the shortage interval $[t_1, T]$, the demand at time t is partially backlogged at a fixed fraction $\delta(0 < \delta < 1)$ of the demand rate. Thus, the backorder level is governed by the differential equation:

$$\frac{dB(t)}{dt} = -a\delta, \quad t_1 \le t \le T \quad (5)$$

Solution of equation (5) is obtained as follows:

$B(t) = -a\delta t + k_3, \quad where \quad k_3$ is a constant.

With the boundary condition $(t_1) = 0$, we have $k_3 = a\delta t_1$

$$\therefore B(t) = -a\delta(t - t_1) \quad (6)$$

Since $I(t)$ must be continuous at $t = t_d$, then from equations (2) and (4), we get

$$Me^{-bt_d} - \frac{a}{b}(1 - e^{-bt_d}) = \frac{a}{(b+\theta)}\left[e^{(b+\theta)(t_1-t_d)} - 1\right]$$

(or)

$$M = \frac{a}{(b+\theta)}\left[e^{(b+\theta)(t_1-t_d)} - 1\right]e^{bt_d} + \frac{a}{b}(e^{bt_d} - 1) \quad (7)$$

Substituting equation (7) into equation (2), we obtain

$$I_1(t) = \left[\frac{a}{(b+\theta)}\left(e^{(b+\theta)(t_1-t_d)} - 1\right)e^{bt_d} + \frac{a}{b}(e^{bt_d} - 1)\right]e^{-bt} - \frac{a}{b}\ (1 - e^{-bt})$$

$$= \left[\frac{a}{(b+\theta)}\left(e^{(b+\theta)(t_1-t_d)} - 1\right)e^{b(t_d-t)} + \frac{a}{b}\left(e^{b(t_d-t)} - e^{-bt}\right)\right] - \frac{a}{b}\ (1 - e^{-bt})$$

$$= \left[\frac{a}{(b+\theta)}\left(e^{(b+\theta)(t_1-t_d)} - 1\right)e^{-b(t-t_d)} + \frac{a}{b}\left(e^{-b(t-t_d)} - e^{-bt}\right)\right] - \frac{a}{b}\ (1 - e^{-bt}) \quad (8)$$

Substituting $t = T$in equation (6), we obtain the maximum amount of demand backlogged per cycle which is given by

$$N = -B(t)$$

$$N = a\delta(T - t_1) \quad (9)$$

Combining equations (7) and (9), we obtain the order quantity, Q, as

$$Q = M + N$$

$$= \frac{a}{(b+\theta)}\left[e^{(b+\theta)(t_1-t_d)} - 1\right]e^{bt_d} + \frac{a}{b}(e^{bt_d} - 1) + a\delta(T - t_1) \quad (10)$$

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The inventory holding cost per cycle is given by

$$C_h = \int_0^{t_d} I_1(t)\,dt + \int_{t_d}^{t_1} I_2(t)dt$$

$$= \int_0^{t_d} \left(\left[\frac{a}{(b+\theta)} \left(e^{(b+\theta)(t_1-t_d)} - 1\right) e^{-b(t-t_d)} + \frac{a}{b}\left(e^{-b(t-t_d)} - e^{-bt}\right) \right] - \frac{a}{b}\ (1 - e^{-bt}) \right)\ dt + \frac{a}{(b+\theta)} \int_{t_d}^{t_1} \left[e^{(b+\theta)(t_1-t)} - 1\right] dt$$

$$= \frac{a}{b(b+\theta)} \left(1 - e^{(b+\theta)(t_1-t_d)}\right)(1 - e^{bt_d}) + \frac{a}{b^2}(e^{bt_d} - bt_d - 1) + \frac{a}{(b+\theta)^2} \left(e^{(b+\theta)(t_1-t_d)} - (b+\theta)(t_1 - t_d - 1)\right) \quad (11)$$

Total amount of items which deteriorate during the cycle time is given by

$C_d = I_2(t_d) -$ Demand during the time interval $[t_d, t_1]$

$$= \frac{a}{(b+\theta)} \left[e^{(b+\theta)(t_1-t_d)} - 1\right] - \left(\int_{t_d}^{t_1} a\ dt + \frac{ab}{b+\theta} \int_{t_d}^{t_1} \left[e^{(b+\theta)(t_1-t_d)} - 1\right] dt \right)$$

$$= \frac{a}{(b+\theta)} \left[e^{(b+\theta)(t_1-t_d)} - 1\right] - a(t_1 - t_d) - \frac{ab}{(b+\theta)^2} \left(e^{(b+\theta)(t_1-t_d)} - (b+\theta)(t_1 - t_d) - 1\right)$$

$$= \frac{a\theta}{(b+\theta)^2} \left\{e^{(b+\theta)(t_1-t_d)} - (b+\theta)(t_1 - t_d) - 1\right\} \quad (12)$$

The shortage cost is given by

$$S_c = \int_{t_1}^{T} \left(-B(t)\right) dt$$

$$= a\delta \int_{t_1}^{T} (t - t_1)\ dt$$

$$= \frac{a\delta}{2} (T - t_1)^2 \quad (13)$$

The lost sales cost is given by

$$C_l = \int_{t_1}^{T} a(1-\delta) dt$$

$$= a(1-\delta)(T - t_1) \quad (14)$$

Total inventory cost per unit time is obtained as:

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$$TC(t_1,T)=\frac{1}{T}\{OC+IHC+DC+SC+LC\}$$

$$=\frac{1}{T}\{C_o+\mu_1C_h+\mu_2C_d+\mu_3S_c+\mu_4C_l\}$$

$$=\frac{C_0}{T}+\frac{\mu_1}{T}\left\{\frac{a}{b(b+\theta)}\left(1-e^{(b+\theta)(t_1-t_d)}\right)(1-e^{bt_d})\right\}+\frac{\mu_1}{T}\left\{\frac{a}{b^2}(e^{bt_d}-bt_d-1)\right\}$$
$$+\frac{a(\mu_1+\theta\mu_2)}{(b+\theta)^2T}\left\{e^{(b+\theta)(t_1-t_d)}-(b+\theta)(t_1-t_d-1)\right\}+\frac{\mu_3}{T}\left\{\frac{a\delta}{2}(T-t_1)^2\right\}$$
$$+\frac{\mu_4}{T}\{a(1-\delta)(T-t_1)\} \qquad (15)$$

The necessary conditions for the total system cost per unit time in equation (15) to be minimized are

$$\frac{\partial}{\partial t_1}(TC(T,t_1)=0 \text{ and } \frac{\partial}{\partial T}(TC(T,t_1)=0 \qquad (16)$$

$$\frac{\partial}{\partial t_1}(TC(T,t_1)=-\frac{a\mu_1}{bT}(1-e^{bt_d})e^{(b+\theta)(t_1-t_d)}+\frac{a\mu_1}{(b+\theta)T}\left(e^{(b+\theta)(t_1-t_d)}-1\right)$$
$$+\frac{a\mu_2\theta}{(b+\theta)T}\left(e^{(b+\theta)(t_1-t_d)}-1\right)-\frac{\mu_3a\delta}{T}(T-t_1)-\frac{\mu_4a}{T}(1-\delta)=0$$

(17)

$$\frac{\partial}{\partial T}(TC(T,t_1)=-\frac{C_0}{T^2}-\frac{\mu_1}{T^2}\left\{\frac{a}{b(b+\theta)}\left(1-e^{(b+\theta)(t_1-t_d)}\right)(1-e^{bt_d})+\frac{a}{b^2}(e^{bt_d}-bt_d-1)\right\}$$
$$-\frac{a(\mu_1+\theta\mu_2)}{(b+\theta)^2T^2}\left\{e^{(b+\theta)(t_1-t_d)}-(b+\theta)(t_1-t_d)-1\right\}+\frac{\mu_3}{T^2}\left\{\frac{a\delta}{2}(T^2-t_1^2)\right\}$$
$$+\frac{\mu_4}{T^2}\{a(1-\delta)t_1\}=0$$

(18)

Our objective is to determine optimum value of t_1 and T to minimize TC. The values of t_1 for which

$\frac{\partial TC}{\partial t_1}=0$ and $\frac{\partial TC}{\partial T}=0$ Satisfying the condition

$$\left\{\left(\frac{\partial^2TC}{\partial t_1^2}\right)\left(\frac{\partial^2TC}{\partial T^2}\right)-\left(\frac{\partial^2TC}{\partial t_1\partial T}\right)^2\right\}>0$$

The optimal solution is obtained by using Mathematica software. This has been illustrated by the following numerical example.

IV. NUMERICAL EXAMPLE

We consider the following parameters values for $a=190, b=0.7,\ \theta=0.2, \delta=0.2,$

$C_0=80, \mu_1=0.5, \mu_2=2, \mu_3=2.5, \mu_4=1.2, t_d=0.1.$

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We obtain the optimal value of $t_1 = 0.724$, $T = 1.028$ $Q = 224.12$ and minimum total cost $(TC) = 153.04$.

V.CONCLUSION

In this paper, an EOQ model for non-instantaneous deteriorating items with stock-dependent demand, allowing shortages and fixed backlogging rate is presented. The impact of stock dependent demand rate, constant rate of deterioration and partial backlogging parameters on order quantity, maximum inventory level and total system cost per unit time are reported and proved by the numerical illustrations. For future research, the model can be improved by combining inflation, time value of money, variable holding cost, and so on.

VI. REFERENCES

1. Chang, C.T. ,Teng J.T. & Goyal, S. K., (2010), "Optimal Replenishment policies for noninstantaneous deteriorating items", *International Journal of Production Economics*, 123(1) 62-68
2. Chang, C., Chen, Y., Tsai T. and Wu, S. (2010). Inventory models with stock-and price-dependent demand for deteriorating items based on limited shelf space. *Yugoslav Journal of operational Research*, 20(1), 55-69.
3. Chang, J. H. and Lin, F. W. (2010). A partial backlogging inventory model for non-instantaneous deteriorating items with stock-dependent consumption rate under inflation. *Yugoslav Journal of Operations Research*, 20 (1), 35 – 54.
4. Giri, B. C., and Chaudhuri, K. S. (1998). Deterministic models of perishable inventory with stock-dependent demand rate and nonlinear holding cost. *European Journal of Operational Research*, 105, 467-474.
5. Jain, S., Kumar M. and Advan, P. (2008). An inventory model with inventory level-dependent demand rate, deterioration, partial backlogging and decrease in demand. *International Journal of Operations Research*, 5(3), 154-159.
6. Mahata, G. C. and Goswani, A. (2009). Fussy EOQ models for deteriorating items with stock dependent & non-linear holding costs. *International Journal of Applied Mathematics and Computer Sciences*, 5 (2), 94-98.
7. Padmanabhan, G. and Vrat, P. (1995). EOQ models for perishable items under stock dependent selling rate. *European Journal of Operational Research*, 86, 281– 292.
8. Uthayakumar, R., and Geetha, K. V. (2009). A Replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging. *Tamsui Oxford Journal of Mathematical Sciences*, 25(3), 313-332.
9. Wee, H. M. (1995). A deterministic lot-size inventory model for deteriorating items with shortages and a declining market. *Computers and Operations Research*, 22, 345-356.
10. Wu, K.S., Ouyang, L.Y. and Yang, C.T. (2006). An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. *International Journal of Production Economics*, 101, 369–384.

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EFFECT of CHEMICAL REACTION on MHD FLOW of a VISCO-ELASTIC FLUID THROUGH POROUS MEDIUM with SLIP CONDITION

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Abstract– This paper investigates that the effect of chemical reaction and slip effects MHD flow of a visco elastic fluid through porous medium. The governing nonlinear coupled equations are converted into ordinary differential equations by similarity transformation. These equations are solved using perturbation technique. The two-dimensional boundary layer flow over a vertical plate with slip conditions near the stagnation-point is analyzed in the presence of magnetic field. It is found that the elasticity of the fluid in the presence of destructive chemical reaction causes thinning of the velocity boundary layer and also noted that the velocity and concentration profiles increase on decreasing the chemical reaction parameter.

Keywords– Porous medium, visco-elastic fluid, Slip condition, suction velocity.

I. INTRODUCTION

Magnetohydrodynamics used in Geophysics i.e., the fluid core of the earth and other planets is theorized to be a huge Magnetohydrodynamics dynamo that generates the earth's magnetic field due to the motion of the molten rock. Acharya [1] studied the free convection and mass transfer flow through a porous medium in the presence of uniform magnetic field bounded by a vertical infinite surface with constant suction velocity and constant heat flux. Ahmed [2] discussed the effects of thermal radiation, soret, dufour, etc. on a free and forced convective developed boundary layer flow of a semi-infinite vertical porous plate. Mujumdar [4] considered an infinite vertical plate under the action of a constant pressure gradient is subjected to an external magnetic field of constant strength in the direction perpendicular to the plate and to the direction of flow. N P Singh [6], the induced magnetic field and constant heat flux is concerned with uniform transverse magnetic field under the flow of viscous fluid past a semi-infinite vertical porous plate. Sajid [7] solved the incompressible viscous fluid of a stretching sheet by using general slip boundary condition. Unsteady flow of conducting thin fluid filled with porous medium through a channel investigated the effect of slip condition, transverse magnetic field and radiative heat transfer by Hamza [8].

II. FORMULATION OF THE PROBLEM

The unsteady free convective flow of a visco-elastic fluid past an infinite vertical porous plate in a porous medium with time dependant oscillatory suction as well as permeability in presence of a transverse magnetic field is considered. Let x'- axis be along the plate in the direction of the flow and y'-axis normal to it. Let us consider the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison with the applied transverse magnetic field. The basic flow in the medium is therefore entirely due to the buoyancy force caused by the temperature difference between the wall and the medium. It is assumed that initially, at $t' \leq 0$, the plate as well as fluid are at the same temperature and also concentration of the species is very low so that the

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soret and dufour effects are neglected. When $t' > 0$, the temperature of the plate is instantaneously raised to T_w' and the concentration of the species is set to C_w'.

Let the permeability of the porous medium and the suction velocity be of the form

$$K_P(t') = K_p'(1+\varepsilon e^{i\omega' t'}) \quad \text{-----> (1)}$$

and

$$v(t') = -v_0(1+\varepsilon e^{i\omega' t'}) \quad \text{-----> (2)}$$

where $v_0 > 0$ and $\varepsilon < 1$ are positive constants . Under the above assumption with usual Boussineq's approximation, the governing equations and boundary conditions are given by

$$\rho\left(\frac{\partial u'}{\partial t'}+v\frac{\partial u'}{\partial y'}\right)=\mu\frac{\partial^2 u'}{\partial y'^2}+\rho g\beta(T'-T_\infty)+\rho g\beta^*(C'-C_\infty)$$
$$-\sigma B_0^2 u'-\frac{\mu u'}{K_p'(1+\varepsilon e^{i\omega' t'})}-k_0\left(\frac{\partial^3 u'}{\partial t'\partial y'^2}+v\frac{\partial^3 u'}{\partial y'^3}\right)$$
$$\text{-----> (3)}$$

$$\frac{\partial T'}{\partial t'}+v\frac{\partial T'}{\partial y'}=k\frac{\partial^2 T'}{\partial y'^2}+S'(T'-T_\infty) \quad \text{-----> (4)}$$

$$\frac{\partial C'}{\partial t'}+v\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-K_c'(C'-C_\infty) \quad \text{-----> (5)}$$

$u = u_{slip} = h\left(\frac{\partial u}{\partial y}\right)$, $T' = T_\infty + \varepsilon(T_w - T_\infty)e^{i\omega' t'}$, $C = C_\infty + \varepsilon(C_w - C_\infty)e^{i\omega' t'}$ at y=0 ,

$u \to U_\infty = U_0(1+\varepsilon e^{i\omega' t'})$, $T' \to T_\infty$, $C' \to C_\infty$ as $y \to \infty$

$$\text{-----> (6)}$$

In boundary layer approximation we have assumed μ and k_o to be of same order of magnitude. This will ensure that the viscosity and elasticity effects are of equal importance in determining the flow characteristics. Introducing the non-dimensional quantities,

$$y=\frac{v_0 y'}{v},\ t=\frac{v_0^2 t'}{4v},\ \omega=\frac{4v\omega'}{v_0^2},\ u=\frac{u'}{v_0},\ T=\frac{T'-T_\infty}{T_w-T_\infty}$$

$$C=\frac{C'-C_\infty}{C_w-C_\infty},\ S=\frac{vS'}{v_0^2},\ K_p=\frac{v_0^2 K_p'}{v^2},\ M^2=\frac{\sigma B_0^2 v}{\rho v_0^2},\ pr=\frac{v}{k},\ K_c=\frac{vK_c'}{v_0^2},\ Gc=\frac{vg\beta'(C_w-C_\infty)}{v_0^3},\ Gr=\frac{v\beta' g(T_w-T_\infty)}{v_0^3},$$

$$Sc=\frac{v}{D},\ R_c=\frac{v_0^2 k_0}{4\rho v^2}$$

$$\text{-----> (7)}$$

The equations (3), (4), (5) and (6) reduce to following non-dimensional form:

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$$\rho\left(\frac{\partial u'}{\partial t'}+v\frac{\partial u'}{\partial y'}\right)=\mu\frac{\partial^2 u'}{\partial y'^2}+\rho g\beta(T'-T_\infty)+\rho g\beta^*(C'-C_\infty)$$
$$-\sigma B_0^2u'-\frac{\mu u'}{K_p^{'}(1+\varepsilon e^{i\omega' t'})}-k_0\left(\frac{\partial^3 u'}{\partial t'\partial y'^2}+v\frac{\partial^3 u'}{\partial y'^3}\right)$$

$$\left(\frac{\partial u'}{\partial t'}+v\frac{\partial u'}{\partial y'}\right)=\frac{\mu}{\rho}\frac{\partial^2 u'}{\partial y'^2}+g\beta(T'-T_\infty)+g\beta^*(C'-C_\infty)-\frac{\sigma}{\rho}B_0^2u'$$
$$-\frac{\mu u'}{\rho K_p^{'}(1+\varepsilon e^{i\omega' t'})}-\frac{k_0}{\rho}\left(\frac{\partial^3 u'}{\partial t'\partial y'^2}+v\frac{\partial^3 u'}{\partial y'^3}\right)$$

$$\frac{\partial u^{'}}{\partial t^{'}}=\frac{1}{4}\frac{v_0^3}{v}\left(\frac{\partial u}{\partial t}\right)$$

$$v\frac{\partial u^{'}}{\partial y^{'}}=-\frac{v_0^3}{v}\left(\frac{\partial u}{\partial y}\right)(1+\varepsilon e^{i\omega t})$$

$$\frac{\mu}{\rho}\frac{\partial^2 u^{'}}{\partial y^{'2}}=\frac{\partial^2 u}{\partial y^2}\frac{v_0^3}{v}$$

$$g\beta(T^{'}-T_\infty)=g\beta T(T_w-T_\infty)$$

$$g\beta^*(C^{'}-C_\infty)=g\beta^* C(C_w-C_\infty)$$

$$-\frac{\sigma}{\rho}B_0^2u^{'}=-M^2u\frac{v_0^3}{v}$$

$$-\frac{vu^{'}}{k_p^{'}(1+\varepsilon e^{i\omega^{'}t^{'}})}=-\frac{v_0^3u}{vk_p(1+\varepsilon e^{i\omega t})}$$

$$-\frac{k_0}{\rho}\frac{\partial^3 u^{'}}{\partial t^{'}\partial y^{'2}}=-R_c\frac{\partial^3 u}{\partial t\partial y^2}\frac{v_0^3}{v}$$

$$-\frac{k_0}{\rho}\frac{v\partial^3 u^{'}}{\partial y^{'3}}=4R_c\frac{\partial^3 u}{\partial y^3}\frac{v_0^3}{v}(1+\varepsilon e^{i\omega t})$$

$$\frac{1}{4}\frac{\partial u}{\partial t}\frac{v_0^3}{v}-(1+\varepsilon e^{i\omega t})\frac{\partial u}{\partial y}\frac{v_0^3}{v}=\frac{\partial^2 u}{\partial y^2}\frac{v_0^3}{v}$$
$$+g\beta T(T_w-T_\infty)+g\beta^* C(C_w-C_\infty)$$
$$-\frac{u}{K_p(1+\varepsilon e^{i\omega t})}\frac{v_0^3}{v}-M^2u\frac{v_0^3}{v}$$
$$-R_c\left(\frac{\partial^3 u}{\partial t\partial y^2}\frac{v_0^3}{v}-4(1+\varepsilon e^{i\omega t})\frac{\partial^3 u}{\partial y^3}\frac{v_0^3}{v}\right)$$

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$$\frac{1}{4}\frac{\partial u}{\partial t}-(1+\varepsilon e^{i\omega t})\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+GrT+GcC-\frac{u}{K_p(1+\varepsilon e^{i\omega t})}-M^2u$$

$$-R_c\left(\frac{\partial^3 u}{\partial t\partial y^2}-4(1+\varepsilon e^{i\omega t})v\frac{\partial^3 u}{\partial y^3}\right)$$

----> (8)

$$\frac{\partial T'}{\partial t'}+v\frac{\partial T'}{\partial y'}=k\frac{\partial^2 T'}{\partial y'^2}+S'(T'-T_\infty)$$

$$\frac{\partial T'}{\partial t'} = \frac{v_0^2}{4v}(T_w-T_\infty)\frac{\partial T}{\partial t}$$

$$v\frac{\partial T'}{\partial y'} = -\frac{v_0^2}{v}(1+\varepsilon e^{i\omega t})\frac{\partial T}{\partial y}(T_w-T_\infty)$$

$$k\frac{\partial^2 T'}{\partial y'^2} = \frac{vv_0^2}{\Pr v^2}\frac{\partial^2 T}{\partial y^2}(T_w-T_\infty)$$

$$S'(T'-T_\infty) = \frac{STv_0^2}{v}(T_w-T_\infty)$$

$$\frac{\partial T'}{\partial t'}+v\frac{\partial T'}{\partial y'} = k\frac{\partial^2 T'}{\partial y'^2}+S'(T'-T_\infty)$$

$$\frac{v_0^2}{4v}(T_w-T_\infty)\frac{\partial T}{\partial t}-\frac{v_0^2}{v}(1+\varepsilon e^{i\omega t})\frac{\partial T}{\partial y}(T_w-T_\infty) = \frac{vv_0^2}{\Pr v^2}\frac{\partial^2 T}{\partial y^2}(T_w-T_\infty)+\frac{STv_0^2}{v}(T_w-T_\infty)$$

$$\frac{1}{4}\frac{\partial T}{\partial t}-(1+\varepsilon e^{i\omega t})\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 u}{\partial y^2}+ST$$

-----> (9)

$$\frac{\partial C'}{\partial t'}+v\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C}{\partial y'^2}-K_c'(C'-C_\infty)$$

$$\frac{\partial C'}{\partial t'} = \frac{1}{4}\frac{v_0^2}{v}\left(\frac{\partial C}{\partial t}\right)(C_w-C_\infty)$$

$$v\frac{\partial C'}{\partial y'} = -\frac{v_0^2}{v}\left(\frac{\partial C}{\partial y}\right)(1+\varepsilon e^{i\omega t})(C_w-C_\infty)$$

$$D\frac{\partial^2 C'}{\partial y'^2} = \frac{\partial^2 C}{\partial y^2}\frac{v_0^2}{vSC}(C_w-C_\infty)$$

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$$-K_c^{'}(C^{'}-C_\infty) = -K_c\frac{v_0^2}{\nu}(C_w-C_\infty)$$

$$\frac{1}{4}\frac{v_0^2}{\nu}\left(\frac{\partial C}{\partial t}\right)(C_w-C_\infty)-\frac{v_0^2}{\nu}\left(\frac{\partial C}{\partial y}\right)(1+\varepsilon e^{i\omega t})(C_w-C_\infty)$$

$$= \frac{\partial^2 C}{\partial y^2}\frac{v_0^2}{\nu SC}(C_w-C_\infty)-K_c\frac{v_0^2}{\nu}(C_w-C_\infty)$$

$$\frac{1}{4}\frac{\partial C}{\partial t}-(1+\varepsilon e^{i\omega t})\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+K_c C$$

----> (10)

u = u_{slip} = h$\frac{\partial u}{\partial y'}$, T =$1+\varepsilon e^{i\omega' t'}$, C = $1+\varepsilon e^{i\omega' t'}$ at $y=0,$

$u \to U_\infty = 1+\varepsilon e^{i\omega' t'}, T' \to 0, C' \to 0$ as $y \to 0$

-----> (11)

III. SOLUTION of the PROBLEM

In view of periodic suction, temperature and concentration at the plate, let the velocity, temperature, concentration in the neighbourhood of the plate be

$$u(y,t)=u_0(y)+\varepsilon u_1(y)e^{i\omega t},$$ -----> (12)

$$T(y,t)=T_0(y)+\varepsilon T_1(y)e^{i\omega t},$$ -----> (13)

$$C(y,t)=C_0(y)+\varepsilon C_1(y)e^{i\omega t}.$$ -----> (14)

Substituting equations (12) – (14) into (8) - (10) and equating the non-harmonic and harmonic terms, we get

$$R_c u_0'''+u_0''+u_0'-\left(M^2+\frac{1}{K_p}\right)u_0=-GrT_0-GcC_0,$$

-----> (15)

$$R_c u_1'''+(1-R_c i\omega)u_1''+u_1'-\left(M^2+\frac{1}{K_p}+\frac{i\omega}{4}\right)u_1$$

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$$= -R_c u_0''' + u_0' - GrT_1 - GcC_1 - \frac{u_0}{K_p},$$

-----> (16)

$$T_0'' + \Pr T_0' + \Pr S T_0 = 0,$$ -----> (17)

$$T_1'' + \Pr T_1' + \Pr(S - \frac{i\omega}{4})T_1 = -\Pr T_0',$$

----> (18)

$$C_0'' + ScC_0' - K_c ScC_0 = 0,$$ ----> (19)

$$C_1'' + ScC_1' - (\frac{i\omega}{4} + K_c)ScC_1 = -ScC_0'.$$

----> (20)

The boundary conditions now reduce to

at y = 0 , $u_0 = hu_0'$, $u_1 = hu_1'$, $T_0 = T_1 = 1$, $C_0 = C_1 = 1$

at y→∞ , $u_0 = u_1 = 1$, $T_0 = T_1 = 0$, $C_0 = C_1 = 0$ -----> (21)

The equations (15) and (16) are of third order but two boundary conditions are available. Therefore the perturbation method has been applied using R_c (R_c<<1) , the elastic parameter as the perturbation parameter.

$$u_0 = u_{00}(y) + R_c u_{01}(y) + o(R_c^2)$$

$$u_1 = u_{10}(y) + R_c u_{11}(y) + o(R_c^2)$$. ------> (22)

Inserting equation (22) into (15) and (16) and equating the coefficients of R^0_c and R_c to zero we have the following sets of ordinary differential equations:

A. ZEROTH ORDER EQUATIONS

$$u_{00}'' + u_0' - (M^2 + \frac{1}{K_p})u_{00} = -GrT_0 - GcC_{0,}$$ -----> (23)

$$u_{01}'' + u_{01}' - (M^2 + \frac{1}{K_p})u_{01} = -u_{00}'''$$. -----> (24)

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B. FIRST ORDER EQUATIONS

$$u_{10}'' + u_{10}' - (M^2 + \frac{1}{K_p} + \frac{i\omega}{4})u_{10} = -u_{00}' - GrT_1 - GcC_1 - \frac{u_{00}}{K_p},$$

-----> (25)

$$u_{11}'' + u_{11}' - (M^2 + \frac{1}{K_p} + \frac{i\omega}{4})u_{11} = -u_{10}''' - i\omega u_{10}'' - u_{00}''' - \frac{u_{01}}{K_p}.$$

-----> (26)

The corresponding boundary conditions are,

$$u_{00} = u_{01} = 0, \quad \text{as} \quad y = 0,$$

$$u_{10} = u_{11} \to 0, \quad \text{as } y \to \infty .$$

-----> (27)

Solving these differential equations with the help of boundary conditions we get,

$C_0'' + \text{Sc}C_0' + \text{Sc}K_cC_0 = 0$

$m^2 + \text{Sc m} - \text{Sc}K_c = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-Sc \pm \sqrt{Sc^2 + 4ScK_c}}{2}$

$C_0 = Ae^{\alpha_1 y} + Be^{-\alpha_1 y}$

Boundary conditions

y = 0, $C_0 = 1$

1 = A+B

$y \to \infty$, $C_0 = 0$

A=0

$C_0 = e^{-\alpha_1 y}$

$T_0'' + \text{Pr}C_0' + \text{PrS}C_0 = 0$

$m^2 + \text{Pr m} - \text{PrS} = 0$

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$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$m = \frac{-Pr \pm \sqrt{Pr^2-4SPr}}{2}$

$T_0 = Ae^{m_1 y} + Be^{-m_1 y}$

Boundary conditions

y=0 , $T_0 = 1$

1=A+B

$y \to \infty$, $T_0 = 0$

A = 0

$T_0 = e^{-m_1 y}$

$C_1'' + ScC_1' - (\frac{i\omega}{4} + K_c)ScC_1 = -ScC_0'.$

$m^2 + \text{Sc m} - (\frac{i\omega}{4} + K_c)\text{Sc} = 0$

$m = \frac{-Sc \pm \sqrt{Sc^2+(i\omega+4K_c)Sc}}{2}$

$C_1 = Ae^{m_2 y} + Be^{-m_2 y}$

Particular integral $= \frac{Sc\alpha_1}{m^2 + \text{Sc m} - (\frac{i\omega}{4}+K_c)\text{Sc}} e^{-\alpha_1 y}$

$= \frac{Sc\alpha_1}{\alpha_1^2 - \text{Sc}\alpha_1 - (\frac{i\omega}{4}+K_c)\text{Sc}} e^{-\alpha_1 y}$

$C_1 = Ae^{m_2 y} + Be^{-m_2 y} + \beta_1 e^{-\alpha_1 y}$

Boundary conditions

y=0, $C_1 = 0$

$0=A+B+\beta_1$

$y \to \infty$, $C_1 = 0$

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A=0

B= - β_1

$$C_1 = \beta_1(e^{-\alpha_1 y} - e^{-m_2 y})$$

$$T_1'' + \Pr T_1' + \Pr(S - \frac{i\omega}{4})T_1 = -\Pr T_0',$$

m^2 + Pr m – $(S - \frac{i\omega}{4})$ Pr = 0

m = $\frac{-Pr \pm \sqrt{Pr^2 - 4PrS + i\omega Pr}}{2}$

$$T_1 = Ae^{-m_3 y} + Be^{m_3 y}$$

-Pr T_0' = Pr $e^{-m_1 y}$ m_1

Particular integral = $\frac{Prm_1}{m_1^2 + \Pr m_1 - (S - \frac{i\omega}{4})\Pr}$ $e^{-m_1 y}$

= $\frac{4Prm_1}{4m_1^2 + 4\ \Pr m_1 - 4\ \mathrm{PrS} - \mathrm{i}\omega\mathrm{Pr}}$ $e^{-m_1 y}$

= $\frac{4m_1 i}{\omega}$ β_2 $e^{-m_1 y}$

Boundary conditions

y=0 , T_1 = 0

1 = A + B + $\frac{4m_1 i}{\omega}$ β_2

$y \to \infty$, T_1 = 0

B=0

A = 1 - $\frac{4m_1 i}{\omega}$ β_2

$$T_1 = e^{-m_3 y} + \frac{4im_1}{\omega}\beta_2(e^{-m_1 y} - e^{-m_3 y})$$

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$$u_{00}'' + u_0' - (M^2 + \frac{1}{K_p})u_{00} = -GrT_0 - GcC_0$$

$m^2 + m - a_1 = 0$

$m = \frac{-1 \pm \sqrt{1+4a_1}}{2}$

$u_{00} = Ae^{-m_4 y} + Be^{m_4 y}$

$-Gr\ T_0 = -\ Gr\ e^{-m_1 y}$

$-Gr\ C_0 = -\ Gc\ e^{-\alpha_1 y}$

$$P.I = -\frac{Gre^{-m_1 y}}{m_1^2 - m_1 - a_1} - \frac{Gre^{-\alpha_{11} y}}{\alpha_1^2 - \alpha_1 - a_1}$$

$$= A_1 e^{-m_1 y} + A_2 e^{-\alpha_1 y}$$

$$u_{00} = Ae^{-m_4 y} + Be^{m_4 y} + A_1 e^{-m_1 y} + A_2 e^{-\alpha_1 y}$$

Boundary conditions

y=0 , $u_{00} = 0$

$0 = A+B+A_1+A_2$

$y \to \infty$, $u_{00} = 0$

$A = -A_1-A_2$

$= A_3$

$$u_{00} = A_3 e^{-m_4 y} + A_1 e^{-m_1 y} + A_2 e^{-\alpha_1 y}$$

$$u_{01}'' + u_{01}' - (M^2 + \frac{1}{K_p})u_{01} = -u_{00}'''$$

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$m^2 + m - a_1 = 0$

$m = \frac{-1 \pm \sqrt{1+4a_1}}{2}$

$u_{00} = Ae^{-m_4 y} + Be^{m_4 y}$

$-u'''_{00} = A_3 m_4^3 e^{-m_4 y} + A_1 m_1^3 e^{-m_1 y} + A_2 \alpha_1^3 e^{-\alpha_1 y}$

$\text{P. I} = \frac{A_3 m_4^3 e^{-m_4 y}}{m_4^2 - m_4 - \alpha_1} + \frac{A_1 m_1^3 e^{-m_1 y}}{m_1^2 - m_1 - \alpha_1} + \frac{A_2 \alpha_1^3 e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_1}$

$= A_4 e^{-m_4 y} + A_5 e^{-m_1 y} + A_6 e^{-\alpha_1 y}$

$u_{01} = A_4 e^{-m_4 y} + A_5 e^{-m_1 y} + A_6 e^{-\alpha_1 y} + Ae^{-m_4 y} + Be^{m_4 y}$

Boundary conditions

y=0 , $u_{01} = 0$

$0 = A + B + A_4 + A_5 + A_6$

$y \to \infty$, $u_{01} = 0$

B = 0

$A = - A_4 - A_5 - A_6$

$A = A_7$

$u_{01} = A_4 e^{-m_4 y} + A_5 e^{-m_1 y} + A_6 e^{-\alpha_1 y} + A_7 e^{-m_4 y}$

$u''_{10} + u'_{10} - (M^2 + \frac{1}{K_p} + \frac{i\omega}{4}) u_{10} = -u'_{00} - GrT_1 - GcC_1 - \frac{u_{00}}{K_p},$

$m^2 + m - a_2 = 0$

$m = \frac{-1 \pm \sqrt{1+4a}}{2}$

$u_{10} = Ae^{-m_5 y} + Be^{m_5 y}$

$-u'_{00} = m_4 A_3 e^{-m_4 y} + m_1 A_1 e^{-m_1 y} + \alpha_1 A_2 e^{-\alpha_1 y}$

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$$-GrT_1 = -Gre^{-m_3 y} - \frac{Gr\beta_2 4im_1 e^{-m_1 y}}{\omega} - \frac{Gr\beta_2 4im_1 e^{-m_3 y}}{\omega}$$

$$-GcC_1 = -Gc\beta_1 e^{-\alpha_1 y} + Gc\beta_1 e^{-m_2 y}$$

$$-\frac{u_{00}}{K_p} = \frac{1}{K_p}\left[-A_3 e^{-m_4 y} - A_1 e^{-m_1 y} - A_2 e^{-\alpha_1 y}\right]$$

$$\text{P. I} = \frac{m_4 A_3 e^{-m_4 y}}{m_4^2 - m_4 - a_2} + \frac{m_1 A_1 e^{-m_1 y}}{m_1^2 - m_1 - a_2} + \frac{\alpha_1 A_2 e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - a_2}$$

$$- \frac{\beta_2 Gre^{-m_3 y}}{m_3^2 - m_3 - a_2} - \frac{\beta_2 4Grm_1 i/\omega e^{-m_1 y}}{m_1^2 - m_1 - a_2} + \frac{\beta_2 4Grm_1 i/\omega e^{-m_3 y}}{m_3^2 - m_3 - a_2} - \frac{\beta_1 Gce^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - a_2} + \frac{\beta_1 Gce^{-m_2 y}}{m_2^2 - m_2 - a_2} -$$

$$\frac{A_3/K_p\, e^{-m_4 y}}{m_4^2 - m_4 - a_2} - \frac{A_1/K_p\, e^{-m_1 y}}{m_1^2 - m_1 - a_2} - \frac{A_2/K_p\, e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - a_2}$$

$$\text{P.I} = A_{11}e^{-m_4 y} + A_8 e^{-m_1 y} + A_{12}e^{-\alpha_1 y} + A_{10}e^{-m_3 y} + A_9 e^{-m_2 y}$$

$$u_{10} = A_{11}e^{-m_4 y} + A_8 e^{-m_1 y} + A_{12}e^{-\alpha_1 y} + A_{10}e^{-m_3 y} + A_9 e^{-m_2 y} + Be^{m_5 y} + Ae^{-m_5 y}$$

Boundary conditions

y=0, $u_{10} = 0$

$0 = A + B + A_{11} + A_8 + A_{10} + A_{12} + A_9$

$y \to \infty$, $u_{10} = 0$

B=0

$A = - A_{11} - A_8 - A_{10} - A_{12} - A_9$

$A = A_{13}$

$$u_{10} = A_{11}e^{-m_4 y} + A_8 e^{-m_1 y} + A_{12}e^{-\alpha_1 y} + A_{10}e^{-m_3 y} + A_9 e^{-m_2 y} + A_{13}e^{-m_5 y}$$

$$u_{11}'' + u_{11}' - (M^2 + \frac{1}{K_p} + \frac{i\omega}{4})u_{11} = -u_{10}''' - i\omega u_{10}'' - u_{00}''' - \frac{u_{01}}{K_p}.$$

$m^2 + m - a_2 = 0$

$m = \frac{-1 \pm \sqrt{1+4a_2}}{2}$

$$u_{11} = Be^{-m_5 y} + Ae^{m_5 y}$$

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$$-u_{00}''' = A_3 m_4^3 e^{-m_4 y} + A_1 m_1^3 e^{-m_1 y} + A_2 \alpha_1^3 e^{-\alpha_1 y}$$

$$-u_{10}''' = m_5^3 A_{13} e^{-m_5 y} + m_1^3 A_8 e^{-m_1 y} + m_2^3 A_9 e^{-m_2 y} + m_3^3 A_{10} e^{-m_3 y} + m_4^3 A_{11} e^{-m_4 y} + \alpha_1^3 A_{12} e^{-\alpha_1 y}$$

$$-\frac{u_{01}}{K_p} = -\frac{A_7}{K_p} e^{-m_4 y} - \frac{A_4}{K_p} e^{-m_4 y} - \frac{A_5}{K_p} e^{-m_1 y} - \frac{A_6}{K_p} e^{-\alpha_1 y}$$

$$-i\omega u_{10}'' =$$

$$-i\omega m_5^2 A_{13} e^{-m_5 y} - i\omega m_1^2 A_8 e^{-m_1 y} - i\omega m_2^2 A_9 e^{-m_2 y} - i\omega m_3^2 A_{10} e^{-m_3 y} - i\omega m_4^2 A_{11} e^{-m_4 y} - i\omega \alpha_1^2 A_{12} e^{-\alpha_1 y}$$

P.I = $\frac{A_3 m_4^3 e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} + \frac{A_1 m_1^3 e^{-m_1 y}}{m_1^2 - m_1 - \alpha_2} + \frac{A_2 \alpha_1^3 e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_2} - \frac{A_7 / K_p\, e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} - \frac{A_4 / K_p\, e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} - \frac{A_6 / K_p\, e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_2} - \frac{A_5 / K_p\, e^{-m_1 y}}{m_1^2 - m_1 - \alpha_2} + \frac{A_8 m_1^3 e^{-m_1 y}}{m_1^2 - m_1 - \alpha_2} + \frac{A_{13} m_5^3 e^{-m_5 y}}{m_5^2 - m_5 - \alpha_2} + \frac{A_9 m_2^3 e^{-m_2 y}}{m_2^2 - m_2 - \alpha_2} + \frac{A_{10} m_3^3 e^{-m_3 y}}{m_3^2 - m_3 - \alpha_2} + \frac{A_{11} m_4^3 e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} + \frac{A_{12} \alpha_1^3 e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_2} - \frac{i\omega A_{13} m_5^2 e^{-m_5 y}}{m_5^2 - m_5 - \alpha_2} - \frac{i\omega A_8 m_1^2 e^{-m_1 y}}{m_1^2 - m_1 - \alpha_2} - \frac{i\omega A_9 m_2^2 e^{-m_2 y}}{m_2^2 - m_2 - \alpha_2} - \frac{i\omega A_{10} m_3^2 e^{-m_3 y}}{m_3^2 - m_3 - \alpha_2} - \frac{i\omega A_{11} m_4^2 e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} - \frac{i\omega A_{12} \alpha_1^2 e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_2}$

$u_{11} = Ae^{m_5 y} + Be^{-m_5 y} + \frac{A_3 m_4^3 e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} + \frac{A_1 m_1^3 e^{-m_1 y}}{m_1^2 - m_1 - \alpha_2} + \frac{A_2 \alpha_1^3 e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_2} - \frac{A_7 / K_p\, e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} - \frac{A_4 / K_p\, e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} - \frac{A_6 / K_p\, e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_2} - \frac{A_5 / K_p\, e^{-m_1 y}}{m_1^2 - m_1 - \alpha_2} + \frac{A_8 m_1^3 e^{-m_1 y}}{m_1^2 - m_1 - \alpha_2} + \frac{A_{13} m_5^3 e^{-m_5 y}}{m_5^2 - m_5 - \alpha_2} + \frac{A_9 m_2^3 e^{-m_2 y}}{m_2^2 - m_2 - \alpha_2} + \frac{A_{10} m_3^3 e^{-m_3 y}}{m_3^2 - m_3 - \alpha_2} + \frac{A_{11} m_4^3 e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} + \frac{A_{12} \alpha_1^3 e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_2} - \frac{i\omega A_{13} m_5^2 e^{-m_5 y}}{m_5^2 - m_5 - \alpha_2} - \frac{i\omega A_8 m_1^2 e^{-m_1 y}}{m_1^2 - m_1 - \alpha_2} - \frac{i\omega A_9 m_2^2 e^{-m_2 y}}{m_2^2 - m_2 - \alpha_2} - \frac{i\omega A_{10} m_3^2 e^{-m_3 y}}{m_3^2 - m_3 - \alpha_2} - \frac{i\omega A_{11} m_4^2 e^{-m_4 y}}{m_4^2 - m_4 - \alpha_2} - \frac{i\omega A_{12} \alpha_1^2 e^{-\alpha_1 y}}{\alpha_1^2 - \alpha_1 - \alpha_2}$

$$u_{11} = A_{18} e^{-m_4 y} + A_{15} e^{-m_1 y} + A_{19} e^{-\alpha_1 y} + Ae^{m_5 y} + Be^{-m_5 y} + A_{14} e^{-m_5 y} + A_{20} e^{-m_4 y} + A_{16} e^{-m_2 y} + A_{17} e^{-m_3 y}$$

Boundary conditions

y=0, u_{11}= 0

0 = A + B + A_{15} + A_{16} + A_{17} + A_{18} + A_{19} + A_{14} + A_{20}

$y \to \infty$, u_{11} = 0

A = 0

B = - A_{15} – A_{16} - A_{17} - A_{18} - A_{19} – A_{14} - A_{20}

B = A_{21}

$$u_{11} = A_{21} e^{-m_5 y} + A_{14} e^{-m_5 y} + A_{15} e^{-m_1 y} + A_{16} e^{-m_2 y} + A_{17} e^{-m_3 y} + A_{18} e^{-m_4 y} + A_{19} e^{-\alpha_1 y} + A_{20} e^{-m_4 y}$$

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$$u(y,t)=\{A_3e^{-m_4y}+A_1e^{-m_1y}+A_2e^{-\alpha_1 y}+R_c(A_7e^{-m_4y}+A_4e^{-m_4y}+A_5e^{-m_1y}+A_6e^{-\alpha_1 y})\}$$
$$+\varepsilon\{(A_{13}e^{-m_5y}+A_8e^{-m_1y}+A_9e^{-m_2y}+A_{10}e^{-m_3y}+A_{11}e^{-m_4y}+A_{12}e^{-\alpha_1 y})$$
$$+R_c(A_{21}e^{-m_5y}+A_{14}e^{-m_5y}+A_{15}e^{-m_1y}+A_{16}e^{-m_2y}$$
$$+A_{17}e^{-m_3y}+A_{18}e^{-m_4y}+A_{19}e^{-\alpha_1 y}+A_{20}e^{-m_4y})\}$$

$$T(y,t)=e^{-m_1y}+\varepsilon\left(e^{-m_3y}+\frac{4im_1}{\omega}\beta_2(e^{-m_1y}-e^{-m_3y})\right)e^{i\omega t}$$

$$C(y,t)=e^{-\alpha_1 y}+\varepsilon\beta_1(e^{-\alpha_1 y}-e^{-m_2y})e^{i\omega t}$$

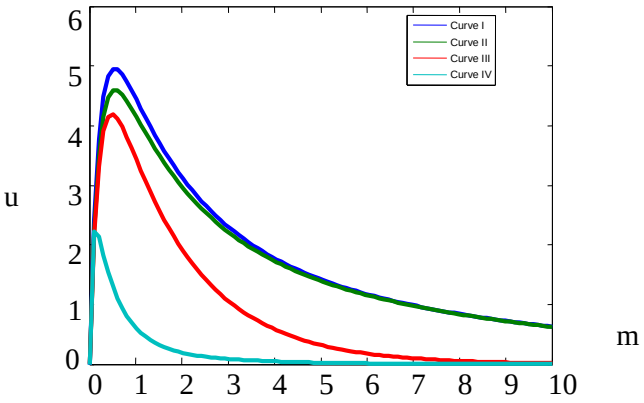

Fig. 1 Velocity Profile

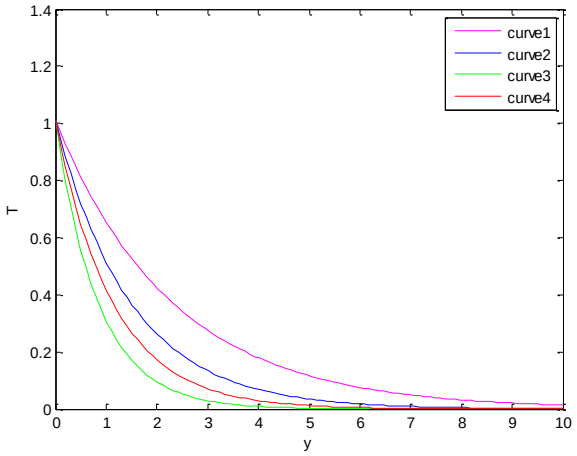

Fig. 2 Temperature

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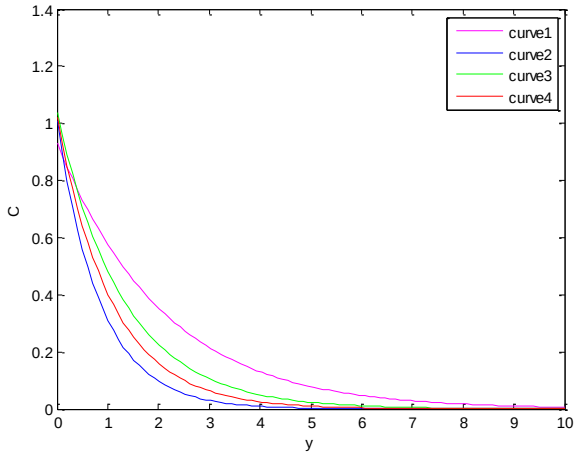

Fig. 3 Concentration

IV. RESULTS and DISCUSSIONS

In fig.1, the elastic parameter causes a significant decrease in velocity near the plate. Moreover, curves II and III shows a further decrease due to the presence of chemical reaction. Thus it may be concluded that the resistive forces due to magnetic field as well as heavier species yield significant reduction in velocity distribution at all points resulting in a thinner boundary layer.

Fig.2 represents the temperature distribution for various values of Pr and S. It is seen that an increase in Pr either in the presence of source/sink leads to decrease the thermal boundary layer thickness. Then it is evident that sharp fall of temperature is marked in the fluid layers close to the surface with low diffusion and in the presence of the sink.

In Fig.3, concentration profiles are displayed for various values of schmidt number (Sc) corresponding to diffusing chemical species of common interest in air. It is seen that fall of concentration occurs for heavier species in the presence of chemical reaction.

V. CONCLUSION

The elasticity of the fluid in the presence of destructive chemical reaction causes thinning of the velocity boundary layer. Presence of sink contributes to periodic motion giving rise to flow instability. Presence of chemical reaction enhances the rate of mass transfer which is a desired consequence of the flow of reacting species. The flow of reacting species in aqueous solution that is with higher Sc and Pr produces flow reversal. Presence of heat source and slow rate of thermal diffusion counteract each other in presence of chemical reaction.

REFERENCES

[1] Acharya M., Dash G.C. and Singh L.P., "*Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux*", Indian J. pure Appl. Math, 31(1)(2000), 1-18.
[2] Ahmed N., "*MHD free and forced convection with mass transfer from an infinite vertical porous plate*", Journal of Energy, Heat and Mass Transfer, 32(2010), 55-70.

Vol. 9 Issue 1, (Special Issue HICAAMMC- January 2019)
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[3] Ferdows M., Sattar M.A. and Siddiqui M.N.A., "*Numerical approach on parameters of the thermal radiation interaction with convection in boundary layer flow at a vertical plate with variable suction*", Thammasat Int. J. of Science and Technology, 9(2004), 19-28.

[4] Majumder M.K. and Deka R.K., "*MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation*", Rom. Jour. Phys., 52(5-7) (2007) , 565-573.

[5] Muthucumaraswamy R., Sundarraj M. and Subhramanian V., "*Unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion*", Int. J. Appl. Math. and Mech., 5(6)(2009) , 51-56.

[6] Singh N.P. and Atul K. Singh, "*MHD effects on heat and mass transfer in flow of a viscous fluid with induced magnetic field*", Ind. J. Pure Appl. Phys, 38(2000), 182-189.

[7] M. Sajid, N. Ali, Z. Abbas and T. Javed, "Stretching flows with general slip boundary condition", nt. J. Mod. Phys. B 2010.24: 5939-5947.

[8] M. M. Hamza , B.Y Isah , H. Usman ,"Unsteady Heat Transfer to MHD Oscillatory Flow through a Porous Medium under Slip Condition", Int. J. Com. App. (0975 – 8887) Vol 33– No.4, Nov. 2011.

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APPLICATIONS OF ROMAN DOMINATION

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ABSTRACT

A theory becomes meaningful if it has applications in real life. Almost all branches in graph theory have developed mainly due to variety of applications and motivation derived from the applications of Mathematical method or Technique to model or represent a real problem is called mathematical modeling. We make models either for representing an object or phenomenon or to solve an underlining problem to obtain a better solution. In this paper we discuss a few modeling situation of some variations of Roman domination and simplification or result that is offered by models.

KEY WORDS: Roman domination in graph, Roman domination number.

INTRODUCTION:

The concept of domination in graph theory originated from a chess board problem. It goes back to 1850's. Chess players were interested in the minimum number of queens such that every square on the chess board either contains a queen or is attacked by a queen. On chess board a queen move either vertically or horizontally or diagonally. All other moves of queen are invalid.

Domination is widely applies in many other fields .It is being used in facility location problems, where the number of facilities (e,g hospitals, fire stations etc) is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility.

Concept from domination also appear in problems involving finding sets of representatives , in monitoring communication or electrical networks and in land surveying (e.g, minimizing the number of places a surveyor must stand in order to take high measurement for an entire region). Here we discuss various applications of Roman domination and its variations. Graph theoretical originations are generally utilized in branch of knowledge and model various application program in various fields. In chemistry, it incorporates an investigation of molecules, construction of bonds and the study of atoms. Roman domination theory is utilized in biology and safeguarding of endeavors where a vertex corresponds to regions where certain species exist and the edges compares to movement way.

APPLICATIONS IN MILITARY AND STRATEGIC PLANNING

Due to the huge expenses of setting up military camps at a strategically important place, some places are selected to setup camps. Selection of such places are made after considering the total cost of maintaining the troops at the stations and the cost of defending the places which could be targeted by enemies.

Defense centers should be selected so that troops could defend all other places effectively from the places where they are based at. We can minimize the cost of defense by minimizing the number of centers. Thus the problem is equivalent the problem to find the

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minimal Roman dominating function of the underlying graph. It is based on the oversimplified assumption that total cost of maintaining defense center at a vertex in same.

We can restate the problem in a more realistic way. Let the cost of setting up a defense station at the vertex vi be s_i and running cost be r_i. The total cost of v_i is $t_i = s_i + r_i$. It is assumed that cost of a military operation from a vertex to another vertex is same between any pairs of vertices.

The total cost of Roman dominating set $R = \{V_1, V_2, \ldots\ldots V_S\}$ is $C_R = \sum_{i=1}^{s} t_1$. The best strategy of the problem is the Roman dominating function for which C_R is minimum. This minimum cost corresponds to a minimal Roman dominating function.

APPLICATIONS IN THE ANALYSIS OF SOCIAL RELATIONS

Social network analysis is a well established area of intensive research in social science .Graph theoretic methods are proved to be very effective in modeling problems in social science.

Edge vertex Roman domination can be used to analyze the social relations among individuals and to select representatives of a group subject to sum constrains. Members of a group usually have different opinions and they divide among themselves based on their opinion. Thus two or more subgroups are formed. Good relations among two members can be represented by the presence of an edge and the absence of edge indicates that the members hate with each other. We want to select some representative of the group subject to the following conditions.

- Atleast one member should represent each opinion group.
- Every member of the group must have good relation with atleast one representative.
- All representatives should have good relationship mutually.
- Number of representative should be minimum

In almost all situations people divide into two based on their opinion. If the graph modeling the problem has a partition of vertex set into V_1 and V_2 and their exists an edge e = uv such that

- $u \in V_1$, $v \in V_2$.
- u is connected to all vertices in V_2.
- v is connected to all vertices in V_1.

That is the associated graph has an edge vertex Roman dominating function f such that f(uv) =2 and f(a) = 0 for all remaining edges in G. Since some roads between regions may be one –way only, we can apply Roman domination to directed graphs. We may redefine Roman dominating function as $f : V \rightarrow \{0,1,2\}$ such that every vertex v for which f(v) = 0 has a neighbor u with f(u) =2 and there exists an arc uv.

APPLICATIONS IN SCHOOL BUS ROUTING

Most school in the country provides school buses for transporting children to and from school also operates under certain rules. One of which usually states that no child shall have to walk farther than, say one quarter km to a bus pickup point. Thus, they must construct a route for each bus that gets within on quarter km of every child in its assigned area. No bus ride can take more than some specified number of minutes, and limits on the number of children that a bus can carry at any one time.

COMPUTER COMMUNICATION NET WORK

Consider a computer network modeled by a graph G = (V, E) for which vertices represents computers and edges represent direct links between pairs of computers. Graphs are utilized in designing the database. Graph database utilizes a representation of a graph with nodes, edges and properties to represent and store data. This graph structure plays an important role in the designing database because it provides quick process implementation

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using different functions and properties of the graph. A graph database uses as, providing index fee adjacency system to store, interconnections can be evaluated using the tool, powerful tool for graph-like-query, Graph databases are often faster for associative datasets that map more directly to the structure of object-oriented applications.

OPERATING SYSTEM

Many practical problems can be solved within the help of graph data structures in the fielding systems such as job scheduling, resource allocation problems. For example graph coloring concept can be applied in job scheduling problems in CPU, jobs are assumed as nodes of the graph and edge between two jobs that cannot be executed simultaneously and there will be one to one relationship between the feasible scheduling of graph.

DATA MINING

Roman dominating theory is the major applications area of graph theory in data mining. Graph mining represents the relational aspects of data. The theoretically based approaches to graph-based data mining. They are sub graphs, sub graph isomorphism, graph invariants mining measures and solution methods. For example automated text analysis and text mining methods have received a great deal of attention because of the remarkable increase of digital documents. Currently we can store, organize and retrieve information in text documents automatically without looking at printed documents. Automated text analysis and text mining are becoming more and more in computer applications.

WEB DESIGNING

Web designing can be represented as a graph where the web pages are symbolized as vertex and the hyperlinks between then are symbolized as edges in the graph. This representation is acknowledged as a web graph. It discovers the interesting information. Another application is web community where the vertices represent classes of objects and each vertex represents one type of objects. In graph theory, such a graph is called a complete bipartite graph. Website development using graph representation in website utility of evaluation and link structure and covering all connected component and provide easy detection.

NETWORK SYSTEMS AND SECURITY

Roman dominating theory accepts a vital part in various territories. Graph theory assumes a critical part of the investigation of these essential issues in mobile ad-hoc networks (MANETs). MANETs dependably brings the consideration of research group. The graph theory concepts identified to connectivity, scalability, routing, and topology controls in MANETs. The solutions of MANETs in graph theory are graph spanners, the proximity of graph. By utilizing graph spanners one can decide a few graph spanners, which are helpful in planning the certain class of routing algorithms, the investigation of network clustering, partitioning, and network topology control. The concept of proximity is done in devising algorithms to construct the graph spanners locally. The studies of proximity graphs assume a vital role in topology control, availability of the network.

SOFTWARE ENGINEERING

The graph has many applications in software engineering. For example, during requirements specification data flow diagrams are used where vertices represent transformations and edges represents the data flow. Amid the design stage, the graphical outline is utilized for depicting relations among modules, while amid testing ,the control flow of a program related with McCabe's multifaceted nature measure which utilizes directed graphs for tending to the sequence of executed instructions and etc. Indeed even software process management has additionally utilization of network diagrams which involves graph algorithms.

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IMAGE PROCESSING

Image analysis is the methodology by which information from images is extracted. Image analysis is mainly preferred on digital image processing techniques. The image processing techniques can be improved using a graph -theoretic approach. The graph-based methods for image processing are segmentation, filtering, classification and clustering. The applications of the graph in image processing are to find edge boundaries using graph search algorithms in segmentation.

1. To compute the alignment of the picture.
2. Discovering mathematical constraints such as entropy by using spanning tree.
3. Shortest path algorithm is utilized to compute the distance between the interior pixels.

ARTIFICIAL INTELLIGENCE

To develop a model to change word " motion issues" into an algorithmic shape keeping in mind the end goal to be prepared by an intelligence tutoring system (ITS).The qualities of motion issues ought to be ordered first and next, proposing a model for the classification was passed out . With a specific end goal to tackle all categories of the issues, graph theory including reverse and forward tying methods of article intelligence were utilized. By embrace of graph theory into the motion problems and set forward some affirmation that the model unravels roughly the greater part of the motion issues.

FACILITY LOCATION PROBLEMS

Roman dominating sets in graphs are natural models for facility location problems in operation research. Facility location problems are concerned with the location of one or more facilities in a way that optimizes a certain objective such as minimizing transportation cost , providing equitable service to customers and capturing the largest market share.

CODING THEORY

The concept of Roman domination is also applied in coding theory .If one defines a graph, the vertices of which are the n-dimensional vectors with co-ordinates chosen from {1,2,…….p}, p > 1, and two vertices are adjacent if they differ in one coordinate ,then the sets of vectors which are (n, p) covering sets, single error correcting codes , or perfect covering sets are all dominating sets of the graph with determined additional properties.

CONCLUSION

The main aim of this paper is to present the importance of Roman domination theoretical ideas in various areas of science and Engineering for researches that they can use Roman domination in graph theoretical concepts for the research. An overview is presented especially to project the idea of graph theory. So the graph theory section of each paper is given importance than the other section. Researches may get some information related to Roman domination theory and its applications in various fields and can get some ideas related to their field of research.

REFERENCES

1. B.Tosuni,"Graph theory and computer science -An overview"., International journal of Academic Research and reflection,Vol.3, No.4,2015.
2. P.Pranav and P.Chirag, "Various graph and their applications in Real World", International journal of engineering Research and Technology, Vol.2, Issue 12, December 2013.
3. Haynes, T.W., Hedetniemi, S.T. and Slater, P.J., Fundamentals of domination in graphs; Marcel Dekkar, Inc-New York (1998).
4. E.W.Chambers, B.Kinnersley, N.Prince and D.B.West, External problems for Roman Domination. SIAM J. Discrete Math. 23 (2009)

International Journal of Management, IT & Engineering
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5. E.J.Cockayne, P.M. Dreyer, S.M. Hedetniemi and S.T. Hedetniemi, Roman domination in graphs, Discrete Math .278 (2004).
6. M.A, Henning and S.T. Hedetniemi, Defending the Roman Empire-A new strategy ,Discrete Math, 266 (2003).

International Journal of Management, IT & Engineering
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The lost sales and full backordering sustainable economic production quantity models under cap and trade policy

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Abstract

In recent years, organizations are paying emphasis on green and sustainable operations for different socio-economic reasons, such as government regulations, financial benefits, competitive advantage, ISO certification, customer satisfaction. They are integrating their operations to curb carbon emissions. Carbon emissions play the central role in global warming. Manufacturing firms are significant contributors to carbon emissions. In many countries, regulatory authorities are taking actions to reduce emissions. Carbon emission trading or cap-and-trade is generally accepted as one of the most effective market- based mechanisms to curb the amount of carbon emissions which has been broadly adopted by UN, EU and many governments to control the amount of carbon emission. Facing the cap and trade, firms can optimize their strategic decisions such as supply chain design or operations decisions in production, transportation and inventory to reduce carbon emissions. In this paper, we develop sustainable economic production quantity models with lost sales and full backordering under cap and trade policy. The objective is to maximize the total profit of the SEPQ inventory system which comprises setup, production, inventory holding, backordering, goodwill loss,obsolescence and emission costs. Finally, the formulated models are explained with numerical examples.

Keywords: Sustainable economic production quantity model(SEPQ), carbon emissions, cap-and-trade, full backordering, lost sales, profit maximization.

1.Introduction

Global warming is a major threat to our planet. It poses severe risk to the nature, human health and well-being. It has many catastrophic effects, such as rise of sea level, disruption in ecosystems, flood, drought, storm etc. The reason for global warming is emission of greenhouse gases (GHGs), such as Carbon dioxide (CO_2), methane, nitrous oxide etc. Among GHGs, CO_2 plays main role for global warming because of it's high heat-trapping property and long survival period in atmosphere. Main causes of GHG emissions are human activities, such as burning of fossil fuel for industrialization, transportation, generating electricity and deforestation for urbanization. In addition since the world economy depends upon the fossil fuel for industrial activities, the GHG emissions cannot be stopped completely. The primary task to mitigate global warming is to reduce the emission rate significantly.

Environmental pollution is a burning issue in recent time. Continuous emissions of greenhouse gases into the atmosphere due to rapid industrial progression have raised the threat to environment and existence of human civilization. As we know, carbon emissions are generated in almost all activities of firms, e.g., procurement, production, inventory holding, order processing, transportation and some others. Generally, carbon emissions from different activities are generated in different ways. For example, emissions from procurement are generated only when procurement activity is implemented, usually irrelevant to the procured quantity; while emissions from inventory holding depend upon the inventory quantity and inventory time. In production process, if the production lotsize is too small, lot of emissions are generated from frequent setups; otherwise, if the production lotsize is too large, lot of emissions are generated from inventory.

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Sustainable development is becoming a key issue for companies worldwide. Industry is one of the major contributors of GHG emissions. Industry emissions can be reduced by efficient use of modern technologies, efficient inventory control and wise managerial decisions during industrial activities. The intergovernmental panel on climate change reports that global warming is likely caused by carbon emissions. The view that human activities are responsible for increase in global temperature and environmental pollution has driven governments and different regulatory bodies to take initiatives and set realistic targets to curb the total amount of carbon emissions. Cap and trade is one of the most effective mechanism adopted by many countries to reduce the amount of emission. The cap and trade mechanism is a sensible regulation policy to reduce GHG emission. In this policy, the regulatory body first sets a maximum allowable emission limit (cap) from all possible sources, which is lowered over time, and then this amount is distributed or auctioned among the polluting entities as permits/allowances. A polluting entity must purchase permits/allowances. Polluters can trade the unused permits. The EU emissions trading system (EU ETS) is the world's first major carbon market.

The rest of the paper is organized as follows: Section 2 presents the literature review. Section 3 provides fundamental notations and assumptions. Section 4 describes the model formulation. Section 5 illustrates a numerical example. The paper concludes in Section 6. A list of references is also provided.

2.Literature review

Recently, a few works have explored sustainability issues in EOQ models. Inman (2002) revealed a number of primary propositions to set research guidelines in the field of environmentally consious operations management. Turkay (2008) analyzed five different approaches : Carbon Tax, Carbon Offsets, Direct Accounting, Cap & Trade, and Direct Cap in a lot-sizing model by considering business carbon footprint in the model. Bouchery et al. (2010) composed a basic sustainable lot-sizing model under direct accounting approach. In this approach sustainability issues are translated into costs and modeled as a part of the total cost function but other approaches focus more on governmental policies. A responsible EOQ model with some short list of environmental costs are proposed by Bonney and Jaber (2011). Wahab et al. (2011) incorporated environmental issues and concentrated especially on transportation emission costs in order to establish an optimal strategy by calculating fixed and variable carbon emission costs. Bouchery et al. (2012) suggested a multi-objective EOQ model that minimizes the cost and environmental damages. Absi et al. (2012) modeled a single-item multi-sourcing lot-sizing problems with fixed and variable carbon emissions so that each sourcing mode includes source location and transportation models. Glock et al. (2012) described a supply chain including a single supplier and single manufacturer and studied tradeoffs between demand, sustainability, costs and profit. Chen et al. (2013) explored the effects of parameters of carbon emission in lot sizing models in supply chain management(SCM) and showed the effect of carbon emissions in their work. Ozlu (2013) considered an EOQ model under cap and trade, carbon cap and carbon tax approaches. He investigated a retailer's joint decision on inventory replenishment and investment for carbon emission reduction. Digiesi et al. (2013) flourished an EOQ model considering environmental aspects with demand uncertainty. Gurtu et al. (2014) discussed the effect of emissions and changes in fuel price in as two echelon supply chain.

Over the last few years, Battini et al. (2014) described on one hand, the ordering, holding of inventory and obsolescence costs and on the other hand, emissions of obsolescence, transportation and holding of inventory costs and suggested a new model with direct accounting approach to calculate sustainable economic order quantity (called S-EOQ). Digiesi et al. (2015) inspected sustainability issues and effects in spare parts logistics. They included repair/replacement costs, such as scrapping cost, in their

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sustainable EOQ model. Hammami et al. (2015) considered production, transportation and holding of inventory emission costs with carbon taxes and direct cap approaches. They consolidated carbon emissions to other production-inventory costs in a multi-echelon system with fixed due dates. Scheel (2016) elaborated a framework called Sustainable Wealth based on Innovation and Technology (SWIT) that considers sustainable wealth sharing in a community. Taleizadeh (2014a) discussed an EOQ model with partial backordering and advance payments for an evaporating item. Taleizadeh (2014b) developed an EOQ model with consecutive payments for deteriorating items. Pentico et al. (2009) studied an EPQ model under partial backordering approach. Datta (2017) analyzed a production-inventory model under carbon tax system. He also suggested that emissions can be reduced to some extent by capital investment on green technology. He et al. (2015) discussed that firms under cap- and-trade mechanism receive a free carbon emissions cap during a finite time period and can trade the cap with other firms in the same carbon market. Guowei Hua, Han Qiao and Jian Li (2011) examined the optimal decisions of retailers under cap and trade. Arindam Ghosh, J.K. Jha & S.P. Sarmah (2016) enhanced a two-echelon serial supply chain with different carbon policies. Juanjuan Qin, Xiaojian Bai and Liangjie Xia (2015) developed the sustainable trade credit and inventory policies with demand related to credit period and the environmental sensitivity of the consumers under carbon cap-and-trade and carbon tax regulations. Ata Allah Taleizadeh, Vahid Reza Soleymanfar and Kannan Govindan (2017) considered shortage issues in the sustainable EPQ inventory problem.

This paper is an extension of "**Sustainable economic production quantity models for inventory systems with shortage**" by Ata Allah Taleizadeh, Vahid Reza Soleymanfar & Kannan Govindan. In this paper we incorporated cap and trade policy in the modelling of SEPQ with lost sales and full backordering. The purpose of cap and trade is to create a market price for emissions or pollutants that did not previously exist and address possible negative externalities.

3.Notations and Assumptions

In order to derive the mathematical models, we use the following notations and assumptions.

Parameters

D	Annual demand rate (unit/year)
P	Maximum annual rate of production (unit/year)
s	Price of a product unit ($/unit)
s'	Scrap price per unit ($/unit)
C_p	Unit production cost ($/unit)
C_s	Setup cost ($/setup)
C_i	Cost of holding a unit of inventory in a time unit ($/unit)
C_b	Backordering cost of a product unit of inventory in a time unit ($/unit)
C_g	Goodwill loss of an unsatisfied demand ($/unit)
C_l	Lost sale cost per unit $\left(C_l = \left(s - C_p\right) + C_g\right)$ ($/unit)
β	Backordered portion of stock-outs (percent)

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α	Obsolescence rate of inventory (percent)
b	Required space for each unit of product (cubic meters per unit)
a	The weight of an obsolete inventory (ton per unit)
C_{ei}	The average emission cost of carbon for inventory holding ($/m^3)
C_{eo}	Average disposal, waste collection, and emission cost of carbon for inventory obsolescence ($/ton)
C_{ep}	The emission cost of carbon for manufacturing each unit ($/unit)
ρ	buying/selling price of per unit of carbon
Z	Carbon emissions cap

Decision variables

T	The inventory cycle or time between two consecutive orders (time)
F	The fraction of period length with positive inventory level (percent)

Dependent variables

I	The highest quantity of inventory (unit/year)
$\bar{I}$	The annual average level of inventory (unit/year)
S	The highest quantity of shortage (unit/year)
B	The highest quantity of backordered $(B = \beta S)$ (unit/year)
$\bar{B}$	The annual average quantity of backordered (unit/year)
$\Pi(T,F)$	Total profit function (denoted by $\Pi_{LS}(T,F)$ for the lost sale SEPQ model, and by $\Pi_B(T,F)$ for the SEPQ model with full backordering) ($/year)
TS	Function of total sales ($/year)
CF_p	Cost function of production ($/year)
CF_{ep}	Cost function of "emission of production" ($/year)
CF_s	Set up cost function ($/year)
CF_i	Inventory holding cost function ($/year)
CF_{ei}	Cost function of "emission of inventory holding" ($/year)
CF_{obs}	The function of obsolescence cost of inventory ($/year)
CF_{eo}	Cost function of "emission of inventory obsolescence" ($/year)
CF_b	Backordering cost function ($/year)
CF_g	Goodwill loss cost function ($/year)

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$TE(T,F)$ Total emission cost function ($/year)

The assumptions of this model are:

1. The company deals with a single product.
2. The problem is modeled in one period to find the optimal values of decision variables since all of the periods are similar.
3. Mode of transportation is single to transport all products to the customers.
4. Demand is deterministic.
5. Finite production rate and the total production capacity is given.
6. The decisions are made in a single period and not in multiple periods; therefore, the carbon emission cap is fixed without considering the effect of time.
7. The selling price and buying cost per unit of carbon is same.

4.Mathematical model

Cap and Trade

Under cap-and-trade policy firms are allowed to emit carbon within a specified level over a planning horizon (e.g., one year), which is called cap. If the firm crosses the cap during its operations, then the firm has to buy carbon credits from other firms. If the firm emits lower than the specified level, it earns carbon credit that can be sold to other firms.

4.1 Modelling of the SEPQ with lost sales under cap and trade

The total emission cost function involving the emission of inventory holding (CF_{ep}), emission of inventory obsolescence (CF_{eo}) and emission of production (CF_{ep}) is given by

$$TE(T,F) = CF_{ep} + CF_{ei} + CF_{eo}$$

$$= C_{ep}D + C_{ei}b\bar{I} + \alpha a\bar{I}C_{eo} \quad (1)$$

The profit function of the SEPQ with lost sales is $\Pi_{LS}(T,F)$. Here, $\rho[Z - TE(T,F)]$ represents the emissions cost or revenue results from buying or selling permits depending upon the usage of given permits. The profit function is given by

$$\Pi_{LS}(T,F) = TS - CF_p - CF_s - CF_i - CF_{obs} - CF_g + \rho[Z - TE(T,F)]$$

$$= sD - C_pD - \frac{C_s}{T} - C_i\bar{I} - \alpha(s - s')\bar{I} - C_gD(1-F) + \rho\left[Z - C_{ep}D - C_{ei}b\bar{I} - \alpha a\bar{I}C_{eo}\right] \quad (2)$$

Where, from Pentico et al. (2009),

$$\bar{I} = \frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) \quad (3)$$

Substituting $\bar{I}$ into the profit function (Equation (2)) we have:

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$$\Pi_{LS}(T,F) = sD - C_pD - \frac{C_s}{T} - C_i\frac{DTF^2}{2}\left(1-\frac{D}{P}\right) - \alpha(s-s')\frac{DTF^2}{2}\left(1-\frac{D}{P}\right) - C_gD(1-F) + \rho Z - \rho C_{ep}D - \rho C_{ei}b\frac{DTF^2}{2}\left(1-\frac{D}{P}\right) - \rho\alpha a\frac{DTF^2}{2}\left(1-\frac{D}{P}\right)C_{eo} \quad (4)$$

To simplify the notation, we define

$$C_i' = C_i\left(1-\frac{D}{P}\right) \quad (5)$$

$$C_{ei}' = C_{ei}\left(1-\frac{D}{P}\right) \quad (6)$$

$$s'' = (s-s')\left(1-\frac{D}{P}\right) \quad (7)$$

$$C_{eo}' = C_{eo}\left(1-\frac{D}{P}\right) \quad (8)$$

Hence the above profit function (4) changes as below

$$\Pi_{LS}(T,F) = sD - C_pD - \frac{C_s}{T} - C_i'\frac{DTF^2}{2} - \alpha s''\frac{DTF^2}{2} - C_gD(1-F) + \rho Z - \rho C_{ep}D - \rho C_{ei}'b\frac{DTF^2}{2} - \rho\alpha aC_{eo}'\frac{DTF^2}{2} \quad (9)$$

To find the optimal policy of the system,taking the partial derivative of profit function (9) with respect to period length and setting this derivative equal to zero, we get

$$T = \frac{1}{F}\sqrt{\frac{2C_s}{\omega D}} \quad (10)$$

Where,

$$\omega = C_i' + \alpha s'' + \rho bC_{ei}' + \rho\alpha aC_{eo}' \quad (11)$$

Using (11) in (9), the profit function becomes

$$\Pi_{LS}(T,F) = (s - C_p - \rho C_{ep})DF - \frac{C_s}{T} - \frac{\omega DTF^2}{2} + \rho Z - C_gD(1-F) \quad (12)$$

Substituting T in (12) we obtain

$$\Pi_{LS}(F) = \left[(C_l - \rho C_{ep})D - \sqrt{2C_s\omega D}\right]F + \rho Z - C_gD \quad (13)$$

Now, $\Pi_{LS}(F)$ is a linear function with respect to the variable F. The maximum profit is determined by taking into account the slope of the function $\Pi_{LS}(F)$. Thus, we have:

1. If $(C_l - \rho C_{ep})D \geq \sqrt{2C_s\omega D}$, then the maximum profit is obtained when $F^* = 1$. This profit is given by
$$\Pi^* = \Pi_{LS}(F^*) = \left[(C_l - \rho C_{ep})D - \sqrt{2C_s\omega D}\right] + \rho Z - C_gD \quad (14)$$

   In this case, the optimal inventory cycle is

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$$T^* = \sqrt{\frac{2C_s}{\omega D}} \qquad (15)$$

2. If $(C_l - \rho C_{ep})D < \sqrt{2C_s\omega D}$, then the maximum profit is obtained when $F^* = 0$ and the optimal inventory cycle is $T^* = \infty$. It means no inventories are carried and there are always lost sales.

4.2.Modelling of the SEPQ with full backordering under cap and trade

The profit function of the SEPQ with full backordering is $\Pi_B(T,F)$. Here, $\rho[Z - TE(T,F)]$ represents the emissions cost or revenue results from buying or selling permits depending upon the usage of given permits. The profit function is given by

$$\Pi_B(T,F) = TS - CF_p - CF_s - CF_i - CF_{obs} - CF_b + \rho[Z - TE(T,F)] \qquad (16)$$

Using (1) in (16), we get

$$\Pi_B(T,F) = sD - C_pD - \frac{C_s}{T} - C_i\bar{I} - \alpha(s - s')\bar{I} - C_b\bar{B} + \rho\left[Z - C_{ep}D - C_{ei}b\bar{I} - \alpha a\bar{I}C_{eo}\right]$$

(17)

Where, from Pentico et al.(2009)

$$\bar{I} = \frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) \qquad (18)$$

$$\bar{B} = \frac{DT(1-F)^2}{2}\left(1 - \frac{D}{P}\right) \qquad (19)$$

Substituting equations (18) and (19) into Equation (17), the profit function becomes

$$\begin{aligned}\Pi_B(T,F) = {} & (s - C_p - \rho C_{ep})D - \frac{C_s}{T} - C_i\frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) - \alpha(s - s')\frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) \\ & - C_b\frac{DT(1-F)^2}{2}\left(1 - \frac{D}{P}\right) + \rho Z - \rho C_{ei}b\frac{DTF^2}{2}\left(1 - \frac{D}{P}\right) \\ & - \rho\alpha a C_{eo}\frac{DTF^2}{2}\left(1 - \frac{D}{P}\right)\end{aligned} \qquad (20)$$

Substituting equations (5) to (8) and using (11) in Equation (20) we have

$$\Pi_B(T,F) = (s - C_P - \rho C_{ep})D - \frac{C_s}{T} + \rho Z - \frac{\omega DTF^2}{2} - C_b\frac{DT(1-F)^2}{2}\left(1 - \frac{D}{P}\right) \qquad (21)$$

Maximizing the objective function presented in Equation (21) is similar to minimizing the following function.

$$\pi_1(T,F) = \frac{\lambda_1}{T} + T[\lambda_2F^2 - 2\lambda_3F + \lambda_3] \qquad (22)$$

Where the new parameters λ_1, λ_2 and λ_3 are

$$\lambda_1 = C_s > 0 \qquad (23)$$

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$$\lambda_2 = \frac{\omega D}{2} + \left(1 - \frac{D}{P}\right) C_b D/2 > 0 \quad (24)$$

$$\lambda_3 = \left(1 - \frac{D}{P}\right) C_b D/2 > 0 \quad (25)$$

Setting the first partial derivatives of $\pi_1(T,F)$ with respect to F and T equal to zero, we obtain the optimal values of decision variables. So we have

$$F = \frac{\left(1 - \frac{D}{P}\right) C_b}{\omega + \left(1 - \frac{D}{P}\right) C_b} \quad (26)$$

The optimum length of the period is:

$$T = \sqrt{\frac{2C_s\left(\omega + C_b\left(1 - \frac{D}{P}\right)\right)}{\omega\left(1 - \frac{D}{P}\right) C_b D}} \quad (27)$$

Finally, the maximum profit $\Pi_B(T,F)$ can be determined by substituting results of equations (26) and (27) in Equation (21).

5.Numerical example

In this section we provide numerical example for the illustration of above proposed models. The parameters used are given as

$D = 40$ units/year, P =100 units/year, s =10 \$/unit, s' =5 \$/unit, $C_P = 7$ \$/unit, α =10%, $C_s = 20$ \$/order, $C_i = 2.5$ \$/unit, $C_b = 3$ \$/unit, $C_g = 1$ ton/unit, $b = 1.7$ m^3/unit, $a = 2$ ton/unit, $C_{ei} = 0.55$ \$/m^3, $C_{eo} = 13$ \$/ton, $C_{ep} = 0.3$ \$/unit, $\rho = 0.2$, $Z = 60$ ton/year, $C_l = (s - C_p) + C_g = 4$\$/unit.

From equation (11) we get $\omega = 1.5 + 0.1 * 3 + 0.2 * 1.7 * 0.33 + 0.2 * 0.1 * 2 * 7.8 = 2.2242$ \$/unit

Example 1: Suppose that $\beta = 0$. In this case we are in the lost sales SEPQ model under cap and trade. The optimal policy depends on the values $(C_l - \rho C_{ep})D = 157.6$, and $\sqrt{2C_s\omega D} = 59.655$. As $(C_l - \rho C_{ep})D > \sqrt{2C_s\omega D}$, then the optimal policy is given by $\boldsymbol{F^* = 1}$ and $\boldsymbol{T^* = 0.6705}$. From equation (14) the maximum profit is $\boldsymbol{\Pi_{LS}^* = 69.945}$ **\$/year**.

Example 2: Suppose that $\beta = 1$. In this case, we are in the full backordering SEPQ model under cap and trade. The optimal policy is given by equations (26) and (27). Thus, we have $\boldsymbol{F^* = 0.447}$ and $\boldsymbol{T^* = 1}$ **year**. From equation (21), the maximum profit is $\boldsymbol{\Pi_B^* = 89.703}$ **\$/year**.

6.Conclusion

In this paper, a profit maximization SEPQ inventory model is discussed under cap and trade policy with two different shortage situations. Carbon cap and trade gives the flexibility to trade carbon in the market and also encourage organization to emit less.Therefore, cap-and-trade is an effective method to reduce emissions from various sources and thereby increase the total profit or to minimize the total cost of the system. Numerical examples are provided to demonstrate the proposed models.

References

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1. N Absi, S Dauzere-Peres, S Kedad-Sidhoum, B Penz, C Rapine. The Single-Item Green Lot-Sizing Problem with Fixed Carbon, Third International Workshop on Lot-Sizing (IWLS).Rotterdam: Erasmus University Rotterdam, 2012.
2. Arindam Ghosh, J.K. Jha & S.P. Sarmah. Optimizing a two-echelon serial supply chain with different carbon policies. International journal of sustainable engineering, 2016, 9(6): 363-377.
3. D Battini, A Persona, F Sgarbossa. A Sustainable EOQ model: Theoretical formulation andapplications, International Journal of Production Economics, 2014, 149: 145-153.
4. Y Bouchery, A Ghaffari, Z Jemai, Y Dallery. How to Incorporate Sustainability Issues intoTraditional Operations, Research Report, Laboratoire Génie Industriel, Paris: Ecole CentraleParis, 2010.
5. Y Bouchery, A Ghaffari, Z Jemai, Y Dallery. Including Sustainability Criteria into InventoryModels. European Journal of Operational Research, 2012, 222: 229-240.
6. M Bonney, M Jaber. Environmentally responsible inventory models: non-classical.International Journal of Production Economics, 2011, 133(1): 43-53.
7. X Chen, S Benjaafar, A Elomri. The carbon-constrained EOQ, Operations Research Letters,2013, 41: 172-179.
8. T K Datta. Effect of green technology investment on a production-inventory system with carbon tax. Advances in operations research, (2017),http://doi: org/ 10.1155/2017/4834839.
9. S Digiesi, G Mossa, S Rubino. A sustainable EOQ model for repairable spare parts under uncertain demand, IMA Journal of Management Mathematics, 2015, 26(2): 185-203.
10. S Digiesi, G Mossa, G Mummolo. A Sustainable Order Quantity Model under UncertainProduct Demand, 7th IFAC Conference on Manufacturing Modelling, Management andControl. St. Petersburg: International Federation of Automatic Control, 2013, 664-669.
11. C H Glock, M Y Jaber, C Searcy. Sustainability strategies in an EPQ model with price andquality sensitive demand. The International Journal of Logistics Management, 2012, 23(3):340-359.
12. A Gurtu, M Y Jaber, C Searcy. Impact of fuel price and emissions on inventory policies,Appl. Math. Modelling, 2014, doi: http://dx.doi.org/10.1016/j.apm.2014.08.001.
13. R Hammami, I Nouira, Y Frein. Carbon emissions in a multi-echelon production-inventorymodel with lead time constraints, International Journal of Production Economics, 2015, 164:292-307.
14. P He, W Zhang and X Xu. Production lot sizing and carbon emission under cap and trade and carbon tax regulations. Journal of cleaner production (2015),103: 241-248.
15. G Hua, H Qiao & L Jian.Optimal Order Lot Sizing and Pricing with CarbonTrade (M arch 22, 2011). Available at SSRN: http://dx.doi.org/10.2139/ssrn.1796507.
16. R A Inman. Implications of environmental management for operations, Production Planning& Control: The Management of Operations, 2002, 13(1): 47-55.
17. Juanjuan Qin, Xiaojian Bai & Liangjie Xia. Sustainable trade credit and replenishment policies under the cap-and-trade and carbon tax regulations. Sustainability (2015),7(12): 16340-16361.
18. H Ozlu. Joint Decisions on Inventory Replenishment and Emission Reduction InvestmentUnder Different Emission Regulations, MSc. Thesis. Graduate School of Engineering andScience, Bilkent University, 2013.
19. D W Pentico, M J Drake, C Toews. The deterministic EPQ with partial backordering: A newapproach, Omega, 2009, 37: 624-636.

International Journal of Management, IT & Engineering
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20. C Scheel. Beyond Sustainability: Transforming industrial zero-valued residues intoincreasing economic returns, Journal of Cleaner Production, 2016, doi:10.1016/j.jclepro.2016.05.018.
21. A A Taleizadeh. An Economic Order Quantity Model with Partial Backordering and Advance Payments for an Evaporating Item, International Journal of Production Economic,2014a, 155: 185–193.
22. A A Taleizadeh. An Economic Order Quantity Model with Consecutive Payments for Deteriorating Items, Applied Mathematical Modeling, 2014b, 38: 5357–5366.
23. A A Taleizadeh, V R Soleymanfar, K Govindan. Sustainable economic production quantity models for inventory systems with shortage, Journal of cleaner production(2017), 174: 1011-1020.
24. M Turkay. Environmentally conscious supply chain management, Process SystemsEngineering. Weinheim: WILEY-VCH, 2008, 45-86.
25. M I M Wahab, S M H Mamun, P Ongkunaruk. EOQ models for a coordinated two-levelinternational supply chain considering imperfect items and environmental impact,International Journal of Production Economics, 2011, 134(1): 151-158.

International Journal of Management, IT & Engineering
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A Study on an Integrated Three Echelon Inventory Model with Quality Improvement Along with the Screening Errors and SSMD Policy in the Transportation

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ABSTRACT:

Each Business individuals want is to hold their customer's for their item, with the goal that they plan to expand their capital investment to deliver the great nature of the items for fulfilling the customer's interest. In any case, anyway every shipment to purchaser contains a small amount of defective items. This proposed paper is to address the three echelon inventory model with the quality enhancement alongside the likelihood of screening errors. And furthermore for the nature advantageous this proposed model focuses the single setup multiple delivery process in the transportation for lessening the total system cost. Mathematical model and the numerical example are given to show the proposed model.

Keywords:

Three echelon, inventory, supply chain, SSMD policy, Transportation, Carbon emission

Introduction:

An inventory network is an arrangement of associations, individuals, exercises, data, and assets engaged with moving an item or administration from manufacturer to customer. In today's competitive business world, most of the companies face the challenges, in order to reduce the total system cost and provide the quality product to their customers. Reducing the total system cost and retain the customers are the important key features of supply chain management in the supply chain. Integrated inventory model is set more interested in today academicians and the research scholars. In 1976 Goyal [3] has introduced a single-supplier, single retailer integrated inventory model, following him Banerjee in year the 1986 [1] has extended the Goyal's model and assumed that the joint economic lot size model. Goyal, [10] extended the Banerjee's model with vendor's production quantity as an integer multiple of buyer's ordering quantity.

Cardenas Barron et al., (2012) [2] considered an integrated production inventory model with three layer supply chain and developed an improved heuristic approach to obtain the solutions. Sarkar et al., (2016b) [11] introduced the effect of variable transportation and carbon emission in a three-echelon supply chain model. In this work the author has introduced a SSMD policy instead of SSSD policy.

The second stream of this research is an imperfect production process. The vendor believes that the every shipment to the buyer is a good quality product. But in nature every shipment to the buyer contains a fraction of imperfect items. Imperfect quality products are the major drawback for the industrialists. To overcome these issues the vendor has increased their capital investment to produce the good quality products. Otherwise they can sell the imperfect items or buy the new products in a local market. Reducing setup cost and improving product quality play an important role in modern competitive business world

Porteus (1985) [6], Rosenblatt, and Lee (1986) [8] who are first authors has analyzed a relation between lot size and quality imperfection. Porteus (1986) [7] has considered the effect of investment in reducing the setup cost and quality improvement cost jointly. Boney and Jaber et al., (2011) [5] has introduced an imperfect production inventory model and also they discussed about the screening errors, during the

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inspection process. Sarkar et al., (2016a) [12] developed a two echelon supply chain inventory model with flexible setup and deterioration of products. Many scholars developed the imperfect inventory model into the different scenarios.

The third stream of this research work is the lead time. The lead time distribution is essential for obtaining the managerial decisions of a supply chain. Therefore the manager of every industry sector faces many difficulties to collect all necessary information about the lead time demand distribution. Scarf (1958) [16], who was the first author to established the min-max distribution free approach for the newsvendor problem without any information about the distribution of lead time demand except known mean and standard deviation. Sarkar and Majumder (2013b) [11] developed an integrated vendor-buyer model with vendor's setup cost reduction and distribution free approach. This paper is the extension of Sarkar and Majumder et.al., (2017) [15] and This proposed work is considers a three echelon inventory model with SSMD policy and also this proposed work is analyses a screening errors of accepting the defective items and rejecting the non – defective items.

The entire article is divided into following manner. Section 1 contains introduction and literature review. Section 2 includes notations to develop the mathematical model. Section 3 consists of the mathematical model of the proposed model along with the screening errors. Section 4 contains numerical example for proposed model, finally concludes the paper in section 5.

2.2 Notation

The following notation is used to develop the model:

S : setup cost f vendor per setup ($/setup)
Q : quantity ordered by buyer (unit)
R :reorder point of buyer(days)
L : length of lead time for buyer(unit)
m_1 : Number of shipments delivered by the supplier to the manufacturer, per each manufacturer's raw material cycle
m_2 : Number of shipments delivered by the manufacturer to the retailer, per each retailer's cycle
θ : Probability that the production process may go out-of-control.
D : average demand per unit time of buyer (units/years)
A_S : Setup/Order cost for the supplier
Ab : ordering cost of buyer per order ($/order)
P : vendor's production rate (units/year)
θ_0 : initial probability that the system may go out-0f-control
S_0 : initial setup cost of vendor per setup ($/setup)
s : replacement cost per unit defective item($/unit)
c_v : unit production cost paid by vendor ($/unit)
C_b : unit purchasing cost paid by buyer ($/unit),($C_b > c_v$)
ρ : Actual fraction of defectives
N_1 : Probability of committing type I error
N_2 : Probability of committing type II error
r_v : Unit holding cost of vendor per unit time ($/unit/year)
r_b : Unit holding cost of buyer per unit time ($/unit/year)

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π : Unit backlogging cost for buyer($/unit)
X : Lead time demand having cumulative distribution function (c.d.f) F with mean DL and standard deviation
σ : Standard deviation of demand per unit time (units/week)
C_s : Production /Purchasing cost per item for Supplier
h_s : Financial holding cost per item for Supplier
s_s : Physical (storage) holding cost per item for Supplier
α' : Amount of raw material required to produce one finished product
F : fixed transportation cost ($/shipment) for the supplier & Manufacturer
v : variable transportation cost ($/unit) for the supplier & Manufacturer
C_{fcs} : fixed carbon emission cost ($/shipment/year)
C_{vcs} : variable carbon emission cost ($/unit/year)
C_{fcm} : fixed carbon emission cost ($/shipment/year)
C_{vcm} : variable carbon emission cost ($/unit/year)
f_1 : fuel cost for the supplier
f_2 : fuel cost for the manufacturer
d : distance travelled (/km)

3. Mathematical Model:

The Mathematical model of this proposed model demonstrates the integrated cost functions of each player of the supply chains. The individual cost function of the supplier is per unit time is demonstrated as follows:

Setup cost for the supplier is $= \frac{A_s D}{Q}$

Production cost for the supplier is $= C_S \alpha' D$

Holding cost for the supplier is $= \frac{(m_1 - 1)}{2}(h_s + S_s)\frac{\alpha' m_2 QD}{Pm_1}$

Fuel cost is $= \frac{f_1 dD}{m_2 Q}$

Fixed transportation cost is $= \frac{m_1 FD}{m_2 Q}$

Variable transportation cost is $= \frac{vD}{m_2}$

Fixed and variable carbon emission cost $= C_{fcs} m_1 + C_{vcs} D$

So the cost function of the supplier is as follows,

$$\frac{A_S D}{Q} + C_S \alpha' D + \frac{(m_1 - 1)}{2}(h_S + S_S)\frac{\alpha' m_2 QD}{Pm_1} + \frac{(m_1 F + f_1 d)D}{m_2 Q} + C_{fcs} m_1 + C_{vcs} D + \frac{VD}{m_2}$$

-

(1)

The individual cost function of the manufacturer per unit time is demonstrated as follows:

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Vendor has conducts screening process and makes the screening error of accepting the Defective items and rejecting the non-defective items, so the cost function of manufacturer is defined as follows,

Manufacturers expected setup cost per unit time is = $\frac{SD}{m_2 Q}$

Expected holding cost per unit time = $r_v C_v \frac{Q}{2}\left[m_2\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]$

Expected annual cost for the imperfect items = $\frac{sDm_2Q\theta}{2}$

Investment for vendor's setup cost reduction is $I_S = B(\ln S_0 - \ln S)$

Investment in quality improvement of the product is $I_\theta = b(\ln \theta_0 - \ln \theta)$

Fuel cost is = $\frac{f_2 dD}{m_2 Q}$

Fixed transportation cost is = $\frac{m_2 FD}{m_2 Q}$

Variable transportation cost is = $\frac{vD}{m_2}$

Fixed and variable carbon emission cost = $C_{fcs}m_2 + C_{vcs}D$

Screening cost = $dD + C_a \rho Q N_2 + C_r(1-\rho)QN_1$

So the cost function of the manufacturer per unit time is demonstrated as follows,

$$\alpha\left[B\left(\ln S_0 - \ln S\right)+b\left(\ln\theta_0-\ln\theta\right)\right]+\frac{SD}{m_2Q}+r_vC_v\frac{Q}{2}\left[m_2\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+$$

$$\frac{SDm_2Q\theta}{2}+dD+C_a\rho QN_2+C_r\left(1-\rho\right)QN_1+C_{fcm}m_2+C_{vcm}D+\frac{\left(m_2F+f_2d\right)D}{m_2Q}+\frac{VD}{m_2}$$

-

(2)

Similarly the individual cost function of the buyer is shown as follows,

Ordering cost per unit time is = $\frac{A_b D}{Q}$

Lead time crashing cost $\frac{DC(L)}{Q}$

Expected holding cost per unit time is = $r_b C_b\left[\frac{Q}{2}+R-DL\right]$

The expected shortage cost is = $\frac{\pi\sigma\sqrt{L}\psi(k)D}{Q}$

So the overall cost function of the buyer is demonstrated as follows,

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$$\frac{A_b D}{Q}+\frac{DC(L)}{Q}+r_b C_b\left[\frac{Q}{2}+R-DL\right]+\frac{\pi\sigma\sqrt{L}\psi(k)D}{Q} \quad - (3)$$

Joint Optimization:

Considering the joint optimization each player's individual cost are compounded into together, it will explained as follows,

$$JATC(Q) = \frac{A_S D}{Q}+C_S\alpha' D+\frac{(m_1-1)}{2}(h_S+S_S)\frac{\alpha' m_2 QD}{Pm_1}+\frac{(m_1F+f_1 d)D}{m_2 Q}+C_{fcs}m_1+C_{vcs}D+\frac{VD}{m_2}$$

$$+\quad \alpha\left[B(\ln S_0-\ln S)+b(\ln\theta_0-\ln\theta)\right]+\frac{SD}{m_2 Q}+r_v C_v\frac{Q}{2}\left[m_2\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]$$

$$+\frac{SDm_2 Q\theta}{2}+dD+C_a\rho N_2 D+C_r(1-\rho)DN_1+C_{fcm}m_2+C_{vcm}D+\frac{(m_2F+f_2 d)D}{m_2 Q}+\frac{VD}{m_2}$$

$$+\frac{A_b D}{Q}+\frac{DC(L)}{Q}+r_b C_b\left[\frac{Q}{2}+R-DL\right]+\frac{\pi\sigma\sqrt{L}\psi(k)D}{Q} \quad - (4)$$

Differentiate the above equation with respect to "Q", we get the optimal order quantity of Q^*

$$Q^* = \sqrt{\frac{2D\left[A_S+\frac{(m_1F+f_1 d)}{m_2}+\frac{S}{m_2}+A_b+C(L)+\pi\sigma\sqrt{L}\psi(k)+\frac{(m_2F+f_2 d)}{m_2}\right]}{\frac{(m_1-1)}{2}(h_s+S_s)\frac{\alpha' m_2 D}{Pm_1}+r_v C_v\left[m_2\left(1-\frac{D}{P}\right)-1+\frac{2D}{P}\right]+sDm_2\theta+r_b C_b}} \quad -$$

(5)

4. Numerical Example:

The following parameter values are used for illustrating the proposed model, D =600 units/year, P =2000 units /year, α' =1, A_s =264 $/order, C_s=20 $/unit, h_s=3 $/unit/year, s_s =2 $/unit/year, m_1 =1, m_2= 2, F =0.2/shipment ,v =0.1/unit ,d=10 km, $f_1 = f_2$ = 71 per liters , $C_{fcs}m_1$ = 0.2 /shipment/year, $C_{fcm}m_2$ =0.2/shipment/year, C_{vcs} = 0.1/unit/year C_{vcm} =0.1/ unit/year, C_a = 500, d= 0.5, C_r = 5, ρ =0.02, θ_0=0.002, θ=0.00000924, π = $50 per unit, σ = 7 units/ week , A_b =$ 200 /order , C_b = $100 /unit , S_0 = $1500 / setup, C_v = $70 /unit , r_b = $0.2/unit/year , r_v =$0.2 /unit/year , α =0.1/dollar/year,

B=18,00 0and b=400, S =576.76 ,k=0.92

The value of the optimal order quantity is, Q^* = 177 and the total cost is $JATC(Q)$ = 21236.

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5. Conclusion

This paper considered a three echelon inventory model with quality improvement of the product. Furthermore this paper analyzes the SSMD policy in the transportation services it would be a convenient tool for the business people to reduce the overall system cost as well as diminishing the environmental pollution due to the transportation activities. Finally this paper investigates the screening errors of accepting the defective items and rejecting the non – defective items. Mathematical model and the numerical example are also helpful for the better understanding.

References:

1. Banerjee, A. (1986) 'A joint economic-lot-size model for purchaser and vendor', Decision Sciences, Vol. 17, No. 3, pp.292–311.

2. Cardenas-Barron, L.E., Teng, J.T., Trevi~no-Garza, G. and Lou, K.R. (2012) 'An improved algorithm and solution on an integrated production-inventory model in a three-layer supply chain', International J. of Production Economics, Vol. 136, No. 2, pp.384–388.

3. Goyal, S.K. (1976) 'An integrated inventory model for a single supplier-single customer problem', International J. of Production Research, Vol. 15, No. 1, pp.107–111.

4. Goyal, S.K. (1988) A joint economic-lot-size model for purchaser and vendor: a comment, Decision Sciences, Vol. 19, No. 1, pp.236–241.

5. Khan M, Jaber MY, Bonney M(2011) An economic order quantity (EOQ) for items with imperfect quality and inspection errors. International Journal of Production Economics 133:113-118

6. Porteus, E.L. (1985) 'Investing in reduced setups in the EOQ model', Management Science,Vol. 31, No. 8, pp.998–1010.

7. Porteus, E.L. (1986) 'Optimal lot sizing, process quality improvement and setup cost reduction', Operations Research, Vol. 34, No. 1, pp.137–144.

8. Rosenblatt, M.J. and Lee, H.L. (1986) 'Economic production cycles with imperfect production processes', IIE Transactions, Vol. 18, No. 1, pp.48–55.

9. Sarkar, B. (2012) 'An EOQ model with delay in payments and stock dependent demand in the presence of imperfect production', Applied Mathematics and Computation, Vol. 218, No. 17,pp.8295–8308.

10. Sarkar, B. (2016) 'Supply chain coordination with variable backorder, inspections, and discount policy for fixed lifetime products', Mathematical Problems in Engineering, Vol. 2016, Article ID 6318737, 14pp, doi:10.1155/2016/6318737.

11. Sarkar, B. and Majumder, A. (2013b) 'Integrated vendor-buyer supply chain model with vendor's setup cost reduction', Applied Mathematics and Computation, Vol. 224, pp.362–371.

12. Sarkar, B., Sett, B.K., Roy, G. and Goswami, A. (2016a) 'Flexible setup cost and deterioration of products in a supply chain model', *International J. of Applied and Computational Mathematics*, Vol. 2, No. 1, pp.25–40.

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13. Sarkar, B., Ganguly, B., Sarkar, M. and Pareek, S. (2016b) 'Effect of variable transportation and carbon emission in a three-echelon supply chain model', Transportation Research Part E: Logistics and Transportation Review, Vol. 91, pp.112–128.

14. Sarkar, B., Majumder, A., Sarkar, M., Dey, B.K. and Roy, G. (2017) 'Two-echelon supply chain model with manufacturing quality improvement and setup cost reduction', J. of Industrial and Management Optimization, Vol. 13, No. 2, pp.1085–1104.

15. Sarkar, B., Majumder, A., Guchhait .R., (2017), Manufacturing quality improvement and setup cost reduction in a vendor – buyer supply chain model , European Journal of Industrial Engineering, Vol.11, No.5, pp. 588 - 612

16. Scarf, H. (1958) A min-max solution of an inventory problem, in Arrow, K., Karlin, S. and Scarf, H. (Eds.): Studies in the Mathematical Theory of Inventory and Production, Stanford University Press, Stanford, California, 910pp.

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INTUITIONISTIC FUZZY SOFT MATRIX THEORY AND ITS APPLICATION IN MEDICAL DIAGNOSIS

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Abstract

In this paper, the patients having the symptoms of cancer like Bladder Cancer, Kidney Cancer and Lymphoma Cancer have been collected. The awareness for the different types of cancer symptoms in the field of medical diagnosis can be made using intuitionistic fuzzy soft matrix. Here the intuitionistic fuzzy soft matrix is applied in Decision making problems. The proposed algorithms are used to finding Cancer affected patients.

Key words: Soft set, Fuzzy soft set, Fuzzy soft matrix (FSM), intuitionistic fuzzy soft set (IFSS), intuitionistic fuzzy soft matrix (IFSM), Value matrix, Score matrix

Introduction

The main concept of fuzzy set theory was introduced by Zadeh ([**1**]). It was explained by Moldostov ([**11**]).Fuzzy set considered as general mathematical and it is not clearly defined the object. This theory was developed into several basic notions of soft set theory. Maji ([**5, 6**]) developed the theory again. Soft matrices representation of the soft sets and it was construct the soft max – min decision making method. This matrix is a representation of a fuzzy soft set and it was successfully applied to the proposed notion in fuzzy soft matrix **([3]).**Thus, there are many "decision can make" Fuzzy matrices are can use to study of Sanchez's approach of medical diagnosis which has been made to apply to solving decision problem. Thus, fuzzy soft matrices are used to construct a decision making problem ([**4, 8, 9, 10, 11, 14**]). Rajeshwari and Dhanalakshmi ([**9**]) both are revealed the similarity between two fuzzy soft set based on its distance. Fuzzy soft matrices are introduced in agriculture by Sarala & Rajkumari ([**14**]) and also fuzzy soft matrices have introduced the orphans and also significant character of fuzzy soft matrices. In ([**3**]) Borah and in **([3])**Neog Extended fuzzy soft matrix concept and its application. In **([4])**, Chetia Proposed intuitionistic fuzzy gentle matrix theory. Accordingly, ([**9**]), Rajarajeswari proposed new definitions for intuitionistic fuzzy gentle matrices and its kinds. Also extended and implemented some operations on it.

Preliminaries

In this section, to recall some basic notion of Intuitionistic fuzzy soft set theory and Intuitionistic fuzzy soft Matrices (IFSM).

Soft Set [14]

Let U be an initial universe set and E be a set of parameters. Let P (U) denotes the power set of U. Let $A \subseteq E$. A pair (F, E) is called a soft set over U, where F_A is a mapping given by $F: E \rightarrow P(U)$ such that $F(e) = \varphi$ if $e \notin A$. Here F is called approximate function of the soft set (F, E). The set F (e) is called e – approximate value set which consist of related objects of the parameter $e \in E$.

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Example 1

Let U= {u_1, u_2, u_3, u_4} be set of four varieties of cloths and E= {High Quality (e_1), Medium Quality (e_2), Low Quality (e_3)} be the set of parameters. If E= {e_1, e_2} $\subseteq$ E. Let F (e_1) = {u_1, u_2, u_3, u_4} and F(e_2) = {u_1, u_2, u_3}. Then we write the soft set (F, E) = {(e_1, {u_1, u_2, u_3, u_4}), (e_2, { u_1, u_2, u_3})} over U which describe the "Quality of cloths" which Mrs. John is going to bye. We may represent the soft set in the following form

Table 1

U	e_1	e_2	e_3
u_1	1	1	0
u_2	1	1	0
u_3	1	1	0
u_4	1	0	0

Fuzzy Soft Set [14]

Let U be an initial universe set and E be a set of parameters. Let A $\subseteq$ E. A pair (F, A) is called a fuzzy soft set (FSS) over U, where F is a mapping given by, F: A $\rightarrow$ I^u, where I^u denotes the collection of all fuzzy subsets of U.

Example 2

Consider the example 1 here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp number 0 and 1, which associate with each element a real number in the interval [0,1]. Then (F, A) = {F (e_1) = {(u_1,0.7), (u_2,0.8), (u_3,0.2), (u_4.0.5)},F (e_2) = {(u_1,0.3), (u_2,0.6), (u_3,0.8)} Is the fuzzy soft set representing the "Quality of Cloths" which Mrs. John is going to buy. We may represent the fuzzy soft set in the following form

Table 2

U	e_1	e_2	e_3
u_1	0.7	0.3	0.0
u_2	0.8	0.6	0.0
u_3	0.2	0.8	0.0
u_4	0.5	0.0	0.0

Fuzzy soft matrices [6]

Let (F, E) be a fuzzy soft set over U. Then a subset of U $\times$ E is uniquely define by R_A = {(u, e): e $\in$ A, u $\in$ F_A(e)} which is called a relation form of (F, E), Now the characteristic function of R_A is written by , μ_{R_A} : u $\times$ E $\rightarrow$ [0,1] such that μ_{R_A} (u, e) is in [0,1] is the membership value of the object u $\in$ U for each e $\in$ E If [μ_{ij}] = μ_{R_A} (u_i, e_j), we can define a matrix

$$[\mu_{ij}]_{m\times n} = \begin{pmatrix} \mu 11 & \mu 12 & . & . & \mu 1n \\ \mu 21 & \mu 22 & . & . & \mu 2n \\ . & . & . & . & . \\ . & . & . & . & . \\ \mu m1 & \mu m2 & . & . & \mu mn \end{pmatrix}$$

Which is called a fuzzy soft matrix of order m$\times$n corresponding to the fuzzy soft set (F_A, E) over U.

Example 3

Consider the example 2 the matrix representation is

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$$\begin{pmatrix} 0.7 & 0.3 & 0.0 \\ 0.8 & 0.6 & 0.0 \\ 0.2 & 0.8 & 0.0 \\ 0.5 & 0.0 & 0.0 \end{pmatrix}$$

Intuitionistic fuzzy soft set [9]

Let U be the Universal set and E be the set of parameters. Let A $\subseteq$ E. A pair (F, A) is called Intuitionistic fuzzy soft set over U, where F is a mapping given by F: A $\rightarrow$ I^u, where I^u denotes the collection of all Intuitionistic fuzzy subsets of U.

Example 4

Suppose that U = {s_1, s_2, s_3, s_4} is a set of students and E = {e_1, e_2, e_3} is a set parameters, which stands for result, activity, sports performance respectively. Consider the mapping from parameters set A $\subseteq$ E to the set of all intuitionistic fuzzy subsets of power set *U*. Then soft set (F, A) describes the character of the students with respect to the given parameters, for finding the best student of an academic year. Consider, A ={e_1, e_2} then Intuitionistic fuzzy soft set is

(F, A) = {F(e_1) = {(s_1, 0.8, 0.1), (s_2, 0.3, 0.6), (s_3, 0.8, 0.1), (s_1, 0.5, 0.1)},

F(e_2) = {(s_1, 0.6, 0.1), (s_2, 0.3, 0.5), (s_3, 0.2, 0.1), (s_1, 0.7, 0.1)}}.

Intuitionistic fuzzy soft Matrix [9]

Let U = {c_1, c_2, c_3, ……c_m} be the Universal set and E be the set of parameters given by E= {e_1, e_2, e_3,….e_n}. Let A $\subseteq$ E and (F, A) be a Intuitionistic fuzzy soft set in the fuzzy soft class (U, E). Then Intuitionistic fuzzy soft set (F, A) in a matrix form as

$A_{m\times n} = [a_{ij}]_{m\times n}$ i = 1, 2, 3…….m, j = 1, 2, 3,……., n ,where $a_{ij} = \begin{cases} \left(\mu_j(c_i), \vartheta_j(c_i)\right) \ if\ c_i\ in\ A \\ (0,1) if\ otherwise \end{cases}$

Example 5

Consider the example 4 the matrix representation is

$$\begin{pmatrix} (0.8,0.1) & (0.6,0.1) \\ (0.3,0.6) & (0.3,0.5) \\ (0.8,0.1) & (0.2,0.1) \\ (0.5,0.1) & (0.7,0.1) \end{pmatrix}$$

Product of fuzzy soft matrices [9]

If A= $\lfloor a_{ij} \rfloor \in IFSM_{m\times n}$, B = $\lfloor b_{jk} \rfloor \in IFSM_{n\times p}$, then we define A.B, product of A and B as A.B = $\lfloor c_{ik} \rfloor_{m\times p}$

= (max min (μ_{Aj}, μ_{Bj}), min max (ν_{Aj}, ν_{Bj})) foe all *i,j*

Example 6

Let A = $\begin{pmatrix} (0.8,0.1) & (0.4,0.5) \\ (0.7,0.3) & (0.4,0.6) \end{pmatrix}$ and B = $\begin{pmatrix} (0.6,0.3) & (0.8,0.2) \\ (0.7,0.3) & (0.5,0.5) \end{pmatrix}$ are two Intuitionistic fuzzy soft Matrices (IFSM), then the product of these two matrices is

A.B = $\begin{pmatrix} (0.6,0.3) & (0.8,0.2) \\ (0.6,0.3) & (0.7,0.3) \end{pmatrix}$

Remark: A.B $\neq$ B. A

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Intuitionistic fuzzy soft Complement [9]

If A= $\lfloor a_{ij} \rfloor \in \text{IFSM}_{m\times n}$, $a_{ij} = \left(\mu_j(c_i), \vartheta_j(c_i)\right)$. Then A^c is called an Intuitionistic fuzzy soft Complement matrix if $A^c = \lfloor b_{ij} \rfloor \in \text{IFSM}_{m\times n}$ $b_{ij} = \left(\vartheta_j(c_i), \mu_{j_j}(c_i)\right)$ for all i ,j

Example 7

A = $\begin{pmatrix}(0.8,0.1) & (0.4,0.5)\\ (0.7,0.3) & (0.4,0.6)\end{pmatrix}$ be Intuitionistic fuzzy soft Matrices (IFSM), then the complement of this matrix is

$A^c = \begin{pmatrix}(0.1,0.8) & (0.5,0.4)\\ (0.3,0.7) & (0.6,0.4)\end{pmatrix}$

Value Matrix [9]

If A= $\lfloor a_{ij} \rfloor \in \text{IFSM}_{m\times n}$, where $a_{ij} = \left(\mu_j(c_i), \vartheta_j(c_i)\right)$, then we define the Value matrix of intuitionistic Fuzzy Soft Matrix A is

V (A) = $\lfloor a_{ij} \rfloor = [\mu_j(c_i) - \vartheta_j(c_i)]$ i = 1, 2,…, m; j = 1,2, ……,n.

Score Matrix [9]

If A= $\lfloor a_{ij} \rfloor \in \text{IFSM}_{m\times n}$, $\lfloor b_{ij} \rfloor \in \text{IFSM}_{m\times n}$, then we define Score matrix of A and B as S(A, B) = $\lfloor d_{ij} \rfloor \in \text{IFSM}_{m\times n}$, where $\lfloor d_{ij} \rfloor$

=V (A) – V (B) for all i, j.

Methodology

Let us assume that there is a set of m patients P= {p_1, p_2, p_3,…,p_m} with the set of n symptoms *S* = {s_1, s_2, s_3, s_4,……s_n}related to a set of *k* diseases *D* = {d_1, d_2, d_3,…,d_k}.We apply intuitionistic fuzzy soft set technology to diagnose which patient is suffering from what disease. we construct a intuitionistic fuzzy soft set (*F*, *P*) over *S* where *F* is a mapping $F : P \rightarrow IF^S$, IF^S is the collection of all intuitionistic fuzzy subsets of *S*. This intuitionistic fuzzy soft set gives a relation matrix *A* called patient symptom matrix, then construct another intuitionistic fuzzy soft set (*G*, *S*) over *D* where *G* is a mapping $G : S \rightarrow IF^D$, IF^D is the collection of all intuitionistic fuzzy subsets of *D*. This intuitionistic fuzzy soft set gives a relation matrix *B* called symptom-disease matrix, where each element denote the weight of the symptoms for a certain disease. We compute the complements $(F; P)^c$ and $(G; S)^c$ and their matrices A^c and B^c. Compute *A.B* which is the maximum membership of occurrence of Symptoms of the diseases. Compute $A^c . B^c$ which is the maximum membership of non occurrence of Symptoms of the diseases. Using def and (3.2),Compute $V(A . B)$; $V(A^c . B^c)$ and the Score matrix. Finally find max (S_i),then conclude that the patient p_i is suffering from disease d_j. Incase *max* (S_i) occurs for more than one value, then reassess the symptoms to break the tie.

Algorithm

1. Input the intuitionistic fuzzy soft sets (G, S) and (F, D) then obtain the intuitionistic fuzzy soft matrix A, B corresponding to (G, S), (F, D) respectively.
2. Write the intuitionistic fuzzy soft complement set $(G, S)^c$, $(F, D)^c$ and obtain the intuitionistic fuzzy soft matrix A^c, B^c corresponding to $(G, S)^c$ and $(F, D)^c$ respectively.
3. Evaluate (A.B),($A^c.B^c$), V(A.B),V($A^c.B^c$).
4. Evaluate the Score Matrix.
5. Find P for which max (S_i) for each patient p_i. Then the patient p_i is suffering from disease D_j.

Case Study

Suppose the test results for three patients P = {p_1, p_2, p_3} as the universal set where p_1, p_2 and p_3 represents patients Ragu, Sheena and Swami with the symptoms S = {s_1, s_2, s_3} as the set of symptoms where s_1, s_2 and s_3 represents symptoms Weight loss, Coughing up blood and unexplained night sweets respectively for the case study. Let the possible diseases relating to the above symptoms D = { d_1, d_2, d_3} be Bladder Cancer, Kidney Cancer and Lymphoma Cancer.

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Step 1

IFSS (G, S) over P, Where G is a mapping G: S $\rightarrow I^P$, gives a collection of an approximation of patient symptoms in the hospital.

(G, S) = {G (s_1) = {(p_1, 0.1, 0.2), (p_2, 0.5, 0.4), (p_3, 0.3, 0.6)},

G (s_2) = {(p_1, 0.5, 0.4), (p_2, 0.2, 0.3), (p_3, 0.5, 0.1)}, G (s_3) = {(p_1, 0.3, 0.6), (p_2, 0.5, 0.1), (p_3, 0.6, 0.2)}}.

This intuitionistic fuzzy set is represented by the following intuitionistic fuzzy soft matrix

$$A = \begin{array}{c} \\ p1 \\ p2 \\ p3 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{pmatrix} (0.1,0.2) & (0.5,0.4) & (0.3,0.6) \\ (0.5,0.4) & (0.2,0.3) & (0.5,0.1) \\ (0.3,0.6) & (0.5,0.1) & (0.6,0.2) \end{pmatrix} \end{array}$$

where S= {s_1, s_2, s_3}as universal set where s_1, s_2 and s_3 represents the symptoms like loss, Coughing up blood and unexplained night sweets respectively. Suppose that IFSS (F, D) over S, where F is a mapping F: D $\rightarrow I^P$, gives an approximate value of intuitionistic fuzzy soft medical knowledge of the three diseases and their symptoms. Let,

(F, D) = {F (d_1) = {(s_1, 0.5, 0.3), (s_2, 0.8, 0.1), (s_3, 0.2, 0.3)},

F (d_2) = {(s_1, 0.1, 0.6), (s_2, 0.4, 0.3), (s_3, 0.3, 0.6)},

F(d_3) = {(s_1, 0.7, 0.1), (s_2, 0.5, 0.2), (s_3, 0.4, 0.5)}}.

This intuitionistic fuzzy soft set is represented by the intuitionistic fuzzy soft matrix

$$B = \begin{array}{c} \\ s1 \\ s2 \\ s3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (0.5,0.3) & (0.1,0.6) & (0.7,0.1) \\ (0.8,0.1) & (0.4,0.3) & (0.5,0.2) \\ (0.2,0.5) & (0.3,0.6) & (0.4,0.5) \end{pmatrix} \end{array}$$

Step 2

The intuitionistic fuzzy soft complement matrices are

$$A^c = \begin{array}{c} \\ p1 \\ p2 \\ p3 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{pmatrix} (0.2,0.1) & (0.4,0.5) & (0.6,0.3) \\ (0.4,0.5) & (0.3,0.2) & (0.1,0.5) \\ (0.6,0.3) & (0.1,0.5) & (0.2,0.6) \end{pmatrix} \end{array} \text{ and } B^c = \begin{array}{c} \\ s1 \\ s2 \\ s3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (0.3,0.5) & (0.6,0.1) & (0.1,0.7) \\ (0.1,0.8) & (0.3,0.4) & (0.2,0.5) \\ (0.5,0.2) & (0.6,0.3) & (0.5,0.4) \end{pmatrix} \end{array}$$

Step 3

$$A.B = \begin{pmatrix} (0.1,0.2) & (0.5,0.4) & (0.3,0.6) \\ (0.5,0.4) & (0.2,0.3) & (0.5,0.1) \\ (0.3,0.6) & (0.5,0.1) & (0.6,0.2) \end{pmatrix} . \begin{pmatrix} (0.5,0.3) & (0.1,0.6) & (0.7,0.1) \\ (0.8,0.1) & (0.4,0.3) & (0.5,0.2) \\ (0.2,0.5) & (0.3,0.6) & (0.4,0.5) \end{pmatrix}$$

$$= \begin{array}{c} \\ p1 \\ p2 \\ p3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1 & d_2 & d_3 \end{array} \\ \begin{pmatrix} (0.5,0.3) & (0.4,0.4) & (0.5,0.2) \\ (0.5,0.3) & (0.3,0.3) & (0.5,0.3) \\ (0.5,0.1) & (0.4,0.3) & (0.4,0.2) \end{pmatrix} \end{array}$$

$$A^c.B^c = \begin{pmatrix} (0.2,0.1) & (0.4,0.5) & (0.6,0.3) \\ (0.4,0.5) & (0.3,0.2) & (0.1,0.5) \\ (0.6,0.3) & (0.1,0.5) & (0.2,0.6) \end{pmatrix} . \begin{pmatrix} (0.3,0.5) & (0.6,0.1) & (0.1,0.7) \\ (0.1,0.8) & (0.3,0.4) & (0.2,0.5) \\ (0.5,0.2) & (0.6,0.3) & (0.5,0.4) \end{pmatrix}$$

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$$= \begin{matrix} \\ p1 \\ p2 \\ p3 \end{matrix} \begin{matrix} \begin{matrix} d_1 & d_2 & d_3 \end{matrix} \\ \begin{pmatrix} (0.5,0.3) & (0.6,0.1) & (0.5,0.4) \\ (0.3,0.5) & (0.4,0.4) & (0.2,0.5) \\ (0.3,0.5) & (0.6,0.3) & (0.2,0.5) \end{pmatrix} \end{matrix}$$

$$V(A.B) = \begin{matrix} \\ p1 \\ p2 \\ p3 \end{matrix} \begin{matrix} \begin{matrix} d_1 & d_2 & d_3 \end{matrix} \\ \begin{pmatrix} 0.2 & 0.0 & 0.3 \\ 0.2 & 0.0 & 0.2 \\ 0.4 & 0.1 & 0.2 \end{pmatrix} \end{matrix} \text{ and } V(A^c.B^c) = \begin{matrix} \\ p1 \\ p2 \\ p3 \end{matrix} \begin{matrix} \begin{matrix} d_1 & d_2 & d_3 \end{matrix} \\ \begin{pmatrix} 0.2 & 0.5 & 0.1 \\ -0.2 & 0.0 & -0.3 \\ -0.2 & 0.3 & -0.3 \end{pmatrix} \end{matrix}$$

Step 4

$$S = \begin{matrix} \\ p1 \\ p2 \\ p3 \end{matrix} \begin{matrix} \begin{matrix} d_1 & d_2 & d_3 & \max \end{matrix} \\ \begin{pmatrix} 0.0 & -0.5 & 0.2 & 0.2 \\ 0.4 & 0.0 & 0.5 & 0.5 \\ 0.6 & -0.2 & 0.5 & 0.6 \end{pmatrix} \end{matrix}$$

Step 5

It is clear from the above matrix the patients p_1 and p_2 suffering from d_3 like Lymphoma Cancer and p_3 is suffering from Bladder Cancer.

Conclusion

In this paper, the theory of intuitionistic fuzzy soft matrix in the field of medical diagnosis is identified. Some new concepts such as complement of intuitionistic fuzzy soft matrix based on reference function have been enhanced. The Cancer affected person should be given for awareness about symptoms of the affected body. Future work is required to study whether the notion put forward in this paper yield a fruitful result.

References:

[1] Zadeh .L.A (1965), "Fuzzy sets, *Information and control*",8, pp.338-353.
[2] Atanassov .K.T, *" Intuitionistic fuzzy set"*, Fuzzy sets and systems,Vol.20(1), 1986, pp:86 -87.
[3] Borah M. J, Neog. T. J & Sut. D. K., "*Fuzzy soft matrix theory and its decision making*", International Journal of Modern Engineering Research 2 (2012) 121-127.
[4] B. Chetia & P. K. Das, "*Some results of intuitionistic fuzzy soft matrix theory*", Advances in Applied Science Research 3 (2012) 412-413.
[5] Maji. P. K, Biswas. R & Roy. A. R, "*Fuzzy soft set*", J. Fuzzy Math. 9 (2001) 589 - 602.
[6] P. K. Maji, R. Biswas & Roy. A. R, "*Intuitionistic fuzzy soft sets*", J. Fuzzy Math. 12 (2004)669 - 683.
[7] Majumdar. P & Samantha. S. K, *"Generalised fuzzy soft sets"*, Comput. Math. Appl. 59 (2010) 1425 - 1432.
[8] Neog .T. J & Sut. D. K, *"An application of fuzzy soft sets in decision making problems using fuzzy soft matrices*", International Journal of Mathematical Archive 2 (2012) 2258 - 2263.
[9] Rajarajeswari. P & Dhanalakshmi. P, "*Intuitionistic fuzzy soft matrix theory and its application in medical diagnosis*", Annals of Fuzzy Mathematics and Informatics, Volume 7, No. 5(May 2014) pp: 765-772.
[10] Yang. Y& Chenli Ji, "*Fuzzy soft matrices and their applications*", Lecture notes in Computer Science 7002 (2011) 618 - 627. Intuitionistic fuzzy soft matrix theory and it application in medical diagnosis
[11] Molodtsov .D(1999), "*Soft set theory first results, computer and Mathematics with application"*, 37, pp:19-31
[12] P. K. Maji, R. Biswas & Roy. A. R (2001), *"Fuzzy soft sets The journal of fuzzy mathematics"*, volume 9, No.3, pp:589-602.
[13] Ahmad. B& Kharal. A. (2009) *"On Fuzzy soft sets, Advance in Fuzzy systems"*, pp: 1-6.
[14] Sarala. N & Rajkumari. S. (2014), *"Drug Addiction Effect in Medical Diagnosis by using Fuzzy Soft Matrices"*, International Journal of Current Engineering and Technology, Volume 5, No.1.

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An Approximate Solution of Nonlinear Fractional Fredholm integro-differential equation using Mahgoub Adomian Decomposition Method

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Abstract

In this paper, Mahgoub Adomian decomposition method has been applied to solve the nonlinear Fractional Fredholm integro-differential equations. The proposed method is based on the application of Mahgoub transform to nonlinear Fredholm integro-differential equation of fractional order. The nonlinear term has been handled with the help of Adomian polynomials. Here, the fractional derivatives are described in Caputo sense. The proposed method has been demonstrated with some illustrative examples and it has been given effective and good results.

Keywords: Mahgoub Transform Method, Fractional Fredholm integro-differential Equations, Adomian polynomials.

The Fractional integro-differential equations have been attracted by many researchers and scientists in the last decades. They have been used in the mathematical modelling in various branches of Science and Engineering [1, 2]. Momani [3] have presented the existence theorems on fractional integro-differential equations. Most of the nonlinear integro-differential equations of fractional order do not have exact solution, so some numerical and approximation technique must be used.

Recently, several numerical methods have been proposed to solve Fractional Fredholm integro-differential equations. Some of them are Chebyev polynomial method [4], Homotopy perturbation method [5, 6], Wavelet method [7, 8] and Collocation method [9]. The Adomian decompoisition method (ADM) is one of the powerful methods to find the approximate solution of linear and nonlinear Fractional integro-differential equations. Momani and Qaralleh [10] have applied Adomian polynomials to solve systems of fractional integro-differential equations.

In this paper Mahgoub Adomian Decomposition method (MADM) have been applied to find the analytical and numerical solution of Fractional Fredholm integro-differential equation. The paper is organized as follows. In section 2 we introduce some definitions and properties of the fractional calculus and Mahgoub transform. In section 3 we construct our method to approximate the solution of the Fractional Fredholm integro-differential equations. In Section 4 we present some numerical examples to illustrate the accuracy of our method.

2. Preliminaries and Notations

In this section, fundamental definitions and properties of fractional calculus and Mahgoub transform have been introduced.

***Definition 1*:** A real function $f(t), t > 0$ is said to be in the space $\mathbb{C}_\mu,\ \mu \in \mathbb{R}$ if there exists a real number $p > \mu$ such that $f(t) = t^p f_1(t)$ where $f_1(t) \in \mathbb{C}[0,\infty)$ and it is said to be in the space $\mathbb{C}_\mu^n$ if and only if $f^{(n)} \in \mathbb{C}_\mu,\ n \in \mathbb{N}$.

Definition 2: The Riemann Liouville fractional integral $I^\alpha f(t)$ of order $\alpha \in R, \alpha > 0$ of function $f(t) \in \mathbb{C}_\mu, \mu \geq -1$ is defined as

$$I^\alpha f(t) = \frac{1}{\Gamma\alpha}\int_0^t (t-\tau)^{\alpha-1} f(\tau)d\tau, \qquad t > 0 \tag{1}$$

Definition 3: The Caputo fractional derivative of a function $f(t)$ of order $\alpha \in \mathbb{R}, \alpha > 0$ is given by

$${}^cD^\alpha f(t) = I^{n-\alpha}D^n f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1} f^n(\tau)d\tau,\ t > 0 \tag{2}$$

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where $n-1 \leq \alpha \leq n, n \in \mathbb{N}^+$ and $\Gamma(.)$ denotes the Gamma function.

Definition 4: Mahgoub transform is defined on the set of continuous functions and exponential order. We consider functions in the set A defined by

$$A = \left\{ f(t): |f(t)| < Pe^{\frac{|t|}{\epsilon_i}} if\ t \in (-1)^i \times [0,\infty), i = 1,2;\ \epsilon_i > 0 \right\} \quad (3)$$

where ϵ_1, ϵ_2 may be finite or infinite and the constant P must be finite.

Let $f \in A$, then Mahgoub transform is defined as

$$M[f(t)] = H(u) = u\int_0^\infty f(t)e^{-ut}dt,\ t \geq 0, \epsilon_1 \leq u \leq \epsilon_2 \quad (4)$$

Theorem 5: [11] Let $n \in \mathbb{N}$ and $\alpha > 0$ be such that $n-1 < \alpha \leq n$ and $H(u)$ be the Mahgoub transform of the function $f(t)$, then the Mahgoub transform of Caputo fractional derivative of $f(t)$ of order α is given by

$$M[{}_0^cD_t^\alpha f(t)] = u^\alpha H(u) - \sum_{k=0}^{n-1} u^{\alpha-k} f^{(k)}(0), \quad (5)$$

3. Construction of MADM

Consider the nonlinear Fredholm integro-differential equation of fractional order

$${}^cD^\alpha\, y(t) = p(t)y(t) + g(t) + \lambda\int_0^1 k(t,x)F(y(x))dx \quad (6)$$

for $t \in [0,1]$, with the initial conditions $y^{(i)}(0) = \delta_i,\quad i = 0,1,\dots,n-1, n-1 < \alpha \leq n, n \in \mathbb{N}$.

where $g \in L^2([0,1]), p \in L^2([0,1]), k \in L^2([0,1]^2)$ are known functions, $y(t)$ is the unknown function, D^α is the Caputo fractional differential operator of order α.

Applying Mahgoub transform on both sides of Eqn. (6)

$$M[{}^cD^\alpha\, y(t)] = M[p(t)y(t)] + M[g(t)] + M\left[\lambda\int_0^1 k(t,x)F(y(x))dx\right]$$

Using the above theorem we get

$$M[y(t)] = \sum_{k=0}^{n-1} u^{-k}y^k(0) + \frac{1}{u^\alpha}M[p(t)y(t)] + \frac{1}{u^\alpha}M[g(t)] + \frac{1}{u^\alpha}M\left[\lambda\int_0^1 k(t,x)F(y(x))dx\right] \quad (7)$$

The MADM represents the solution as an infinite series

$$y(t) = \sum_{n=0}^\infty y_n(t) \quad (8)$$

The nonlinear operator is decomposed in Adomian Polynomial as

$$Ny = F(y(x)) = \sum_{n=0}^\infty A_n(y(x)) \quad (9)$$

where A_n is the Adomian polynomials of $y_0, y_1, y_2, \dots, y_n, \dots$ that are given by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^\infty \lambda^i y_i\right)\right]_{\lambda=0}, n = 0,1,2,\dots$$

For the nonlinear function $Ny = F(y)$ the first Adomian polynomials are given by

$$A_0 = F(y_0)$$

$$A_1 = y_1F^{(1)}(y_0),$$

$$A_2 = y_2F^{(1)}(y_0) + \frac{1}{2}{y_1}^2F^{(2)}(y_0),$$

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$$A_3 = y_3F^{(1)}(y_0) + y_1y_2F^{(2)}(y_0) + \frac{1}{3}{y_1}^3F^{(3)}(y_0),$$
$$\vdots$$

Substituting (8) and (9) into (7), we can obtain

$$M[\sum_{n=0}^{\infty} y_n(t)] = \sum_{k=0}^{n-1} u^{-k}y^k(0) + \frac{1}{u^\alpha}M[p(t)\sum_{n=0}^{\infty} y_n(t)] + \frac{1}{u^\alpha}M[g(t)] + \frac{1}{u^\alpha}M\left[\lambda\int_0^1 k(t,x)\sum_{n=0}^{\infty} A_n(y(x))\,dx\right] \quad (10)$$

Comparing both sides of Eqn. (10) yields the following iterative algorithm:

$$M[y_0] = \sum_{k=0}^{n-1} u^{-k}y^k(0) + \frac{1}{u^\alpha}M[g(t)], \quad (11)$$

$$M[y_1] = \frac{1}{u^\alpha}M[p(t)y_0] + \frac{\lambda}{u^\alpha}M\left[\int_0^1 k(t,x)A_0(y(x))dx\right] \quad (12)$$

$$M[y_2] = \frac{1}{u^\alpha}M[p(t)y_1] + \frac{\lambda}{u^\alpha}M\left[\int_0^1 k(t,x)A_1(y(x))dx\right] \quad (13)$$

In general, the recursive relation is given by

$$M[y_{n+1}] = \frac{1}{u^\alpha}M[p(t)y_n] + \frac{\lambda}{u^\alpha}M\left[\int_0^1 k(t,x)A_n(y(x))dx\right] \quad (14)$$

Applying inverse Mahgoub transform to (11)-(14) we get

$$y_0(t) = M^{-1}\left[\sum_{k=0}^{n-1} u^{-k}y^k(0) + \frac{1}{u^\alpha}M[g(t)]\right]$$

$$y_1(t) = M^{-1}\left[\frac{1}{u^\alpha}M[p(t)y_0]\right] + M^{-1}\left[\frac{\lambda}{u^\alpha}M\left[\int_0^1 k(t,x)A_0(y(x))dx\right]\right]$$
$$\vdots$$

$$y_{n+1}(t) = M^{-1}\left[\frac{1}{u^\alpha}M[p(t)y_n]\right] + M^{-1}\left[\frac{\lambda}{u^\alpha}M\left[\int_0^1 k(t,x)A_n(y(x))dx\right]\right]$$

4. Numerical Examples

In this section, we present some examples to show the effectiveness of the Mahgoub Adomian decomposition method for solving the Fredholm integro-differential equations of fractional order.

Example 1

Consider the nonlinear Fractional Fredholm integro-differential equation

$${}^cD^\alpha y(t) = 1 - \frac{1}{4}t + \int_0^1 xt[y(x)]^2dx, \quad (15)$$

for $0 < \alpha \leq 1$ and with the initial condition $y(0) = 0$

Exact solution of Eqn. (15) for $\alpha = 1$ is $y(t) = t$.

Applying the Mahgoub transform to both sides of (15) gives

$$M[{}^cD^\alpha y(t)] = M\left[1 - \frac{1}{4}t\right] + M\left[\int_0^1 xt[y(x)]^2dx\right], \quad (16)$$

Using Theorem 5 and initial condition in Eqn. (16), we get

$$M[y(t)] = \frac{1}{u^\alpha} - \frac{1}{4u^{\alpha+1}} + \frac{1}{u^\alpha}M\left[\int_0^1 xt[y(x)]^2dx\right]$$

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Assuming an infinite series solution of the form (8) and (9) in the above equation, we have

$$M[\sum_{n=0}^{\infty} y_n] = \frac{1}{u^{\alpha}} - \frac{1}{4u^{\alpha+1}} + \frac{1}{u^{\alpha}} M\left[\int_0^1 xt \sum_{n=0}^{\infty} A_n \, dx\right] \quad (17)$$

where the nonlinear operator $F(y) = y^2$ is decomposed as in terms of the Adomian polynomials. The first few Adomian polynomials are

$$A_0 = {y_0}^2,$$

$$A_1 = 2y_0y_1,$$

$$A_2 = 2y_0y_2 + {y_1}^2,$$

$$A_3 = 2y_0y_3 + 2y_1y_2,$$

$$A_4 = 2y_0y_4 + 2y_1y_3 + {y_2}^2,$$

Comparing both sides of Eqn. (17) then we can get the following relation:

$$M[y_0] = \frac{1}{u^{\alpha}}, \quad (18)$$

$$M[y_1] = -\frac{1}{4u^{\alpha+1}} + \frac{1}{u^{\alpha}} M\left[\int_0^1 xtA_0 dx\right] \quad (19)$$

$$\vdots$$

$$M[y_n] = \frac{1}{u^{\alpha}} M\left[\int_0^1 xtA_{n-1} dx\right], n \geq 2. \quad (20)$$

Taking inverse Mahgoub transform in Eqns. (18)-(20) we obtain

$$y_0 = \frac{t^{\alpha}}{\Gamma(1+\alpha)}$$

$$y_1 = -\frac{t^{1+\alpha}}{4\Gamma(2+\alpha)} + M^{-1}\left[\frac{1}{u^{\alpha}} M\left[\int_0^1 xtA_0 dx\right]\right] = 0,$$

$$\vdots$$

$$y_n = M^{-1}\left[\frac{1}{u^{\alpha}} M\left[\int_0^1 xtA_{n-1} dx\right]\right] = 0, n \geq 2.$$

The approximate solution is $y = y_0 + y_1 + y_2 + \cdots$

i.e $y = \frac{t^{\alpha}}{\Gamma(1+\alpha)}$

The exact solution and our approximate solution obtained by our method corresponding to distinct values of t are presented in Table 1. The Approximate solution is very much close to the exact solution. The obtained numerical results for $\alpha = 0.5, 0.75, 0.95$ and $\alpha = 1$ are summarized in Figure 1.

Table 1: Numerical Solution of Eqn. (15) for different values of α

t	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.95$	MADM	
				$\alpha = 1$	Exact
0.1	0.356825	0.193488	0.114506	0.100000	0.100000
0.2	0.504627	0.325407	0.221210	0.200000	0.200000
0.3	0.618039	0.441058	0.325156	0.300000	0.300000
0.4	0.71365	0.547268	0.427350	0.400000	0.400000
0.5	0.797885	0.646967	0.528261	0.500000	0.500000
0.6	0.874039	0.741768	0.628160	0.600000	0.600000

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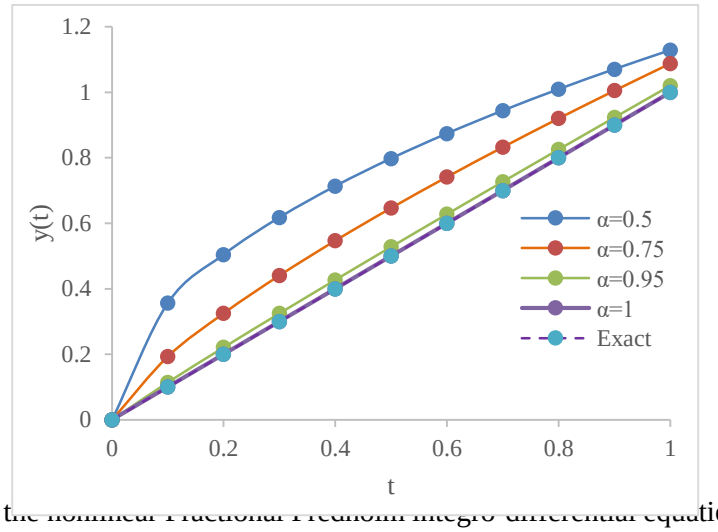
0.7	0.944070	0.832681	0.727227	0.700000	0.700000
0.8	1.009253	0.920391	0.825586	0.800000	0.800000
0.9	1.070474	1.005395	0.923331	0.900000	0.900000
1	1.128379	1.088065	1.020532	1.000000	1.000000

Figure 1: The approximate solution of Eqn. (15) for $\alpha = 0.5, 0.75, 0.95$ and $\alpha = 1$



Example 2 Consider the nonlinear Fractional Fredholm integro-differential equation

$$ {}^{c}D^{5/3}y(t) = g(t) + \int_0^1 (x+t)^2 [y(x)]^3 \, dx, \tag{21} $$

where $g(t) = \frac{6}{\Gamma(1/3)} \sqrt[3]{t} - \frac{t^2}{7} - \frac{t}{4} - \frac{1}{9}$ with the initial condition $y(0) = y'(0) = 0$.

Applying Mahgoub transform and its properties to both sides of Eqn. (21), we have

$$ M\left[{}^{c}D^{5/3}y(t)\right] = M[g(t)] + M\left[\int_0^1 (x+t)^2 [y(x)]^3 \, dx\right], $$

Using Theorem 5 and the initial conditions in the above equation we get

$$ M[y(t)] = u^{-5/3} M[g(t)] + u^{-5/3} M\left[\int_0^1 (x+t)^2 [y(x)]^3 \, dx\right] $$

Assuming an infinite series solution of the form (8) and (9) in the above equation, we have

$$ M[\textstyle\sum_{n=0}^{\infty} y_n] = \frac{2}{u^2} - \frac{2}{7u^{11/3}} - \frac{1}{4u^{8/3}} - \frac{1}{9u^{5/3}} + \frac{1}{u^{5/3}} M\left[\int_0^1 (x+t)^2 \textstyle\sum_{n=0}^{\infty} A_n \, dx\right] \tag{22} $$

where the nonlinear operator $F(y) = y^3$ is decomposed as in terms of the Adomian polynomials. The first few Adomian polynomials are

$$ A_0 = {y_0}^3, $$

$$ A_1 = 3{y_0}^2 y_1, $$

$$ A_2 = 3{y_0}^2 y_2 + 3y_0 {y_1}^2, $$

$$ A_3 = 2{y_0}^2 y_3 + 6y_0 y_1 y_2 + {y_3}^3, $$

$$ \vdots $$

Comparing both sides of Eqn. (22), the components of $y(t)$ can be defined as follows:

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$$M[y_0] = \frac{2}{u^2} \tag{23}$$

$$M[y_1] = -\frac{2}{7u^{11/3}} - \frac{1}{4u^{8/3}} - \frac{1}{9u^{5/3}} + \frac{1}{u^{5/3}} M\left[\int_0^1 (x+t)^2 A_0 d\tau\right] \tag{24}$$

$$\vdots$$

$$M[y_n] = -\frac{1}{u^{5/3}} M\left[\int_0^1 (\tau+t)^2 A_{n-1} d\tau\right] \tag{25}$$

Taking the inverse Mahgoub transform in the Eqns. (23)-(25) we get

$$y_0 = t^2$$

$$y_1 = M^{-1}\left[-\frac{2}{7u^{11/3}} - \frac{1}{4u^{8/3}} - \frac{1}{9u^{5/3}} + \frac{1}{u^{5/3}} M\left[\int_0^1 (x+t)^2 . x^6\, dx\right]\right] = 0$$

$$\vdots$$

$$y_n = 0, \;\; n > 1.$$

The solution is obtained to be $y(t) = \sum_{n=0}^{\infty} y_n = t^2$,

which is the exact solution.

Conclusion

In this paper, Mahgoub Adomian decomposition method has been successfully applied to find the approximate solution of nonlinear Fractional Fredholm integro-differential equation. This method is very reliable and efficient in finding analytical as well as numerical solutions of linear and nonlinear fractional integro-differential equations.

References

[1] S. G. Samko, A. A. Kilbas and O. I. Marichev, Fractional integrals and derivatives: Theory and Applications, Gordon and Breach, 1993.

[2] I. Podlubny, Fractional Differential Equations, Academic Press, San Diego, California, USA, 1999.

[3] S. M. Momani, Local and global existence theorems on fractional integro-differential equations, Journal of Fractional Calculus, 18, pp. 81-86, 2000.

[4] C. Yang, Numerical solution of nonlinear Fredholm integro-differential equations of fractional order by using hybrid of block pulse functions and Chebyshev polynomials. Mathematical Problems in Engineering, pp. 1-11, 2011. DOI: 10.1155/2011/341989.

[5] H. Saeedi and M. F. Samimi, He's homotopy perturbation method for nonlinear Fredholm integro differential equations of fractional order, International Journal of Engineering Research and Applications, 2, pp. 52-56, 2012.

[6] R. K. Saeed and H. M. Sdeq, "Solving a system of linear Fredholm fractional integro-differential equations using Homotopy perturbation method", Australian Journal of Basic and Applied Sciences, 4, pp. 633-638, 2010.

[7] L. Zhu and Q. Fan, Solving fractional nonlinear Fredholm integro-differential equations by the second kind Chebyshev wavelet, Communications in Nonlinear Science and Numerical Simulation, 17, pp. 2333-2341, 2012.

International Journal of Management, IT & Engineering
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[8] H. Saeedi, M. M. Moghadamb, N. Mollahasani and G. N. Chuev, A CAS wavelet method for solving nonlinear Fredholm integro-differential equations of fractional order, Communications in Nonlinear Science and Numerical Simulation, 16, pp. 1154-1163, 2011.

[9] Y. Yang, Y. Chen and Y. Huang, Spectral collocation method for fractional Fredholm integro differential equations, Journal of Korean Mathematical Society, 51, 203-224, 2014.

[10] S. Momani and R. Qaralleh, An efficient method for solving systems of fractional integro-differential equations, Computers and mathematics with applications, 52, pp. 459-470, 2006.

[11] A. Emimal and K. Lydia, Solving nonlinear fractional differential equation using Mahgoub Adomian decomposition method, Journal of Emerging Technologies and Innovative Research, 5, pp. 580-585, 2018.

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An investigation of similarity measures between trapezoidal fuzzy numbers

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Abstract:

The degree of similarity or dissimilarity between the objects plays an important role. Effectiveness of similarity measures will be known only after comparing it with existing one. Similarity measures have many applications in the field of information retrieval, citation analysis, classification, clustering, pattern recognition,risk analysis, decision making, image processing etc. However, it scarcely deals with trapezoidal fuzzy information and multicriteria decision-making problems. A Novel similarity measure proposed by Lakshmana Gomathy Nayagam and Geetha Sivaraman using trapezoidal fuzzy number is precise due to the inclusion of minimal parameters .For this purpose ,a novel similarity measure between trapezoidal fuzzy numbers is used in this paper and compared with the existing results of Jun Ye,2011.It is found that novel similarity measure is simple and effective and it coincides with Chen and Chen's similarity measure(2007).

Keywords: Similarity measures, trapezoidal fuzzy numbers, similarity measures between trapezoidal fuzzy numbers, Novel similarity method, Cosine similarity method, Chen methods.

Introduction:

In many real-world situations, the decision maker cannot provide deterministic alternative values but fuzzy numbers instead. This kind of uncertainty in multicriteria decision making (MCDM) can be modeled using fuzzy set theory and is ideally suited for solving these problems. Bellman and Zadeh (1970) first proposed the fuzzy decision making model. Since then, great numbers of studies have been done on fuzzy multicriteria decision making (FMCDM) (Hwang *et al*, 1981; Chen *et al*, 1992; Xu, 2004; Wang *et al*, 2005; Wu *et al*, 2007), so that the discipline has created several methodologies so far. Chen (2000) extended one
of known classical MCDM method, technique for order preference by similarity to ideal solution (TOPSIS), to develop a methodology for solving multicriteria decision-making problems in fuzzy environment. Recently, Jahanshahloo *et al* (2006) developed the TOPSIS approach to decision making with fuzzy data, where the rating of each alternative and the weight of each criterion are expressed in triangular fuzzy numbers.
He *et al* (2009) proposed the extension of the expected value method for multiple attribute decision making with fuzzy data, in which the preference values take the form of triangular fuzzy numbers and attribute weights are completely unknown. Moreover, Zeng (2006) developed an expected value method for FMCDM problems, in which the criteriaweights and criteria values are trapezoidal fuzzy numbers.
On the other hand, Salton and McGill (1983) proposed a cosine similarity measure between fuzzy sets and applied it to information retrieval of words. Recently, Ye (2011) proposed a cosine similarity measure between intuitionistic fuzzy sets based on the concept of the cosine similarity measure between fuzzy sets and it demonstrated a stronger discrimination among the existing similarity measures

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by the comparisons of a variety of similarity measures for intuitionistic fuzzy sets, and then it was applied to pattern recognition and medical diagnosis. However, the domains of fuzzy sets and intuitionistic fuzzy sets are discrete sets, Trapezoidal fuzzy numbers extend discrete sets to continuous sets and are the extension of fuzzy sets. The advantage of the continuous sets is to maintain the integrity of information; while discrete sets may be loss partial information in the information integration. Therefore, the continuous sets are superior to the discrete sets. Furthermore, the existing cosine similarity measures do not deal with trapezoidal fuzzy numbers. Therefore, this paper will propose a cosine similarity measure for trapezoidal fuzzy numbers and a FMCDM method based on the cosine similarity measure under the conditions that the criterion weights and the evaluated values in the decision matrix are expressed by means of trapezoidal fuzzy numbers. Through the expected weight and the weighted cosine similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. The advantage is that the proposed FMCDM approach has some simple tools and concepts in the fuzzy similarity measure and aggregation approach among the existing ones. An illustrative example shows that the proposed method is simple and effective.

2. Some Preliminaries

This section introduces some definitions and basic concepts related to fuzzy sets, fuzzy numbers, and trapezoidal fuzzy numbers.

Definition 1 (Zadeh, 1965): A fuzzy set A in the universe of discourse $X = \{x1, x2,..., xn\}$ is defined as follows:

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\},$$

which is characterized by membership function $\mu_A(x)$: $X \rightarrow [0, 1]$, where $\mu_A(x)$ indicates the membership degree of the element x to the set A.

Definition 2 (Dubois *et al*, 1983). Let A be an fuzzy number in the set of real numbers R, its membership function is defined as

$$\mu_A(x) = \begin{cases} 0, x < a_1 \\ f_A(x), a_1 \le x \le a_2 \\ 1, a_2 \le x \le a_3 \\ g_A(x), a_3 \le x \le a_4 \\ 0, a_4 < x \end{cases}$$

where $a_1, a_2, a_3, a_4 \in R$, f_A: $[a_1, a_2] \rightarrow [0, 1]$ is a increasing continuous function, $f_A(a_1) = 0$, $f_A(a_2) = 1$, which is called the left side of the fuzzy number A, and
g_A: $[a_3, a_4] \rightarrow [0, 1]$ is a decreasing continuous function, $g_A(a_3) = 1$, $g_A(a_4) = 0$, which is called the right side of the fuzzy number A.
Particularly, if the increasing functions f_A and decreasing functions g_A are linear, then we have trapezoidal fuzzy numbers, which are preferred in practice.
For convenience, the trapezoidal fuzzy number is usually denoted by $A = (a1, a2, a3, a4)$.

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Definition 3 (Dubois *et al*, 1983): A trapezoidal fuzzy number A with four parameters $a_1 \le a_2 \le a_3 \le a_4$ is denoted as $A = (a_1, a_2, a_3, a_4)$ in the set of real numbers R. In this case, its membership function can be given as

$$\mu_A(x) = \begin{cases} 0, x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2 \\ 1, a_1 \le x \le a_2 \\ \dfrac{x - a_4}{a_3 - a_4}, a_3 \le x \le a_4 \\ 0, a_4 < x \end{cases}$$

The trapezoidal fuzzy number degenerates to a triangular fuzzy number when $a_2 = a_3$ holds, which is considered as a special case of the trapezoidal fuzzy number.
The following properties for trapezoidal fuzzy numbers have been given by Zeng (2006).
Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers in the set of real numbers R and r be a positive
scalar number. Then,
$A+B = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$, (4)
$A–B = (a_1–b_1, a_2–b_2, a_3–b_3, a_4–b_4)$, (5)
$rA = (ra_1, ra_2, ra_3, ra_4)$, (6)
$AB = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$. (7)
The expected value (Zeng, 2006) of a trapezoidal fuzzy number
$A = (a_1, a_2, a_3, a_4)$ is

$$E(A) = \frac{1}{4}(a_1+a_2+a_3+a_4)$$

Various Similarity Measures between Trapezoidal Fuzzy Numbers:

Chen Method 1996:

Chen (1996) presented a distance-based similarity measure for trapezoidal fuzzy numbers. Consider two trapezoidal fuzzy number A and B, where $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$. The degree of similarity S(A,B) between A and B can be calculated as

$$S(A,B) = 1 - \frac{\sum_{i=1}^{4} |a_i - b_i|}{4}$$

Chen Method 2001:

Hsieh and Chen proposed similarity measure using the "graded mean integration distance" where the degree of similarity measure S(A,B) between fuzzy numbers A and B can be calculated as follows:

$$S(A,B) = \frac{1}{1 + d(A,B)}$$

Where

$$d(A,B) = |P(A) - P(B)|$$

P(A) and P(B) are the graded mean integration representations of A and B respectively. If A and B are trapezoidal fuzzy numbers, where $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$,then the

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graded mean integration representations P(A) and P(B) of A and B respectively are defined as follows:

$$P(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

$$P(B) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

It is obvious that the larger the value of S(A,B), the more the similarity between the fuzzy numbers *A* and B.

Chen and Chen's Method (2007):

Consider two generalized trapezoidal fuzzy numbers *A* and *B*, where $A=(a_1, a_2, a_3, a_4; W_A)$, $B= (b_1, b_2, b_3, b_4; W_B)$, $0 \le W_A \le 1$, $0 \le W_B \le 1$, $0 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$, and $0 \le b_1 \le b_2 \le b_3 \le b_4 \le 1$. The degree of similarity $S(A, B)$ between the generalized trapezoidal fuzzy numbers *A* and *B* can be calculated as follows (Chen, 2007):

$$S(A,B) = \left[\sqrt[4]{\prod_{i=1}^{4}\left(2 - |a_i - b_i|\right)} - 1\right] X \frac{\min(y_A, y_B)}{\max(y_A, y_B)}$$

Where y_A is calculated by

$$Y_A = \begin{cases} \dfrac{W_A X \dfrac{a_3 - a_2}{a_4 - a_1} + 2}{6}, & if\ a_1 \neq a_4\ and\ 0 \le W_A \le 1, \\ \dfrac{W_A}{2}, & if\ a_1 = a_4\ and\ 0 \le W_A \le 1 \end{cases}$$

Where y_B is calculated by the above formula. The larger value of S(A,B),the greater the similarity measure between the generalized fuzzy numbers A and B.

Cosine Similarity Measure between Trapezoidal Fuzzy Numbers:

In this section we introduce a definition and some concepts of the cosine similarity between fuzzy sets (Salton *et al*, 1983) and propose a cosine similarity measure between trapezoidal fuzzy numbers, then compare the calculation results with Chen (1996) and Chen *et al* (2001, 2007).

Cosine Similarity Measure for Fuzzy Sets

A cosine similarity measure for fuzzy sets (Salton *et al*, 1983) is defined as the inner product of two vectors divided by the product of their lengths. This is nothing but the cosine of the angle between the vector representations of the two fuzzy sets.

Assume that $A = (\mu_A(x_1), \mu_A(x_2),\ldots, \mu_A(x_n))$ and $B = (\mu_B(x_1), \mu_B(x_2),\ldots, \mu_B(x_n))$ are two fuzzy sets in the universe of discourse $X =\{x_1, x_2,\ldots, x_n\}$, $x_i \in X$.

A cosine similarity measure (angular coefficient) between *A* and *B* can be defined as follows (Salton *et al*,1983):

$$C_F(A,B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A^2(x_i)}\sqrt{\sum_{i=1}^{n} \mu_B^2(x_i)}}$$

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where $0 \le C_F(A, B) \le 1$. It is undefined if $\mu_A(x_i) = 0$ and/or $\mu_B(x_i) = 0$ ($i = 1, 2,\ldots, n$). Then, let the cosine measure value be zero when $\mu_A(x_i) = 0$ and/or $\mu_B(x_i) = 0$ ($i = 1, 2,\ldots, n$).

Cosine Similarity Measure for Trapezoidal Fuzzy Numbers:

In this subsection, a cosine similarity measure between trapezoidal fuzzy numbers is proposed based on the concept of the cosine similarity measure for fuzzy sets.

Let $A = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number in the set of real numbers R, the four parameters in A can be considered as a vector representation with the four elements. Thus, a cosine similarity measure for trapezoidal fuzzy numbers is proposed in an analogous manner to the cosine similarity measure (angular coefficient) between fuzzy sets (Salton *et al*, 1983). Assume that there are two trapezoidal fuzzy numbers $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ in the set of real numbers R. Based on the extension of the cosine similarity measure for fuzzy sets, a cosine similarity measure between A and B is proposed as follows:

$$S(A,B) = \frac{\sum_{p=1}^{4} a_p b_p}{\sqrt{\sum_{p=1}^{4} (a_p)^2}\sqrt{\sum_{p=1}^{4} (b_p)^2}}$$

Novel similarity measure of trapezoidal fuzzy numbers:

In this section, a new method for the degree of similarity between trapezoidal fuzzy numbers and prove some properties of the proposed similarity measure.

Let $A=(a_1,a_2,a_3,a_4,W_A)$ and $B=(b_1,b_2,b_3,b_4,W_B)$ be two generalized trapezoidal fuzzy numbers .Then the degree of similarity S(A,B) between the generalized trapezoidal fuzzy numbers A and B is given by

$$S(A,B) = \left[\sqrt[4]{\prod_{i=1}^{4}\left[1-\left|a_i - b_i\right|\right]}\right] X \frac{Min(P(A),P(B)) + Min(W_A, W_B)}{Max(P(A),P(B)) + Max(W_A, W_B)}$$

where P(A) and P(B) are

$$P(A) = \sqrt{(a_2-a_1)^2 + (a_3-a_2)^2 + (a_4-a_3)^2 + W_A^2}$$

$$P(B) = \sqrt{(b_2-b_1)^2 + (b_3-b_2)^2 + (b_4-b_3)^2 + W_B^2}$$

Clearly P(A) and P(B) are the minimum distances of the generalized trapezoidal fuzzy numbers A and B respectively which includes height and scales of generalized trapezoidal fuzzy numbers. The larger the value of S(A,B), the greater the similarity between the fuzzy numbers A and B.

Comparison of Similarity Measures

To illustrate the effectiveness of the comparison of the similarity measures discused in the previous section, we use six sets of trapezoidal fuzzy numbers to compute the results of all methods.

The six sets of trapezoidal fuzzy numbers are shown in Table 1 adapted from Chen (1996) and Chen *et al* (2001, 2007), and then the calculation results of different similarity measures

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are shown as Table 2.By applying Novel method, from Table 2, we can see that the results of Novel method coincides with ones of Chen (1996) and Chen et al. (2001, 2007). Therefore, the Novel method is reasonable.

Table 1. Six sets of trapezoidal fuzzy numbers

Set numbers	Trapezoidal fuzzy numbers
Set 1	A=(0.1,0.2,0.3,0.4), B=(0.1,0.25,0.25,0.4)
Set 2	A=(0.1,0.2,0.3,0.4), B=(0.5,0.65,0.65,0.8)
Set 3	A=(0.1,0.2,0.3,0.4), B=(0.3,0.45,0.45,0.6)
Set 4	A=(0.1,0.2,0.3,0.4), B=(0.1,0.2,0.3,0.4)
Set 5	A=(0.1,0.2,0.3,0.4), B=(0.5,0.6,0.7,0.8)
Set 6	A=(0.1,0.2,0.3,0.4), B=(0.3,0.4,0.5,0.6)

Table 2. Calculation results of different similarity measures

Set numbers	Chen's method (1996)	Chen and Chen's method (2001)	Chen and Chen's method (2007)	Cosine Method	Novel Method
Set 1	0.975	0.8357	0.9499	0.9916	0.9698
Set 2	0.6	0.3086	0.5846	0.9633	0.5967
Set 3	0.8	0.5486	0.7794	0.9774	0.7963
Set 4	1	1	1	1	1
Set 5	0.6	0.36	0.6	0.9689	0.6
Set 6	0.8	0.64	0.8	0.9844	0.8

Conclusions

In this paper, various similarity measure between trapezoidal fuzzy numbers have been studied and compared. We observed that the novel similarity measure coincide with Chen's method (1996).The novel similarity measure leads to more accurate and effective.

References

1. Bellman R. and Zadeh L.A. 1970. Decision making in a fuzzy environment. *Management Science*, Vol. 17B, No. 4, pp. 141-164.
2. Chen S.J. and Hwang C.L. 1992. Fuzzy Multiple Attribute Decision Making: Methods and Applications. *Springer-Verlag*, Berlin.
3. Chen S.M. 1996. New methods for subjective mental workload assessment and fuzzy risk analysis. *Cybernetics and Systems*, Vol.27, No. 5, pp. 449-472.
4. Chen S.J. and Chen S.M. 2001. A new method to measure the similarity between fuzzy numbers. In: *Proceedings of the 10th IEEE international conference on fuzzy systems*, Melbourne, Australia, pp. 208-214.
5. Chen S.J. and Chen S.M. 2007. Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers. *Applied Intelligence*,vol.26, No.1,
6. pp. 1-11.

Vol. 9 Issue 1, (Special Issue HICAAMMC- January 2019)
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Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
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7. Chen C.T. 2000. Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems*, Vol. 114,pp. 1-9.
8. Dubois D. and Prade H. 1983. Ranking fuzzy number in the setting of possibility theory. *Information Sciences*, Vol. 30, pp. 183-224.
9. He Y.Y., Wang Q. and Zhou D.Q. 2009. Extension of the expected value method for multiple attribute decision making with fuzzy data. *Knowledge-Based Systems*, Vol. 22, pp. 63-66.
10. Hwang C.L. and Yoon K. 1981. Multiple Attribute Decision Making: Methods and Applications. *Springer-Verlag*, Berlin. Jahanshahloo G.R., Hossein zadeh Lotfi F. and Izadikhah M. 2006. Extension of the TOPSIS method for decision-making with fuzzy data. *Applied Mathematics and Computation*, Vol. 181, pp. 1544-1551.
11. Salton G. and McGill M.J. 1983. Introduction to modern information retrieval. *McGraw-Hill*, Auckland.
12. Wang Y.M. and Parkan C. 2005. Multiple attribute decision making based on fuzzy preference information on alternatives:ranking and weighting. *Fuzzy Sets and Systems*, Vol. 153, pp. 331-346.
13. Wu Z.B. and Chen Y.H. 2007. The maximizing deviation method for group multiple attribute decision making under linguistic environment. *Fuzzy Sets and Systems*, Vol. 158, pp. 1608-1617.
14. Xu Z.S. 2007. A method for multiple attribute decision making with incomplete weight information in linguistic setting. *Knowledge-Based Systems*, Vol. 20, pp. 719-725.
15. Xu Z.S. 2004. Uncertain Multiple Attribute Decision Making: Methods and Applications. *Tsinghua University Press*, Beijing.
16. Ye J. 2011. Cosine similarity measures for intuitionistic fuzzy sets and their applications. *Mathematical and Computer Modelling*, Vol. 53, No.1-2, pp. 91-97.
17. Zadeh L.A. 1965. Fuzzy sets. *Information Control*, Vol. 8, pp. 338-353.
18. L. A. Zadeh, "Fuzzy Sets," *Information and Control*, vol. 8, pp. 338 –
19. 353, 1965. D. V. Weken, M. Nachtegael, and E. E. Kerre, "Using similarity
20. measures and homogeneity for the comparison of images," *Image and*
21. *Vision Computing*, vol. 22, pp. 695 – 702, 2004.
22. İ. Güler and E. D. Übeyli, "Analysis of Doppler ultrasound signals
23. based on fuzzy similarity index," *Expert Systems with Applications*, vol.
24. 31, pp. 130 – 136, 2006.

25. M. S. Yang and D. C. Lin, "On similarity and inclusion measures
26. between type-2 fuzzy sets with an application to clustering,"
27. *Computers & Mathematics with Applications*, vol. 57, pp. 896 – 907,
28. 2009.

29. H. B. Mitchell, "On the Dengfeng-Chuntian similarity measure and its
30. application to pattern recognition," *Pattern Recognition Letters*,
31. vol.24, pp. 3101 – 3104, 2003.

32. N. Karacapilidis and C. Pappis, "Computer-supported collaborative
33. argumentation and fuzzy similarity measures in multiple criteria
34. decision making," *Computers & Operations Research*, vol. 27,

Vol. 9 Issue 1, (Special Issue HICAAMMC- January 2019)
ISSN: 2249-0558 Impact Factor: 7.119
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
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35. pp. 653– 671, 2000.

36. B. Prados-Suárez, J. Chamorro-Martínez, D. Sánchez, and J. Abad,
37. "Region-based fit of color homogeneity measures for fuzzy image
38. segmentation," *Fuzzy Sets and Systems*, vol. 158, pp. 215 – 229, 2007.

39. R. Saraçoğlu, K. Tütüncü, and N. Allahverdi, "A fuzzy clustering
40. approach for finding similar documents using a novel similarity
41. measure," *Experts Systems with Applications*, vol. 33, pp. 600 – 605,
42. 2007.

43. S. J. Chen and S. M. Chen, "Fuzzy risk analysis based on similarity
44. measures of generalized fuzzy numbers," *IEEE Transactions on Fuzzy*
45. *systems*, vol. 11, pp. 45 – 56, 2003.

46. S. M. Chen, "New methods for subjective mental workload assessment
47. and fuzzy risk analysis," *Cybernetics and Systems: An International*
48. *Journal*, vol. 27, pp. 449-472, 2001.

49. T. Gu and B. Dubuisson, "Similarity of classes and fuzzy clustering,"
50. Fuzzy *Sets and Systems*, vol. 34, pp. 213 – 221, 1990.

51. S. H. Wei and S. M. Chen, "A new approach for fuzzy risk analysis
52. based on similarity measures of generalized fuzzy numbers," *Expert*
53. *Systems with Applications*, vol. 36, pp. 589 – 598, 2009.

54. C. H. Hsieh and S. H. Chen, "Similarity of generalized fuzzy numbers
55. with graded mean integration representation," in *Proceedings of the*
56. *Eighth International Fuzzy Systems Association World Congress*, vol.
57. 2, pp. 551 – 555, 1999.

58. H. S. Lee, "An optimal aggregation method for fuzzy opinions of group
59. decision," *Proceedings of the 1999 IEEE International conference on*
60. *Systems, Man and Cybernetics*, vol. 3, pp. 314 - 319, 1999.

61. S. J. Chen, "A new similarity measure of generalized fuzzy numbers
62. based on Geometric-Mean Averaging operator," in *Proceedings of the*
63. *IEEE International Conference on Fuzzy Systems*, pp. 1879 – 1886,
64. Fuzz-IEEE 2006, Vancouver, Canada, 2006.

65. S. J. Chen, "Similarity measure between Generalized Fuzzy Numbers
66. using Quadratic- Mean operator," *Second International Symposium on*
67. *Intelligent Information Technology Application*, vol. 3, pp. 440 – 444,
68. 2008.

69. D. Yong, S. Wenkang, D. Feng, and L. Qi, "A new similarity measure
70. of generalized fuzzy numbers and its application to pattern
71. recognition," *Pattern Recognition Letters*, vol. 25, pp. 875 – 883,2004.

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A Study on Performance of Hedge Fund in India

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Abstract— Two fast growing emerging markets, India and China are keenly observed for new investment propositions, particularly investment in hedge funds. Presence of systematic institutional framework for hedging, regulatory factors, a well-developed capital economy, liberalized stable economy, rapid reforms, democratic set-up, good information disclosure standards, better return on capital have rather favored India score over China as a superior place for investment in hedge funds. Investment in hedge funds is a cynosure of interest for sophisticated investors, wealthy individuals or families and big institutions. They are class of investors who believe in the finance mantra - higher risk, higher opportunity investments and higher rewards. Yes, investment in India focused hedge funds - for those with an appetite for risks, most willing to take risks in anticipation of explosive reward. Investment in hedge funds in India has been gaining momentum post 2001-2002. From this study the research has made to examine the overall performance of hedge fund in India.

Keywords— Hedge Fund, Performance, Characteristics,

I. INTRODUCTION

Hedge funds are alternative investments using pooled funds that employ numerous different strategies to earn active return, or alpha, for their investors. Hedge funds may be aggressively managed or make use of derivatives and leverage in both domestic and international markets with the goal of generating high returns (either in an absolute sense or over a specified market benchmark). It is important to note that hedge funds are generally only accessible to accredited investors as they require less SEC regulations than other funds. One aspect that has set the hedge fund industry apart is the fact that hedge funds face less regulation than mutual funds and other investment vehicles.

A. Hedge funds in India

Financial experts opine that India has tremendous potential for attracting global investments in hedge funds. The early entrants into the Indian markets have recorded encouraging returns which in turn attracted other hedge fund players to step in. Renaissance Technologies, Vikram Pandit-founded Old Lane, DE Shaw, Och-Ziff Capital Management are some reputed international hedge funds firms in India. Here is a list of hedge funds operating in India.

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- Indea Capital Pte Ltd.
- India Capital Fund.
- India Deep Value Fund
- Absolute India Fund (AIF)
- Fair Value
- Naissance Jaipur (India) Fund
- Avatar Investment Management
- Passport India Fund
- HFG India Continuum Fund
- Monsoon Capital Equity Value Fund
- Karma Capital Management, LLC
- Vasishta South Asia Fund Limited
- Atyant Capital
- Atlantis India Opportunities Fund

B. Importance of the study

Hedge funds can provide benefits to financial markets by contributing to market efficiency and enhance liquidity. Many hedge fund advisors take speculative trading positions on behalf of their managed hedge funds based extensive research about the true value or future value of a security. Hedge funds play a valuable arbitrage role in reducing or eliminating mispricing in financial markets. They are an important source of liquidity, both in periods of calm and stress. They add depth and breadth to our capital market. This study provides some perspectives on hedge funds and the policy implications of their evolving role in the financial system.

C. Statement of the study

Each hedge fund is constructed to take advantage of certain identifiable market opportunities. Hedge funds use different investment strategies and thus are often classified according to investment style. There is substantial diversity in risk attributes and investments among styles. Legally, hedge funds are most often set up as private investment limited partnerships that are open to a limited number of accredited investors and require a large initial minimum investment. Investments in hedge funds are illiquid as they often require investors keep their money in the fund for at least one year, a time known as the lock-up period. Withdrawals may also only happen at certain intervals such as quarterly or bi-annually.

D. Objective of the study

This study mainly focuses on the performance of the hedge fund in India over the years.

II. Hedge Fund

A. The History of the Hedge Fund

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Former writer and sociologist Alfred Winslow Jones's company, A.W. Jones & Co. launched the first hedge fund in 1949. It was while writing an article about current investment trends for Fortune in 1948 that Jones was inspired to try his hand at managing money. He raised $100,000 (including $40,000 out of his own pocket) and set forth to try to minimize the risk in holding long-term stock positions by short selling other stocks. This investing innovation is now referred to as the classic long/short equities model. Jones also employed leverage to enhance returns.

In 1952, Jones altered the structure of his investment vehicle, converting it from a general partnership to a limited partnership and adding a 20% incentive fee as compensation for the managing partner. As the first money manager to combine short selling, the use of leverage, shared risk through a partnership with other investors and a compensation system based on investment performance, Jones earned his place in investing history as the father of the hedge fund.

Hedge funds went on to dramatically outperform most mutual funds in the 1960s and gained further popularity when a 1966 article in Fortune highlighted an obscure investment that outperformed every mutual fund on the market by double-digit figures over the previous year and by high double-digits over the previous five years.

However, as hedge fund trends evolved, in an effort to maximize returns, many funds turned away from Jones' strategy, which focused on stock picking coupled with hedging, and chose instead to engage in riskier strategies based on long-term leverage. These tactics led to heavy losses in 1969-70, followed by a number of hedge fund closures during the bear market of 1973-74.

The industry was relatively quiet for more than two decades until a 1986 article in Institutional Investor touted the double-digit performance of Julian Robertson's Tiger Fund. With a high-flying hedge fund once again capturing the public's attention with its stellar performance, investors flocked to an industry that now offered thousands of funds and an ever-increasing array of exotic strategies, including currency trading and derivatives such as futures and options.

High-profile money managers deserted the traditional mutual fund industry in droves in the early 1990s, seeking fame and fortune as hedge fund managers. Unfortunately, history repeated itself in the late 1990s and into the early 2000s as a number of high-profile hedge funds, including Robertson's, failed in spectacular fashion. Since that era, the hedge fund industry has grown substantially. Today the hedge fund industry is massive—total assets under management in the industry is valued at more than $3.2 trillion according to the 2016 Preqin Global Hedge Fund Report. The number of operating hedge funds has grown as well. There were around 2,000 hedge funds in 2002. That number increased to over 10,000 by 2015. However, in recent years, the number of hedge funds is currently on a decline again according to data from Hedge Fund Research.

B. Hedge funds in India Glossary

Inception date: The date on which the fund starts trading.

Investment manager: The prospectus ought to provide details about the investment manager whether an individual or a group of persons who would oversee investment strategies. The investment manager may also play the roles of fund manager, investment advisor or sponsor.

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Investment manager provides clarification to the investor on any matter related to hedge fund investment.

Fund administrator: Primarily responsible for processing investor's subscriptions and redemption, the fund administrator calculates the value of investor's holding too. Fund administrator may be an individual or a group of persons.

Custodian: The financial entity that holds hedge fund assets. This includes cash in the fund as well as securities.

Prime broker: There can be overlapping of roles and responsibilities with fund administrators. But prime brokers bring in investment and operational perspective to hedge funds. Prime brokers help fund manager or investment manager to decide on allocation of investment funds to different brokers.

Transaction broker: All investment activities are executed through the transaction broker.

Typical hedge fund investment

- Investor chooses and decides hedge fund investment
- Subscription amount is paid to the custodian.
- Custodian confirms receipt of payment to fund administrator.
- Fund administrator instructs issue of share to investor.
- Fund administrator issues reports on hedge fund performance.
- Investment manager instructs custodian to move funds to prime broker for investment in market.
- During the process the prime broker and custodian are in direct contact with fund administrator.

C. Guide to investing in hedge funds in India

- Prior to finalizing investment, take couple of months to know about the hedge fund industry in India. The age of hedge fund industry, the key players, their worth, the operational risks, the pros and cons of investing in hedge funds etc
- Identify potential hedge funds, refer commercial directories or databases. Account for your investment goals, risk tolerance level, amount allocated for investment.
- Get to understand the ground realities of regulatory factors, its implications; how business is run in India all helps.
- Read blogs, financial magazines, websites, news articles, white papers on hedge funds in India. Talk to personnel; preferably interact with hedge fund managers involved with hedge fund investments and those who have already invested in hedge funds.
- Notice annual events like Hedge funds world India to gain an assessment of the burgeoning Indian hedge fund industry.
- Approach wealth manager in wealth management companies, securities broker or licensed investment consultant for advice on hedge fund investments in India.
- Understand terms related to hedge funds, remittance, management fee and performance fee, withdrawal and redemption fees.
- Check the pros and cons of long-term hedge funds vs. short-term hedge funds.

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- Ensure your activities are that of an accredited investor (with a net worth of more than $1 million).
- Involve financial advisor in the process of investing in hedge funds in India.
- Maintain direct communication with hedge fund manager.
- Check if diverse hedge fund strategies and techniques are put to use.
- Receive and file monthly or quarterly updates.
- Engage in data mining, keep track of trends.
- Check with accountant with regard to tax reporting and implications.
- Know your rights, where to seek help in terms of a dissatisfied hedge fund investment operation, or any other complaint in general that doesn't confirm with regulations [1].

B. The structure of the hedge funds market

The Securities and Exchange Board of India (SEBI) issued the SEBI (Alternative Investment Funds) Regulations 2012 (AIF Regulations) on 21 May 2012 with a view to regulate the non-retail asset management segment on a comprehensive basis. The AIF Regulations introduced three categories of AIFs to register and regulate the formation of various types of alternative investment funds. Under the AIF Regulations, hedge funds are categorised as Category III AIFs employing complex or diverse trading strategies. Before the AIF Regulations, there were no specific regulations governing onshore hedge funds.

There was a rapid growth in the hedge funds market in 2017 and as of 30 June 2017, Category III AIFs had raised commitments worth INR150.6 billion, more than twice the commitments received up until June 2016. SEBI has progressively relaxed the regulatory regime for AIFs to attract further investments.

In June 2017, recognising the need for increased institutional participation in commodity derivatives markets in India to create desired liquidity and depth for efficient price discovery and price risk management, SEBI issued a circular permitting Category III AIFs to participate in the commodity derivatives markets subject to conditions such as broadbasing of the portfolio and reporting norms.

SEBI also introduced an online system that can be used to apply for registration and for complying with reporting and filing obligations. All applicants seeking registration must apply online. Existing AIFs must also file their compliance reports on the SEBI portal after activating their online accounts.

C. Regulatory framework

The key regulations include:

- Securities and Exchange Board of India (SEBI) (Alternative Investment Funds) Regulations 2012.
- SEBI (Foreign Portfolio Investor) Regulations 2014.
- SEBI circulars, issued from time to time, prescribing additional compliance and reporting requirements.

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- Foreign Exchange Management Act 1999 and regulations and circulars issued thereunder by Reserve Bank of India (RBI) from time to time, regulating investments by offshore investors/hedge funds into onshore hedge funds.

Regulatory bodies:

SEBI is the securities regulator, which was established under the SEBI Act 1992.

RBI is the central bank of India, which was established under the RBI Act 1949.

D. Hedge funds regulated ensure compliance with general international standards of good practice

Risk:

Onshore hedge funds employing leverage must set up a comprehensive risk management framework that is appropriate to the size, complexity and risk profile of the fund. The private placement memorandum (PPM) must contain information regarding risk management tools.

The onshore hedge fund must also provide a report to the investors detailing the material risks faced by the hedge fund and how the risks are managed, which may include:

- Concentration risk at fund level.
- Foreign exchange risk at fund level.
- Leverage risk at fund and investee company levels.
- Realisation risk (that is, change in exit environment) at fund and investee company levels.
- Strategy risk (that is, change in business strategy) at investee company level.
- Reputation risk at investee company level.
- Extra financial risks including environmental, social and corporate governance risks at fund and investee company levels.

Valuation and pricing:

The PPM must contain a description of the valuation procedure and the method for valuing assets. Details regarding the manner of valuation must also be provided in the annual report circulated to the investors. Further, the net asset value (NAV) must be disclosed to the investors on a quarterly basis for close-ended Category III Alternative Investment Funds (AIFs) and on a monthly basis for open-ended Category III AIFs.

Systems and controls:

The Securities and Exchange Board of India (SEBI) set out the operational, prudential and reporting norms that are applicable to Category III AIFs, which include:

- Maintaining a risk management framework.
- Maintaining appropriate records of transactions performed.

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- Submitting periodic reports to SEBI. The reports must be furnished quarterly by Category III AIFs that do not undertake leverage and monthly by Category III AIFs employing leverage. These reports must be submitted within seven days from the end of the relevant quarter or month.
- Establishing and implementing a liquidity management policy to meet redemption obligations and other liabilities.
- Reporting the amount of leverage at the end of each day to the custodian.

Insider dealing and market abuse:

The SEBI (Prohibition of Insider Trading) Regulations 2015 provide the framework for insider trading. Communication or procurement of unpublished price sensitive information (UPSI) or trading in securities when in possession of UPSI are offences.

Dealing in securities in a fraudulent or unfair manner and manipulating the price of securities is prohibited under the SEBI (Prohibition of Fraudulent and Unfair Trade Practices relating to Securities Market) Regulations 2003.

Hedge funds must comply with these regulations relating to prohibition on insider trading and fraudulent dealing in securities.

Transparency:

To ensure transparency, hedge funds must disclose the following information to the investors:

- Financial, risk management, operational, portfolio and transactional information on fund investments.
- Fees charged by the manager or sponsor or any associate of the manager or sponsor.
- Annual report to investors within 180 days from the year end.

In addition, certain event-based disclosures must be made to investors, as follows:

- Inquiries or legal action by legal or regulatory bodies in any jurisdiction.
- Material liability arising during the tenure of the hedge fund.
- Breach of any provision of the PPM or agreement made with the investor or any other fund documents.
- Change in control of the sponsor or manager or investee company.

Money laundering:

The offence of money laundering is governed by the Prevention of Money-laundering Act 2002 (PMLA). This legislation, among other provisions, deals with the prevention and control of money laundering as well as confiscation and seizure of property obtained from laundered money. Hedge funds are subject to the anti-money laundering obligations imposed by the PMLA as well as other compliance and reporting obligations made applicable by SEBI and the Reserve Bank of India through circulars issued by each of them.

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Short selling:

The SEBI (Alternative Investment Funds) Regulations 2012 do not contain a specific provision pertaining to short selling. However, SEBI permits all classes of investors, including hedge funds, to short sell subject to the SEBI framework on short selling and the framework dealing with securities lending and borrowing.

E. New Regulations for Hedge Funds

Hedge funds are so big and powerful that the SEC is starting to pay closer attention, particularly because breaches such as insider trading and fraud seem to be occurring much more frequently. However, a recent act has actually loosened the way that hedge funds can market their vehicles to investors.

In March 2012, the Jumpstart Our Business Startups Act (JOBS Act) was signed into law. The basic premise of the JOBS Act was to encourage funding of small businesses in the U.S. by easing securities regulation. The JOBS Act also had a major impact on hedge funds: In September 2013, the ban on hedge fund advertising was lifted. In a 4-to-1 vote, the SEC approved a motion to allow hedge funds and other firms that create private offerings to advertise to whomever they want, but they still can only accept investments from accredited investors. Hedge funds are often key suppliers of capital to startups and small businesses because of their wide investment latitude. Giving hedge funds the opportunity to solicit capital would in effect help the growth of small businesses by increasing the pool of available investment capital.

Hedge fund advertising entails offering the fund's investment products to accredited investors or financial intermediaries through print, television and the internet. A hedge fund that wants to solicit (advertise to) investors must file a "Form D" with the SEC at least 15 days before it starts advertising. Because hedge fund advertising was strictly prohibited prior to lifting this ban, the SEC is very interested in how advertising is being used by private issuers, so it has made changes to Form D filings. Funds that make public solicitations will also need to file an amended Form D within 30 days of the offering's termination. Failure to follow these rules will likely result in a ban from creating additional securities for a year or more.

Recent Years: Chasing the S&P:

Since the 2008 crisis, the hedge fund world has entered into another period of less-than-stellar returns. Many funds which previously enjoyed double-digit returns during an average year have seen their profits diminish significantly. In many cases, funds have failed to match the returns of the S&P 500. For investors considering where to place their money, this becomes an increasingly easy decision: why suffer the high fees and initial investments, the added risk, and the withdrawal limitations of hedge funds if a safer, simpler investment like a mutual fund can produce returns that are the same or, in some cases, even stronger?

Major Hedge Funds:

In mid-2018, data provider HFM Absolute Return created a ranked list of hedge funds according to total AUM. This list of top hedge funds includes some companies which hold

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more in AUM in other areas besides a hedge fund arm. Nonetheless, the ranking factors in only the hedge fund operations at each firm.

Paul Singer's Elliott Management Corporation held $35 billion in AUM as of the survey. Founded in 1977, the fund is occasionally described as a "vulture fund," as roughly one third of its assets are focused on distressed securities, including debt for bankrupt countries. Regardless, the strategy has proven successful for multiple decades.

Founded in 2001 by David Siegel and John Overdeck, New York's Two Sigma Investments is near the top of the list of hedge funds by AUM, with more than $37 billion in managed assets. The firm was designed to not rely on a single investment strategy, allowing it to be flexible along with shifts in the market.

One of the most popular hedge funds in the world is James H. Simon's Renaissance Technologies. The fund, with $57 billion in AUM, was launched in 1982, but it has revolutionized its strategy along with changes in technology in recent years. Now, Renaissance is known for systematic trading based on computer models and quantitative algorithms. Thanks to these approaches, Renaissance has been able to provide investors with consistently strong returns, even in spite of recent turbulence in the hedge fund space more broadly.

AQR Capital Investments is the second-largest hedge fund in the world, overseeing just under $90 billion in AUM as of the time of HFM's survey. Based in Greenwich, Connecticut, AQR is known for utilizing both traditional and alternative investment strategies.

Ray Dalio's Bridgewater Associates remains the largest hedge fund in the world, with just under $125 billion in AUM as of mid-2018. The Connecticut-based fund employs about 1700 people and focuses on a global macro investing strategy. Bridgewater counts foundations, endowments, and even foreign governments and central banks among its clientele [2].

F. Hedge funds and its future prospects

As of 2017, his Net worth is estimated to be $5.4 billion USD.

"Hedge funds" are made available only to certain sophisticated or accredited investors and cannot be offered or sold to the general public.

In Simple language, Hedge Fund refers to taking money from the rich and wealthy and then investing that money either in stocks, bonds, etc. often with complex portfolio—construction and risk management techniques, so that the chances of the return on investment of money keeps increasing.

Other Hedge Funds include Bridgewater Associates, AQR Capital Management, Man Group, Two Sigma Investments, etc.

The average day of a Hedge Fund Manager starts at 4 a.m. by rolling over and watching some early morning trading on your laptop and ends at 4 p.m. with the Financial Markets closing and you reading trade publications and research reports.

The level of people you are dealing has huge money and you open a new world of huge investment opportunities to them. As for now, we don't have any laws on setting up a Hedge Fund in India. The average salary of hedge fund Manager in the US is $70,000 plus bonuses

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based on their performance and that of top hedge fund manager can make over $1 billion in compensation.

The job profile requires very strong analytical and decision-making skills. You are always dealing with numbers and stats. The whole hedge fund Industry is mostly 70% Intuition based and 30% data-driven. Hedge Fund Managers are synonymous to mutual fund managers but the difference lies in their clients.

Baby boomer is a term referring to a person who was born between 1946 and 1964. The baby boomer generation makes up a substantial portion of the North American population, representing nearly 20% of the American public.

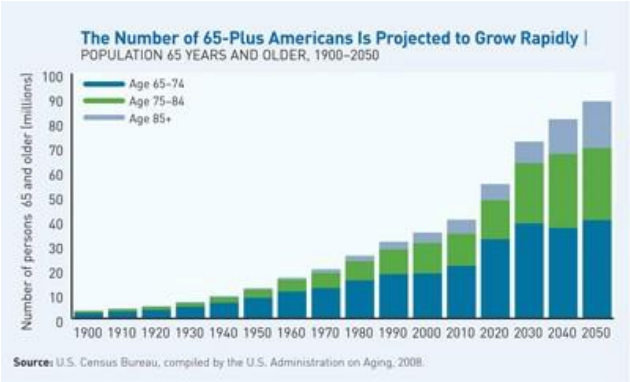

Fig. 1 Growth of Hedge fund

Each day around 10000 baby boomers retire and it has been predicted that about $12 trillion worth of assets will be transferred from greatest generation(born between the 1920s and 30s) to the boomers in the near future. It is expected that these boomers will invest hugely in hedge funds as it is less risky and receive higher returns.

The hedge fund industry holds $2.9 trillion which is greater than India's GDP(2.074 trillion USD (2015)) and considering that most developed countries, including the U.S., have a rapidly aging population, the demand for investment options that generate low-volatility returns over the long term is expected to increase as investors try to match cash flows with long-term liabilities.

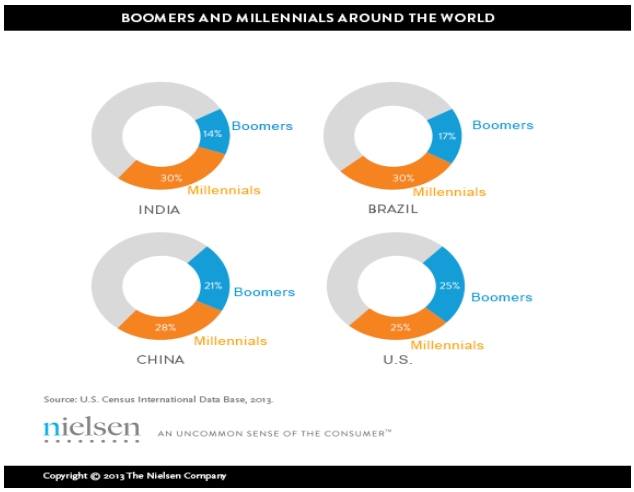

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Fig. 2 Boomers and Millennials around the world

When we talk about the Indian Baby Boomers around 14% of the Indian population lies in Boomers. The Indian Market Scenario keeps fluctuating, so Hedge Fund Investment would be a great industry to invest in. This would not only help the aging population in mitigating risks but also help them gain high returns. Lack of Knowledge and the Socio-Cultural Customs like that of Property and Asset Transfer(Will) restrict the multiplication of money to a great extent. They shouldn't just invest in Hedge Funds but could look at other options like Crowd-Sourcing, VC's and becoming an Angel Investors.

Also the above 3 industries should try to make the on boarding process more relaxed instead of making it stringent. Same thing goes with government policies and regulations.

Though the change of perception has begun but still a lot of work has to be done. With the Gen X coming into the role of Baby Boomers in the next 15–20 years, many new ideas and insights are expected to flourish [3].

III. CONCLUSION

There are many reasons why hedge funds have struggled in recent years, running the gamut from geopolitical tensions around the globe to an overreliance among many funds on particular sectors, including technology, and interest rate hikes by the Fed. Many of the most prominent fund managers have made highly-publicized bad bets which have cost them not only monetarily but in terms of their reputations as savvy fund leaders, too. Notably, the overall size of the hedge fund industry (in terms of assets under management) has not declined significantly during this period, and in fact has continued to grow. There are new hedge funds launching all the time, even as several of the past 10 years have seen record numbers of hedge fund closures. In the midst of growing pressures, some hedge funds are reevaluating aspects of their organization, including the "Two and Twenty" fee structure. According to data from Hedge Fund Research, the last quarter of 2016 saw the average management fee fall to 1.48%, while the average incentive fee fell to 17.4%. In this sense, the average hedge fund is still much more costly than, say, an index or mutual fund, but the fact that the fee structure is changing on average is notable.

REFERENCES

[1] https://www.madrasi.info/hedge-funds-in-India.html
[2] https://www.investopedia.com/terms/h/hedgefund.asp
[3] https://medium.com/@edcbitmesra/hedge-funds-and-its-future-prospects.

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Probabilities of Default of different firms: A comparative study

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ABSTRACT

Probability of default (PD) is the risk of enduring misfortune because of the decrease in the reliability of counterparty in a budgetary exchange. This paper measure the probability of default (PD) of utilizing market-based data which outlines their convenience for monetary reconnaissance. In this paper, the Black-Scholes Model for European Call Option (BSM-CO) is used to measure the PD of various firms and the data has been taken from the term of five years on yearly introduce from 2012 to 2016. This paper shows how d_2 in Black Scholes model help in surveying the PD of the different firm. The central purpose of this paper is whether there are any mean differentiations between the mean contrasts of PD between the organizations using ANOVA.

Key Words: PD, BSM-CO, Merton Model, ANOVA

1. Introduction

Evaluating the PD of a firm is the initial step while surveying the credit exposure and potential misfortunes faced by a firm. For corporate securities, the securities are issued by a firm and there is a plausibility that they will default eventually amid the term of the agreement. In this paper, the BSM-CO is utilized to quantify the probability of default. The Black-Scholes Model (BSM) is the technique for displaying derivatives costs that have been first presented in 1973. The recipe of BSM demonstrates to us proper methodologies to discover the cost of an option contract (call and put option). It tends to be dictated by utilizing basic equation anyway in this examination just the European call option has been utilized [1-3]. In 1974, Merton has shown the model, specifically the Merton Model. It is utilized to gauge the default for the organizations. It is the first basic model since it gives a connection between the default risk and the advantage (capital) structure of the firm [4]. Since the firm will default just when the obligation of the firm is over the estimation of the firm, in this condition the proprietor will put the firm to the obligation holder. Merton has contemplated the firm's value E as a call option on its assets. The inceptions of popular credit risk auxiliary models have developed generally from the theoretical domain. Merton's works turned out to be theoretically broadened and for all intents and purposes actualized by the KMV Corporation. Through this paper, it is expected that [4-5]:

- The underlying asset St supplanted by the value of the firm V.
- The strike price K in a call option supplanted by the Debt D
- The risk free rate of interest is supplanted by the expected growth of the firm.

In such manner, the significance of d_2 has been clarified. It is the interior part of the Black Scholes equation. In any case, one of the intriguing and helpful methodologies N(-d_2) characterizes the likelihood that an option will be practiced in a risk-neutral way instead of real-world probability. It is realized that d_2 is the thought behind the Merton Model. This paper utilizes d_2 of BSM-CO to appraise the distance to default and the PD of various firms. ANOVA has been utilized to demonstrate whether there are contrasts between the mean changes of PD between the organizations. In the year 2017, Amir and

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Anuradha clarify the components influencing the PD [6]. This paper depends on the structural model and the basic model depends on the Black and Scholes Model (1973) and Merton Model (1974) [1-2].

See other references (7-9)

1.1 Methodology

In order to estimate the distance to default and the PD of Jammu and Kashmir Bank, Bank of Baroda, Indian Overseas Bank and Canara bank, we are using the BSM-CO, the internal part of the BSM-CO (d_2). To measure the PD we used the annual reports of above-mentioned firms from the year 2012-2016. Finally, we applied the ANOVA to test whether there is any difference in mean variances of the PD between the firms.

2. Merton Model

2.1 Black Scholes Formula

The Black Scholes Model (BSM) is used to measure the pricing of the derivatives. It was 1st introduced in year 1973. The Robert Merton and Myron Scholes awarded the Nobel Prize for economics in the year 1997 because of the derivative pricing model.

Essentially, BSM formula shows us how to find the price of an option contract (call and put option) can be determined by using simple formula but in this paper only European call option is using. The formula for European call option is

$$C(S,T) = SN(d_1) - Ke^{-rT}N(d_2) - - - - - (1)$$

Where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} - - - - - (2)$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} - - - - - (3)$$

$\mathbf{S}$ Is the present price of the stock, $\mathbf{K}$ is the strike price, $\mathbf{r}$ is the free risk rate interest, $\boldsymbol{\sigma}$ is the volatility of the stock and $\mathbf{N}$ is the CDF function for a standard normal distribution.

2.2 Merton Model- A Simple Concept

The Merton model was the primary structural model which estimates the PD for firms. It accepts that the firm will issue the two debts D and additionally equity E too. Give us a chance to accept that the value of the firm is V at time t. It will shift over the time because of activities by the firm, which does not pay any sort of dividend on the equity or coupon.
The Zero coupon bonds are a piece of an association's debt with ensured reimbursement of amount D at time T. The rest of the firm V at time T will be issued to the investors and the firm will be twisted up. On account of the breeze up the firm, the investors rank is beneath the debt holders.
In the event that the firm will produce the great reserve in such a path thus, to the point that can pay the debt, at that point the investors will get the result of:

$$V - D - - - (3)$$

The firm will default at time T when $V < D$

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In the above case, the bondholders will receive V instead of D and the shareholders will receive nothing.
Consider the both the conditions we get:
The shareholders will receive the payoff of:

$$max(V - D, 0) - - - (5)$$

This is same as the payoff of the European call option, with V as an underlying asset price and D as a Strike price.
Following assumptions would undermine the model efficiency:

1. The firm can default only at time T and not before.
2. Assets of the firm's follow lognormal distribution.
3. On the basis of accounting data, the PD for private firms can be estimated. The model does distinguish between the types of default according to their seniority, convertibility and collaterals.

To estimate the distance to default we need the Back Scholes Formula for a European call option is given by:

$$\mathrm{d}_2 = \frac{\ln\left(\frac{\mathrm{S_t}}{\mathrm{K}}\right) + \left(\mathrm{r} - \frac{\sigma^2}{2}\right)\mathrm{T}}{\sigma\sqrt{\mathrm{T}}} - - - -(6)$$

Replace:

- The risk free rate of interest r by the expected continuously compounded return on value of the firm μ_V.
- The value of the underlying asset $\mathrm{S_t}$ at time t by the value of the firm at time t is V.
- The strike price K by the face value of the debt D.
- The volatility σ by the volatility of the firms value σ_V

The distance to default is given by:

$$\mathrm{d}_2 = \frac{\ln\left(\frac{\mathrm{V}}{\mathrm{D}}\right) + \left(\mu_V - \frac{{\sigma_V}^2}{2}\right)\mathrm{T}}{\sigma_V\sqrt{\mathrm{T}}} - - - (7)$$

Where μ_V is expected rate of return of the firm's asset and D, is the face value of a debt and expected growth of assets is equal to $(\mu_V - \frac{{\sigma_V}^2}{2})$.

The numerator of the equation (7) is really the distance to default; it demonstrates that the distance between expected assets and D as appeared in figure 1. It tends to be figured as a whole of initial distance and growth of that distance within the period T. The equation (7) is the distance to default in wording as a multiplier of Standard Deviation. The distance to default is characterized as how much a firm is far off from the default point. Subsequent to evaluating the distance to default, we can create and gauge the probability of default.

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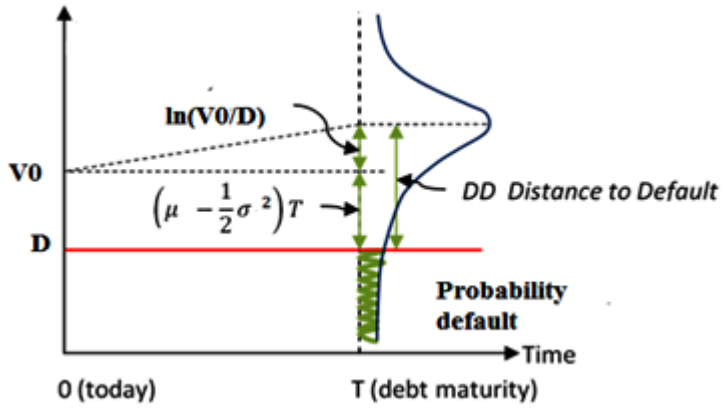

Fig 1: PD

The PD under the risk neutral measure as per the Black Scholes Merton model is given by:

$$\text{Probability default} = \mathrm{N}(-\mathrm{d}_2) = \mathrm{N}\left(-\frac{\ln\left(\frac{\mathrm{V}}{\mathrm{D}}\right) + \left(\mu_V - \frac{{\sigma_V}^2}{2}\right)\mathrm{T}}{\sigma_V\sqrt{\mathrm{T}}}\right) --- (8)$$

Or

$$\text{Probability of default} = 1 - \mathrm{N}(\mathrm{d}_2)$$

The equation (8) is the probability of default that is it is distance between the value of the firm and the value of the debt (V/D) adjust for the expected growth related to asset volatility $\left(\mu_V - \frac{{\sigma_V}^2}{2}\right)$ related to asset volatility.

3. Result and Analysis

We have taken the secondary data of all the firms for 5 years starting from March 2012 to March 2016. The BSM-CO has been used to estimate the PD of all firms using BSM models:

To estimate the PD we need some parameters like:

1. Firm value of Assets, V (Total equity + Debt).
2. Value of a debt, D.
3. Volatility of an Asset, σ
4. Rate of return, μ_V.
5. Time period, T

3.1 Result

Table 1: Probability to Default: Use equation (8) to estimate the PD

Year	Jammu and Kashmir Bank	Indian overseas Bank	Bank of Baroda	Canara Bank
2012	0.268888	0.20148	0.4555564	0.423839
2013	0.2687425	0.2016335	0.4553912	0.4237237
2014	0.268681897	0.202425	0.4552581	0.435759
2015	0.268711978	0.2022066	0.4552265	0.4234976
2016	0.268676859	0.2046978	0.4553076	0.4236178

Note:

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- Taking Total Equity and Value of debt from historical data of JK Bank.
- Firm value of Assets = Total Equity + Debt.
- For simplicity taking volatility as 20%.

The PD of Jammu and Kashmir bank is 26.88% in year 2012 as appeared in table 1, which suggests that the likelihood a JK Bank will default in year 2012 is 26.88% that is 26.88% obligations that a Jammu and Kashmir has not paid obligation.

Figure 2 demonstrates the PD of the considerable number of firms. Each firm is having the distinctive PD at various day and age. The inquiry is: which organization is great so a speculator will give advance to the firm? The appropriate response is straightforward - firm which is having the less PD. E.g. a financial specialist ABC needs to give the advance to a one firm in 2013. The speculator ABC is having four firms where he can put however he will put just in one firm. As appeared in table 1, the PD of Jammu and Kashmir Bank, Indian overseas Bank, Bank of Baroda and Canara Bank is 26.88%, 20.148%, 45.55% and 42.38% individually in year 2012. Based on past data with respect to the four firms the financial specialist will give advance to Indian overseas Bank since it is having less PD as appeared in table 1.

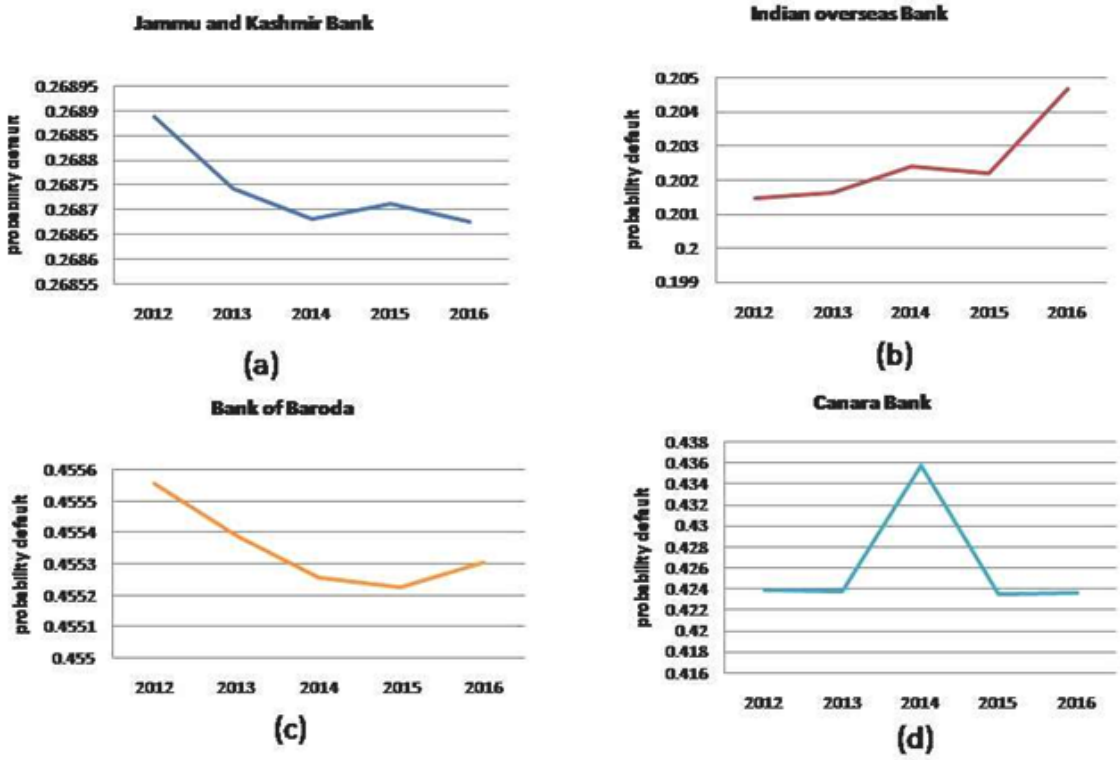

Fig 2: PD of all firms

Take a gander at the figure 2 it is demonstrated that the PD in all organizations increments or abatements. The Indian overseas bank is enhancing according to our outcome. The bank of Baroda diminishes after that it increments. The Jammu and Kashmir diminishes at all that are not a decent sign and the Canara Bank increments first after that it begin diminishing moreover. Presently we will complete a speculation test on all the four firms to watch that there is any contrast between the methods.

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3.2 One-Way ANOVA for 4 Independent Samples

In order to determine whether there is any mean difference between the PD of firms given in table 1. We compare the p-value of all the mean values of parameters with the significance level ($\alpha = 5\%$). The α indicated that the risk of concluding that the parameters are significant different.

In order to verify, we have two cases:

1. If $\mathrm{p-value} > \alpha\,(0.05)$, there is no mean difference (Fail to reject the null hypothesis or accept the null hypothesis).

2. If $-\mathrm{value} \leq \alpha\,(0.05)$, there is mean difference (reject the null hypothesis).

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Factor	4	JK BANK, IOB BANK, BOB, CANARA BANK

The ANOVA is used to explain that the mean response between the firms varies or not. If there are no differences between the mean of all the firms then the F value is around 1. If the F value is large, then there are mean difference between the firms.

Table 2: Basic statistics

Data Summary	Jammu and Kashmir Bank	Indian overseas Bank	Bank of Baroda	Canara Bank	**Total**
N	5	5	5	5	20
Sum	1.3437	1.0124	2.2767	2.1304	6.7633
Mean	0.2687	0.2025	0.4553	0.4261	0.3382
sumsq	0.3611	0.205	1.0367	0.9079	2.5107
SS	0	0	0	0.0001	0.2236
variance	0	0	0	0	0.0118
st. dev.	0.0001	0.0013	0.0001	0.0054	0.1085

Variances and standard deviations are calculated with denominator = n-1.

Figure 3: The standard deviation of all firms

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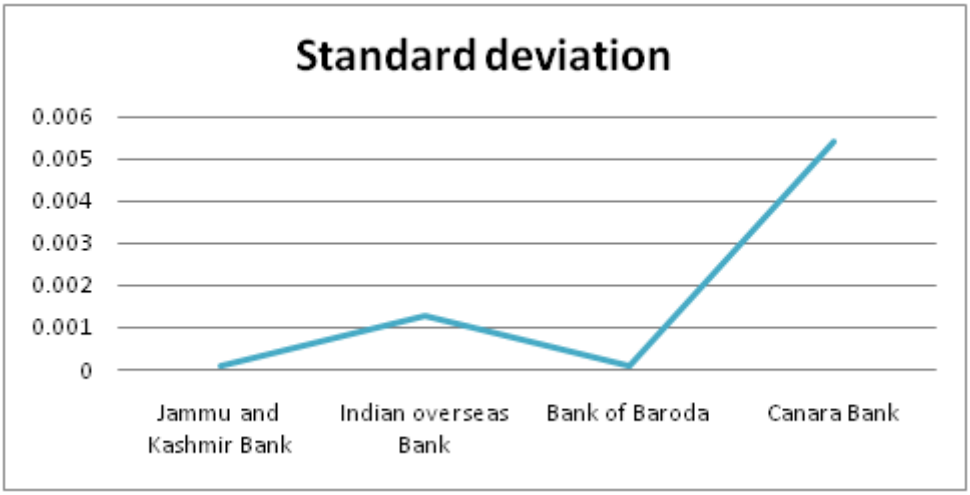

Fig 3: Standard deviation of all firms

The standard deviation of all firms in several years gives the measurement of variance of probability of default.

3.3 ANOVA Summary

Table 3: ANOVA

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	3	0.223450	0.074483	9626.24	0.000
Error	16	0.000124	0.000008		
Total	19	0.223574			

A hypothesis is a test for whether there is any such difference between the treatments is based on the F ratio. It will ask that whether the F ratio for the treatments is unusually high by comparing the F ratio to a kind of standard distribution called an F distribution. The p value for the treatments is the probability of getting such a high F ratio if all the treatments were really identical.

The p value is less that 0.001 which indicates that the there are difference between the treatments among the four firms. In other words we have strong evidence to reject the null hypothesis that mean of PD score varies.

Conclusion

We cannot decide the decision on the basis of overall standard deviation, better and best method is to measure the PD for an investor to invest the money in any firm because the PD will give us the rate of default for a particular firm and the standard deviation of PD for several years gives the measurement of variance of the PD and based on this it is not helpful to calculate or decide which firm is best to invest as per the overall standard deviation of the probability of default. As per ANOVA, we reject the null hypothesis which indicates that there are differences between the mean among the four firms.

Declarations - Availability of data and material

"The data has been collected from annual reports of firms from the year 2012-2016 and the data may not be correct but the procedure that we mentioned in this paper is defined well".

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References

[1] Black, F., & Scholes, M.: The pricing of options and corporate liabilities. Journal of political economy. *81*(3), 637-654 (1973).
[2] Merton, R. C.: On the pricing of corporate debt: The risk structure of interest rates. The Journal of finance. *29*(2), 449-470 (1974).

[3] Hull, J. C.: Options, Futures and Other Derivatives. 7th Ed, First Indian Reprint, Pearson Education (2009).

[4] Saunders, A., and Allen, L.: Credit risk measurement: New approaches to value at risk and other paradigms. New York: John Wiley & Sons, Inc. (2002).

[5] Bohn, J., & Arora, N. I Korablev: Power and Level Validation of the EDF Credit Measure in the US Market. Moody's KMV Working Paper (2005).

[6] Dar, A. A., & Anuradha, N.: Probability default in black scholes formula: A qualitative study. Journal of Business and Economic Development. *2*(2), 99-106 (2017).

[7] Varma, J. R., & Raghunathan, V.: Modelling credit risk in Indian bond markets. Journal of Applied Finance. 6(3), 53-67 (2000).

[8] Bharath, S. T., & Shumway, T.: Forecasting default with the Merton distance to default model. The Review of Financial Studies. *21*(3), 1339-1369 (2008).

[9] Bohn, J. R.: A survey of contingent-claims approaches to risky debt valuation. The Journal of Risk Finance. *1*(3), 53-70 (2000).

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A comparative study on multi criteria decision making methods.

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Abstract

Multi criteria decision making methods are widely used in many fields of research including engineering, production, medicine etc. There are several methods proposed so far to solve these problems. In this paper we have made a comparative analysis under intuitionistic fuzzy environment between the existing methods namely: TOPSIS, VIKOR, and TODIM which are pre dominantly used in many fields. The objective of this paper is not to determine which method is the best one but to find out the different ranking order of alternatives, and the reason behind them.

1. Introduction:

In recent years, multi criteria decision making has been extensively used in many fields such as engineering, production, supplier selection etc. Multi criteria decision making methods in the process of identifying the best alternative from all the feasible alternative. Wide range of methods can be applied for solving MCDM Problems. There is no appropriate solution for this problem. Often, different methods can give us different ranking of alternatives the inconsistency in results occurs of the following reason which are listed they are

1) Algorithm for selection of preferred alternative differ.
2) Techniques use criteria weights differently in their calculations.
3) Algorithm attempt to scale the objectives, thus affecting the weights already chosen.
4) Algorithm introduce additional parameters that affect selection of preferred alternative.

Of the many MCDM methods, five methods are commonly used. Simple additive method (SAW), Weighted product method (WPM), Analytical hierarchy process (AHP), Techniques for order preference by similarity to ideal solution (TOPSIS), a compromise ranking method (VIKOR), TODIM is a recently developed MCDM method which is used widely nowadays other methods such as ANP, PROMETHEE are also used.

In 1965 Zadeh introduced the concepts of fuzzy sets. Atttansov and gargov presented the concept of Intuitionistic fizzy sets. In 1981 Hwang and yoon suggested a method of technique for order preference by similarity to ideal solution (TOPSIS) method to identify the best alternative among the available alternatives by finding the shortest distance from positive ideal solution and fastest distance from the negative ideal solution. The preference ranking organization methods for enrichment evaluation (PROMOTHEE), proposed by Beans & vuicke (1985) which is based on the comparison of each alternative with each other considering the deviations that alternatives show according to each criterion. VIKOR (Vlse kriterijumska optimizalija I kompromisno Resenje) was proposed by opricovic in 1998 which rank the alternatives according to the value of three scalar quantities (S_i, R_i & Q_i) that have to be calculated for each option. The ELECTRE method (ELimination Et Choix Tradrisant la REalite) was proposed by Roy which ranks the relation between the alternatives taken two at a time. T It was proposed by Gomes and Lima with discrete data. Later it was extended by Fan et al to the fuzzy environment. Krohling et al presented the extension of TODIM to the interval valued intuitionistic fuzzy environment. Due to some complexity and the uncertainty, it is impossible to consider all the aspects of the problem. Therefore, the DM's provide us their knowledge and preference. It may be a bias one. In order to overcome this, a specific and considerable weight must be assign to each DM's. Also, however in fuzzy environment, time, pressure, lack of data and limited information DM's cannot provide a weight for each criteria. Quande Qin proposed the TODIM based multi criteria decision making with triangular intuitionistic fuzzy numbers in which they have

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developed the model for assigning the weight vector of DM's for each alternative with respect to each criteria. This paper is organized as follows: section 2, basic concepts of fuzzy set, intuitionistic trapezoidal fuzzy numbers and the operations on trapezoidal numbers were discussed. In section 3, TODIM, TOPSIS and VIKOR methods were discussed. Finally, a numerical example is illustrated to prove the effectiveness of the method.

2. Preliminaries

Definition 2.1: Let $\tilde{a}$ be an intuitionistic trapezoidal fuzzy number, its membership is given by

$$\mu_{\tilde{a}}(x)=\begin{cases}\frac{x-a}{b-a}\,\mu_{\tilde{a}}, & a\le x<b\\ \mu_{\tilde{a}}, & b\le x\le c\\ \frac{d-x}{d-c}\,\mu_{\tilde{a}}, & c<x\le d\\ 0, & Others\end{cases} \tag{1}$$

and its non-membership is given by

$$\gamma_{\tilde{a}}(x)=\begin{cases}\frac{b-x+(x-a_1)\gamma_{\tilde{a}}}{b-a_1}\,\mu_{\tilde{a}}, & a_1\le x<b\\ \gamma_{\tilde{a}}, & b\le x\le c\\ \frac{x-c+\gamma_{\tilde{a}}(d_1-x)}{d-c}\,\mu_{\tilde{a}}, & c<x\le d_1\\ 0, & Others\end{cases} \tag{2}$$

where $0\le \mu_{\tilde{a}} \le 1, 0 \le \gamma_{\tilde{a}} \le 1$ and $\mu_{\tilde{a}} + \gamma_{\tilde{a}} \le 1, a, b, c, d \in R$.

Then $\tilde{a} =< ([a,b,c,d],\mu_{\tilde{a}})[a_1,b,c,d_1],\gamma_{\tilde{a}})>$ is called a intuitionistic trapezoidal fuzzy number.

For convenience let $\tilde{a} = ([a,b,c,d];\mu_{\tilde{a}},\gamma_{\tilde{a}})$

Definition 2.2: Let $\tilde{a}_1 = ([a_1,b_1,c_1,d_1];\mu_{\tilde{a}_1},\gamma_{\tilde{a}_1})$

$\tilde{a}_2 = ([a_2,b_2,c_2,d_2];\mu_{\tilde{a}_2},\gamma_{\tilde{a}_2})$ be two intuitionistic trapezoidal fuzzy number and λ>0, then

1. $\tilde{a}_1 + \tilde{a}_2 = ([a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2];\ \mu_{\tilde{a}_1}+\mu_{\tilde{a}_2}-\mu_{\tilde{a}_1}.\mu_{\tilde{a}_2}, \gamma_{\tilde{a}_1}.\gamma_{\tilde{a}_2})$
2. $\tilde{a}_1.\tilde{a}_2 = \left([a_1.a_2, b_1.b_2, c_1.c_2, d_1.d_2];\ \mu_{\tilde{a}_1}.\mu_{\tilde{a}_2}, \gamma_{\tilde{a}_1}+\gamma_{\tilde{a}_2}-\gamma_{\tilde{a}_1}.\gamma_{\tilde{a}_2}\right)$
3. $\lambda\tilde{a}_1 = ([\lambda\tilde{a}_1, \lambda\tilde{b}_1, \lambda\tilde{c}_1, \lambda\tilde{d}_1];\ 1-(1-\mu_{\tilde{a}_1})^{\lambda}, \gamma_{\tilde{a}_1}{}^{\lambda})$
4. $\tilde{a}_1{}^{\lambda} = ([\tilde{a}_1{}^{\lambda}, \tilde{b}_1{}^{\lambda}, \tilde{c}_1{}^{\lambda}, \tilde{d}_1{}^{\lambda}], \mu_{\tilde{a}_1}{}^{\lambda}, 1-(1-\gamma_{\tilde{a}_1})^{\lambda})$

Definition 2.3: Let $\tilde{a}_1 = ([a_1,b_1,c_1,d_1];\mu_{\tilde{a}_1},\gamma_{\tilde{a}_1})$ and $\tilde{a}_2 = ([a_2,b_2,c_2,d_2];\mu_{\tilde{a}_2},\gamma_{\tilde{a}_2})$ be two intuitionistic trapezoidal fuzzy number, then the normalized Hamming distance between $\tilde{a}_1$ and $\tilde{a}_2$ is defined as follows:

$$d(\tilde{a}_1,\tilde{a}_2) = \frac{1}{8}\left(\left|(1+\mu_{\tilde{a}_1}-\gamma_{\tilde{a}_1})a_1\right| - \left|(1+\mu_{\tilde{a}_2}-\gamma_{\tilde{a}_2})a_2\right| + \left|(1+\mu_{\tilde{a}_1}-\gamma_{\tilde{a}_1})b_1\right| - \left|(1+\mu_{\tilde{a}_2}-\gamma_{\tilde{a}_2})b_2\right| + \left|(1+\mu_{\tilde{a}_1}-\gamma_{\tilde{a}_1})c_1\right| - \left|(1+\mu_{\tilde{a}_2}-\gamma_{\tilde{a}_2})c_2\right| + \left|(1+\mu_{\tilde{a}_1}-\gamma_{\tilde{a}_1})d_1\right| - \left|(1+\mu_{\tilde{a}_2}-\gamma_{\tilde{a}_2})d_2\right|\right) \tag{3}$$

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Definition 2.4 Let $\tilde{a}_j = ([a_j, b_j, c_j, d_j]; \mu_{\tilde{a}_j}, \gamma_{\tilde{a}_j})$ be ITFN set and $w = \{w_1, w_2, \ldots\ldots w_n\}^T$ be the weight vector of $\tilde{a}_j$ satisfying $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, the weighted arithmetic interaction averaging operator of ITFN (ITFN-WAIA) can be defined as follows:

ITFN-WAIA$_w$($\tilde{a}_1, \tilde{a}_2, \ldots\ldots \tilde{a}_n$) = $\sum_{j=1}^{n} w_j \tilde{a}_j$

When $w = (\frac{1}{n}, \frac{1}{n}, \ldots\ldots \frac{1}{n})^T$,

ITFN-WAIA$_w$($\tilde{a}_1, \tilde{a}_2, \ldots\ldots \tilde{a}_n$) = $\frac{1}{n} \sum_{j=1}^{n} \tilde{a}_j$
(4)

Definition 2.5 Let $\tilde{a}_j = ([a_j, b_j, c_j, d_j]; \mu_{\tilde{a}_j}, \gamma_{\tilde{a}_j})$ (j=1,2,…..n) be a collection of ITFNs, and let ITFWAA: $Q^n \to Q$, if

ITFWAAω($\tilde{a}_1, \tilde{a}_2, \ldots\ldots \tilde{a}_n$)=
$\sum_{j=1}^{n} w_j \tilde{a}_j = \left([\sum_{j=1}^{n} w_j a_j, \sum_{j=1}^{n} w_j b_j, \sum_{j=1}^{n} w_j c_j, \sum_{j=1}^{n} w_j d_j]; \; 1 - \prod_{j=1}^{n} \left(1 - \mu_{\tilde{a}_j}\right)^{\omega_j}, \prod_{j=1}^{n} (\gamma_{\tilde{a}_j})^{\omega_j} \right)$ (5)
where $\omega = (\omega_1, \omega_2, \ldots\ldots \omega_n)^T$ be the weight vector of $\tilde{a}_j$ (j=1,2,…n) and $w_j > 0, \sum_{j=1}^{n} w_j = 1$, then ITFWAA is called the intuitionistic trapezoidal fuzzy arithmetic averaging operator (ITFWAA).

2.6: Determination of DM weights and criterion weights:

In most of the multi criteria decision making problems, the weights of decision makers and criteria are not clearly defined. Some cases they assigned equally. But in practical, DM's may not be knowledgeable about all the criteria. In order to overcome this practical inconvenience, Q.Qin et al [6] in 2017 proposed the method for the determination of DM weights and criterion weights.

The method is as follows:

Suppose $\tilde{a}_{ij}^k$ (i=1,2,…..m; j=1,2,….n) is the evaluation values for alternative A_i with respect to criterion C_j provided by p DM's as $\tilde{a}'_{ij} == ([a_j, b_j, c_j, d_j]; \mu_{\tilde{a}_j}, \gamma_{\tilde{a}_j})$ It can be calculated by using the ITFN-WAIA operator in definition 2.4.

The degree of similarity between $\tilde{a}_{ij}^k (i = 1,2, \ldots\ldots m; j = 1,2, \ldots\ldots n)$ and the mean value

$\tilde{a}'_{ij} = ([a_j, b_j, c_j, d_j]; \mu_{\tilde{a}_j}, \gamma_{\tilde{a}_j})$ is defined as s($\tilde{a}_{ij}^k, \tilde{a}'_{ij}$);

$$s(\tilde{a}_{ij}^k, \tilde{a}'_{ij}) = 1 - \frac{d\left(\tilde{a}_{ij}^k, \tilde{a}'_{ij}\right)}{\sum_{k=1}^{p} d\left(\tilde{a}_{ij}^k, \tilde{a}'_{ij}\right)},$$

(6)

$$(k = 1,2, \ldots\ldots p; i = 1,2, \ldots\ldots m; j = 1,2, \ldots\ldots n)$$

Where $d\left(\tilde{a}_{ij}^k, \tilde{a}'_{ij}\right)$ represents the distance between $\tilde{a}_{ij}^k$ and $\tilde{a}'_{ij}$.

Then, the weight for DM D_k for the weight for alternative A_i with respect to the criterion C_j is defined as

$$\lambda_{ij}^k = \frac{s(\tilde{a}_{ij}^k, \tilde{a}'_{ij})}{\sum_{k=1}^{p} s(\tilde{a}_{ij}^k, \tilde{a}'_{ij})},$$

(7)

$$(k = 1,2, \ldots\ldots p; i = 1,2, \ldots\ldots m; j = 1,2, \ldots\ldots n)$$

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Aggregating the individual decision matrix $A^k = [\tilde{a}_{ij}^k]_{mxn}$ in to group decision matrix $G = [\tilde{g}_{ij}]_{mxn}$ by using definition 2.5 To determine the criterion weights for the collective

decision matrix $G = [\tilde{g}_{ij}]_{mxn}$, we denote the mean of evaluation under criterion C_j as $\tilde{g}'_{ij} = ([a_j, b_j, c_j, d_j]; \mu_{\tilde{a}_j}, \gamma_{\tilde{a}_j})$ which can be calculated by utilizing the ITFN-WAIA

Then, we can obtain the weight for the criterion C_j:

$$w_j = \frac{\sum_{i=1}^{m} d\left(\tilde{g}_{ij}, \tilde{g}'_{ij}\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m} d\left(\tilde{g}_{ij}, \tilde{g}'_{ij}\right)} \tag{8}$$

Where $d\left(\tilde{g}_{ij}, \tilde{g}'_{ij}\right)$ denotes the distance between collective evaluation value $\tilde{g}_{ij}$ and the mean value $\tilde{g}'_{ij}$.

3.1 TODIM

The steps involved in TODIM method for solving multi criteria group decision making with intuitionistic trapezoidal fuzzy numbers are as follows:

Step 1: Define the decision matrix $R^k = [\tilde{r}_{ij}^k]_{mxn}$ where r_{ij} are crisp numbers $i \in m, j \in n$.

Step 2: Obtain the weight vector $\lambda_{ij}^k = \{\lambda_{ij}^1, \lambda_{ij}^2, \ldots\ldots\lambda_{ij}^p\}$ of DM D_k for alternative A_i with respect to the criterion C_j using equations (6)-(7).

Step 3: Aggregate the individual decision matrix $R_k = \{\tilde{r}_{ij}^k\}_{mxn}$ in to group decision matrix $G = [\tilde{g}_{ij}]_{mxn}$ using equation (5)

Step 4: Determine the criterion weight $w=(w_1, w_2, \ldots\ldots w_n)$ using (8).

Step 5: Calculate the relative weight w_{jr} of criterion C_j to the reference criterion C_r which is expressed as

$$w_{jr} = \frac{w_j}{w_r}$$

Where $w_r = \max\{\frac{w_j}{j} = 1,2, \ldots . n\}$.
(9)

Step 6: On the basis of the classical TODIM method, the dominance of each alternative A_i over each alternative A_k under the criterion C_j can be obtained by

$$\varphi_j(A_i, A_k) = \begin{cases} \sqrt{\frac{w_{jr}}{\sum_{j=1}^{n} w_{jr}} . d\left(\tilde{g}_{ij}, \tilde{g}'_{ij}\right)} & if\ \left(\tilde{g}_{ij} > \tilde{g}'_{ij}\right) \\ & if\ \left(\tilde{g}_{ij} = \tilde{g}'_{ij}\right) \\ \frac{-1}{\theta}\sqrt{\frac{w_{jr}}{\sum_{j=1}^{n} w_{jr}} . d\left(\tilde{g}_{ij}, \tilde{g}'_{ij}\right)} & if\ \left(\tilde{g}_{ij} < \tilde{g}'_{ij}\right) \end{cases} \tag{10}$$

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Step 7: The dominance degree matrix with respect to the criterion C_j can be constructed as

$$\varphi_j = [\varphi_{ik}^j]_{mxn} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} A_1 & A_2 & \dots & \dots & A_m \end{array} \\ \begin{pmatrix} \varphi_{11}^j & \varphi_{12}^j & \cdots & \cdots & \varphi_{1m}^j \\ \varphi_{21}^j & \varphi_{22}^j & \vdots & \cdots & \varphi_{2m}^j \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \varphi_{m1}^j & \varphi_{m2}^j & & & \varphi_{mn}^j \end{pmatrix} \end{array}$$

(11)

Where $\varphi_{ii}^j = 0$, k=1,2,…..m; j=1,2,…..n.

Step 8: The global dominance degree of each alternative A_i over each alternative A_k by

$$\delta(A_i, A_k) = \sum_{j=1}^{n} \varphi_j(A_i, A_k)$$

(12)

Step 9: By normalizing the global dominance degree matrix, we can obtain the global value for the alternative i according to the following expression:

$$\xi_j = \frac{\sum_{k=1}^{m} \delta(A_i,A_k) - \min_{i\epsilon M}\{\sum_{k=1}^{m} \delta(A_i,A_k)\}}{\max_{i\epsilon M} \sum_{k=1}^{m} \delta(A_i,A_k) - \min_{i\epsilon M}\{\sum_{k=1}^{m} \delta(A_i,A_k)\}}$$

(13)

Step 10: Rank the alternatives and select the best one. The higher the value ξ_j, the better the alternative A_i.

3.2 TOPSIS:

The steps involved in TOPSIS method for solving multi criteria group decision making with intuitionistic trapezoidal fuzzy numbers are as follows:

The first 4 steps in TOPSIS are same as in TODIM which is mentioned in section 3.1

Step 5: The positive ideal solution (PIS) G^+ and the negative ideal solution (NIS) G^- which can be defined as follows:

$G^+ = \{\tilde{g}_1^+, \tilde{g}_2^+, \dots \dots . \tilde{g}_m^+\}$, $G^- = \{\tilde{g}_1^-, \tilde{g}_2^-, \dots \dots . \tilde{g}_m^-\}$

(14)

Where

$$\tilde{g}_1^+ = \left(\left(\max_{i\epsilon M} a_i, \max_{i\epsilon M} b_i, \max_{i\epsilon M} c_i, \max_{i\epsilon M} d_i\right); 1,0\right)$$

$$\tilde{g}_1^- = \left(\left(\min_{i\epsilon M} a_i, \min_{i\epsilon M} b_i, \min_{i\epsilon M} c_i, \min_{i\epsilon M} d_i\right); 0,1\right)$$

The PIS and NIS can be obtained by eq( ) and eq() which are listed in table

Step 6: The separation measures d_i^+ and d_i^- of each alternative from the PIS G^+ and NIS G^- respectively from

$d_i^+ = \sum_{j=1}^{n} w_j \, d_{ij}^+$, i=1,2,……m; j=1,2,…….n

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$d_i^- = \sum_{j=1}^{n} w_j \, d_{ij}^-$, i=1,2,……m; j=1,2,…….n
(15)

Step 7: The relative closeness of an alternative A_i with respect to the PIS G^+ can be calculated as follows:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-}$$
(16)

The larger the closeness coefficient C_i, the better the alternative A_i is closer to the PIS. Thus alternative can be ranked according to the closeness coefficients which are listed in table ().

3.3 VIKOR

The steps involved in VIKOR method for solving multi criteria group decision making with intuitionistic trapezoidal fuzzy numbers are as follows:

The first 4 steps in VIKOR are same as in TODIM which is mentioned in section

Step 5: The best value G^+ and the worst value G^- which can be defined as follows:

$G^+ = \{\tilde{g}_1^+, \tilde{g}_2^+, \ldots\ldots \tilde{g}_m^+\}$, $G^- = \{\tilde{g}_1^-, \tilde{g}_2^-, \ldots\ldots \tilde{g}_m^-\}$
(17)

Where

$$\tilde{g}_j^+ = \left(\left(\max_{i\in M} a_i, \max_{i\in M} b_i, \max_{i\in M} c_i, \max_{i\in M} d_i\right); 1,0\right)$$

$$\tilde{g}_j^- = \left(\left(\min_{i\in M} a_i, \min_{i\in M} b_i, \min_{i\in M} c_i, \min_{i\in M} d_i\right); 0,1\right)$$

Step 6: Compute the values S_i, R_i and Q_i. Three key values of IF-VIKOR method, the group utility value S_i, the individual regret value R_i, and the compromise value Q_i, are computed in light of the intuitive distance measure for each alternative:

$$S_i = \sum_{j=1}^{m} w_j \left(\frac{d\left(\tilde{g}_j^+, g_{ij}\right)}{d\left(\tilde{g}_j^+, \tilde{g}_j^-\right)}\right) \qquad R_i = \max_j w_j \left(\frac{d\left(\tilde{g}_j^+, g_{ij}\right)}{d\left(\tilde{g}_j^+, \tilde{g}_j^-\right)}\right) \qquad Q_i = \gamma\left(\frac{S_i - S^*}{S^- - S^+}\right) + (1-\gamma)\left(\frac{R_i - R^*}{R^- - R^+}\right)$$
(18)

Where $S^- = \max_i S_i$, $S^* = \min_i S_i$ and $R^- = \max_i R_i$, $R^* = \min_i R_i$ where γ is the decision coefficient. Rank the alternatives and derive the compromise solution The smaller the value Q_i, the better the alternative A_i.

4. Example:

Let us suppose there is a risk investment company which wants to invest a sum of money in the best option. There is a panel with five possible alternatives to invest the money. The risk investment company must take a decision according to the following 4 attributes or criteria:

- Risk analysis
- Growth analysis
- Socio-political impact analysis
- Environment impact analysis

The five possible alternatives A_i (i=1,2,3,4,5) to be evaluated using the intuitionistic trapezoidal fuzzy numbers by three decision makers.

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4.1 TODIM

Step 1: The decision matrices provided by three DM's are listed in tables 1-3.

Step 2: The DM weight vector of DM D_k for alternative A_i with respect to the criterion C_j can be calculated using eqs. (6) and (7).

Table 1: The ITFN decision matrix by DM D_1

	C_1	C_2	C_3	C_4
A_1	((0.5,0.6,0.7,0.8);0.5,0.4)	((0.1,0.2,0.3,0.4);0.6,0.3)	;0.6,0.8,0.9);0.3,0.6)	((0.4,0.5,0.6,0.7);0.2,0.7)
A_2	((0.6,0.7,0.8,0.9);0.7,0.3)	((0.5,0.6,0.7,0.8);0.7,0.2)	((0.4,0.5,0.7,0.8);0.7,0.2)	((0.5,0.6,0.7,0.9);0.4,0.5)
A_3	((0.1,0.2,0.4,0.5);0.6,0.4)	((0.2,0.3,0.5,0.6);0.5,0.4)	((0.5,0.6,0.7,0.8);0.5,0.3)	((0.3,0.5,0.7,0.9);0.2,0.3)
A_4	((0.3,0.4,0.5,0.6);0.8,0.1)	((0.1,0.3,0.4,0.5);0.6,0.3)	((0.1,0.3,0.5,0.7);0.3,0.4)	((0.6,0.7,0.8,0.9);0.2,0.6)
A_5	((0.2,0.3,0.4,0.5);0.6,0.2)	((0.3,0.4,0.5,0.6);0.4,0.3)	((0.2,0.3,0.4,0.5);0.7,0.1)	((0.5,0.6,0.7,0.8);0.1,0.3)

Table 2: The ITFN decision matrix by DM D_2

	C_1	C_2	C_3	C_4
A_1	((0.4,0.5,0.6,0.7);0.4,0.3)	((0.1,0.2,0.3,0.4);0.5,0.2)	((0.4,0.5,0.7,0.8);0.2,0.5)	((0.3,0.4,0.5,0.6);0.1,0.6)
A_2	((0.5,0.6,0.7,0.8);0.6,0.2)	((0.4,0.5,0.6,0.7);0.6,0.1)	((0.3,0.4,0.6,0.7);0.6,0.1)	((0.4,0.5,0.6,0.8);0.3,0.4)
A_3	((0.1,0.2,0.3,0.4);0.5,0.3)	((0.1,0.2,0.4,0.5);0.4,0.3)	((0.4,0.5,0.6,0.7);0.4,0.2)	((0.2,0.4,0.6,0.8);0.5,0.2)
A_4	((0.2,0.3,0.4,0.5);0.7,0.1)	((0.1,0.2,0.3,0.5);0.5,0.2)	((0.1,0.2,0.4,0.6);0.2,0.3)	((0.5,0.6,0.7,0.8);0.1,0.5)
A_5	((0.1,0.2,0.3,0.4);0.5,0.1)	((0.2,0.3,0.4,0.5);0.3,0.2)	((0.1,0.2,0.3,0.4);0.6,0.2)	((0.4,0.5,0.6,0.7);0.4,0.2)

Table 3: The ITFN decision matrix by DM D_3

	C_1	C_2	C_3	C_4
A_1	((0.6,0.7,0.8,0.9);0.4,0.5)	((0.2,0.3,0.4,0.5);0.5,0.4)	((0.6,0.7,0.9,1);0.2,0.7)	((0.5,0.6,0.7,0.8);0.1,0.8)
A_2	((0.7,0.8,0.9,1);0.6,0.4)	((0.6,0.7,0.8,0.9);0.6,0.3)	((0.5,0.6,0.8,0.9);0.6,0.3)	((0.6,0.7,0.8,1);0.3,0.6)
A_3	((0.2,0.3,0.5,0.6);0.5,0.5)	((0.3,0.4,0.6,0.7);0.4,0.5)	((0.6,0.7,0.8,0.9);0.4,0.4)	((0.4,0.6,0.8,1);0.5,0.4)
A_4	((0.4,0.5,0.6,0.7);0.7,0.2)	((0.2,0.4,0.5,0.6);0.5,0.4)	((0.2,0.4,0.6,0.8);0.2,0.5)	((0.7,0.8,0.9,1);0.1,0.7)
A_5	((0.3,0.4,0.5,0.6);0.5,0.3)	((0.4,0.5,0.6,0.7);0.3,0.4)	((0.3,0.4,0.5,0.6);0.6,0.2)	((0.6,0.7,0.8,0.9);0.4,0.4)

Step 3: Aggregate the individual decision matrices $R_k = \{\tilde{r}_{ij}^k\}_{mxn}$ in to group decision matrix G= $[\tilde{g}_{ij}]_{mxn}$ using equation (5) and are listed in table 5.

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Step 4: The criterion weight vector can be determined using equation (8), $w = (0.273, 0.243, 0.264, 0.220)$

Step 5: Calculate the relative weight w_{jr} of criterion C_j to the reference criterion C_r, $w_{jr} = (1, 0.888, 0.967, 0.808)$

Step 6: The dominance of each alternative A_i over each alternative A_k under the criterion C_j can be obtained by equation (10) where θ is assumed to be 1.

Table 5: Group Decision Matrix

	C_1	C_2
A_1	((0.5050,0.6050,0.7050,0.8051);0.4391,0.3971)	((0.1398,0.2399,0.3399,0.4399);0.5324,0.2978)
A_2	((0.6085,0.7085,0.8085,0.9085);0.6411,0.2978)	((0.5185,0.6185,0.7185,0.8185);0.6429,0.2029)
A_3	((0.1399,0.2399,0.4143,0.5143);0.5369,0.4062)	((0.2139,0.3139,0.5139,0.6139);0.4393,0.4069)
A_4	((0.3315,0.4315,0.5315,0.6315);0.7521,0.1339)	((0.1406,0.3169,0.4169,0.5406);0.5374,0.3053)
A_5	((0.2288,0.3288,0.4288,0.5288);0.5441,0.2154)	((0.3139,0.4139,0.5139,0.6139);0.3398,0.3035)

	C_3	C_4
A_1	((0.4988,0.5988,0.7988,0.8988);0.2380,0.5934)	((0.3975,0.4975,0.5975,0.6976);0.1381,0.6930)
A_2	((0.4213,0.5213,0.7213,0.8213);0.6430,0.2061)	((0.5015,0.6015,0.7016,0.9016);0.3384,0.4952)
A_3	((0.5080,0.6080,0.7080,0.8080);0.4399,0.2973)	((0.3098,0.5098,0.7098,0.9098);0.4417,0.2967)
A_4	((0.1351,0.3065,0.5065,0.7066);0.2378,0.3985)	((0.5973,0.6972,0.7972,0.8972);0.1378,0.5919)
A_5	((0.2279,0.3279,0.4279,0.5279);0.6516,0.1433)	((0.5077,0.6077,0.7077,0.8077);0.3419,0.2944)

Step 7: Using equation (15) the global dominance degree of each alternative A_i over each alternative A_k are obtained

Step 8: The global value for alternative A_i can be obtained according to equation (13) as follows:

$\xi_1 = 0, \xi_2 = 1, \xi_3 = 0.438, \xi_4 = 0.288, \xi_5 = 0.488$

Step 9: Rank the alternatives in accordance with global value we get $A_2 > A_5 > A_3 > A_4 > A_1$. Therefore, A_2 is the best alternative.

Table 6: Ranking of alternatives with different values of ɵ

δ	$\theta = 1$		$\theta = 1.5$		$\theta = 2$		$\theta = 2.5$	
	ξ	Ranking	ξ	Ranking	ξ	Ranking	ξ	Ranking
A_1	0	5	0	5	0	5	0	5
A_2	1	1	1	1	1	1	1	1
A_3	0.438	3	0.382	3	0.371	3	0.362	3
A_4	0.288	4	0.26	4	0.249	4	0.241	4
A_5	0.488	2	0.46	2	0.435	2	0.415	2

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4.2 TOPSIS

The first 4 steps in TOPSIS are same as in TODIM.

Step 5: The positive ideal solution (PIS) G^+ and the negative ideal solution (NIS) G^- can be obtained by eq (14) which are listed in Table (7)

Step 6: The separation measures d_i^+ and d_i^- of each alternative from the PIS G^+ and NIS G^- respectively from eq (15) which are listed in Table (8) and (9)

Step 7: The relative closeness of an alternative A_i with respect to the PIS G^+ can be calculated by eq(16) that are listed in Table (10)

Table 7: The PIS and NIS

	G^+	G^-
C_1	((0.6085,0.7085,0.8085,0.9085);1,0)	((0.1400,0.2400,0.4143,0.5143),0,1)
C_2	((0.5185,0.6185,0.7185,0.8185),1,0)	((0.1399,0.2399,0.3399,0.4399),0,1)
C_3	((0.5080,0.6080,0.7988,0.8988),1,0)	((0.1351,0.3066,0.4279,0.5279),0,1)
C_4	((0.5972,0.6972,0.7972,0.9098),1,0)	((0.3097,0.4976,0.5976,0.6976),0,1)

Table 8: The distance between each alternative and PIS

	C1	C2	C3	C4
A1	0.4172	0.4896	0.4782	0.6285
A2	0.2491	0.1872	0.2570	0.4651
A3	0.5735	0.4547	0.3275	0.4013
A4	0.3689	0.4508	0.5298	0.5464
A5	0.5068	0.4282	0.4184	0.4059

Table 9: The distance between each alternative and NIS

	C1	C2	C3	C4
A1	0.3413	0.1789	0.2252	0.1219
A2	0.5094	0.4813	0.4464	0.2853
A3	0.1850	0.2138	0.3759	0.3491
A4	0.3896	0.2177	0.1736	0.2040
A5	0.2517	0.2404	0.2850	0.3445

Table 10: The relative closeness of each alternative and the rankings

	d_i^+	d_i^-	C_i	RANK
A1	0.4974	0.2229	0.3095	5
A2	0.2837	0.4366	0.6062	1
A3	0.4418	0.2785	0.3866	2
A4	0.4703	0.2500	0.3471	4
A5	0.4421	0.2782	0.3862	3

The larger the closeness coefficient C_i , is the best alternative. The ranking order of the five alternatives using TOPSIS is $A_2>A_3>A_5>A_4>A_1$. Therefore, A_2 is the best alternative.

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4.3: VIKOR

The first 4 steps in VIKOR are same as in TODIM.

Step 5: The best value G^+ and the worst value G^- which can be calculated by eq (17) which are listed in table (11)

Step 6: Compute the values S_i, R_i and Q_i using (18) that are listed in table (12)

Where $S^- = \max_i S_i$, $S^* = \min_i S_i$ and $R^- = \max_i R_i$, $R^* = \min_i R_i$ where γ is the decision coefficient. Rank the alternatives and derive the compromise solution The smaller the value Q_i, the better the alternative A_i.

Table 11: The best and the worst solution

	G^+	G^-
C_1	((0.6085,0.7085,0.8085,0.9085);1,0)	((0.1400,0.2400,0.4143,0.5143),0,1)
C_2	((0.5185,0.6185,0.7185,0.8185),1,0)	((0.1399,0.2399,0.3399,0.4399),0,1)
C_3	((0.5080,0.6080,0.7988,0.8988),1,0)	((0.1351,0.3066,0.4279,0.5279),0,1)
C_4	((0.5972,0.6972,0.7972,0.9098),1,0)	((0.3097,0.4976,0.5976,0.6976),0,1)

Table 12: The value S_i, R_i and Q_i

	S_i	R_i	Q_i	Rank
A1	0.6918	0.1843	0.8418	3
A2	0.3905	0.1364	0	1
A3	0.6122	0.2064	0.8679	4
A4	0.6556	0.1988	0.8857	5
A5	0.6140	0.1824	0.6994	2

The smaller value of Q_i is the best alternative. The ranking order of the five alternatives using VIKOR is $A_2 > A_5 > A_1 > A_3 > A_4$. Therefore, A_2 is the best alternative.

Conclusion

Thus in this paper, we made a comparative study on the multi criteria decision making methods namely TOPSIS, TODIM and VIKOR. In all the methods, Alternative 2 is the best option although the ranking order of all other alternatives are slightly different. When compared to TOPSIS and VIKOR, TODIM method gives us an accurate solution since TOPSIS and VIKOR method does not consider the psychological behavior in to consideration. Therefore, it can be seen that TODIM method is more accurate and reasonable.

References

[1] K.Attanasov, Intuitionistic fuzzy sets, Fuzzy sets and systems, 20 (1) (1986) 87-96.
[2] K.Attanasov G.Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy sets and systems, 31 (3) (1989) 343-349.
[3] L.F.A.M.Gomes, M.M.P.P. Lima, TODIM: basics and application to multi criteria ranking of projects with environ
[4] L.F.A.M.Gomes, L.A.D.Rangel, An application to the TODIM method to the multi criteria rental evaluation of residential properties, European journal of operations research , 193, (1), (2009), 204-211.
[5] R.A.Krohling, A.G.Pacheco , A.L.Sivireo, IF-TODIM: an intuitionistic fuzzy TODIM to multi criteria decision making, knowledge based systems, 53 (2013) 142-146.
[6] Q.Qin et al, A TODIM based multi criteria group decision making with triangular intuitionistic fuzzy numbers, 55 (2017) 93-107.

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[7] Hwang CL and Yoon k (1981) Multi criteria decision making lecture notes in Economics and Mathematical systems, 186, Springer verlag berlin, Germany.
[8]G.W.Wei, Some arithmetic aggregation operators with intuitionistic trapezoidal fuzzy number and its application to multi criteria decision making, Journal of Computers 5 (2010) 345-351.
[9] L.A.Zadeh, Fuzzy sets, Information control 8 (1965) 338-353.
[10] X.Zhang, P.D.Liu, Method for aggregating triangular intuitionistic fuzzy information and its application to decision making , Technological and Economic Development of Economy 16 (2010) 280-290.
[11] Zanakis.s.h.h Solomon A wishart N and Dublish s (1998) Multi criteria decision making: A simulation comparison of select methods, European journal of operations research, vol 107, No 3 pp 507-529

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DETECTION OF BREAK IN 115-YEAR SOUTH INDIAN RAINFALL SERIES USING STATISTICAL ANALYSIS

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Abstract

Global as well as regional climate has changed due to human activities like unwise land use, release of large quantities of Green House Gases, deforestation etc. The consequences of these activities have led to global warming and climate change. Global warming, in turn has a massive impact on atmospheric events especially on precipitation (rainfall / snowfall) which will be reflected upon the water availability. Hence, it is important to develop an adaptation and mitigation strategies in National and International levels using available precipitation information to overcome the water scarcity in future. In order to develop such strategies, it is necessary to understand the probable break event (defined as an abrupt change in the probability law of the series at a given time) in the yearly and monthly rainfall series. This will help to estimate the future water availability and amount of water requirement to overcome the future water shortage if any.

Khronostat is a statistical model used in this research analysis which was developed by IRD (Research Institute for Development) at the House of Water Sciences of Montpellier. Khronostat includes various statistical tests: the first deals with the randomness and series of the data and the second with the homogenous character of the series, i.e., no breaks. Pettitt test, Hubert Segmentation test, Buishand test, Lee and Heghinian tests are related to the detection of breaks.

The rainfall data from 8 regions of South India namely Tamil Nadu (TN), Telangana (TS), Coastal Andhra Pradesh (CAP), Rayalaseema (RAP), Kerala (KL), Coastal Karnataka (CKA), North Interior Karnataka (NKA) and South Interior Karnataka (SKA) were collected for the time period of 115 years (from 1901 to 2015). The annual and seasonal (Autumn, Spring, Summer and Winter) precipitation were calculated from the monthly data using normal statistical analysis. The four numbers of tests were used in this study to analyze the break event from 115 years of data for all 8 regions. This study provides interesting opportunities to climate experts and water stakeholders in understanding the link between climate instability and water availability, thus contributing to the sustainable management of water resources.

Key Words: *Khronostat Statistical Model, Rainfall data, Climate, Water availability*

Introduction

The climate change is occurring due to human activities such as alteration in coastal areas, urban expansion, industrialization and over exploitation of natural resources. Decreasing of forest area increases the settlements and croplands; and decreasing in cropland increases the urban settlements which is one of the indicators for agriculture land crisis (Barry et al., 2017). The land use and land cover changes by new buildings, roads and other infrastructure development decreases the forest area and crop land and also decreases the groundwater level without runoff penetration into the ground. Ogungbenro and Morakinyo (2014) stated that information of prolonged variability in

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averaged annual rainfall is important in understanding the groundwater resources and hydrological sector. Changes in land use and land change are closely linked with climate change and it is important to evaluate the climate change through rainfall and temperature analysis (Barry et al., 2017). Conway et al., (2009) stated that changes in rainfall are magnified in the runoff response and this level of variability presents significant challenges to water resources management. Jun and Bantin (2017) clearly highlighted that rainfall and river flow rates are correlated with each other. The decline in rainfall decreases the flow of water in the river. River basin, rainfall series and extensive river flow records are used to characterize and improve understanding of spatial and temporal variability (Conway et al., 2009). Grelle et al., (2016) stated that the impact of climate change on the water resource of the river shows by decreasing level of precipitations and flow rate; no action can stop the variability of climate factors other than necessary management of water resources.

Water is an essential production factor in agriculture for both crop production and livestock production. Agriculture is a key sector for a developing country and adaptation will certainly be necessary to address climate-related challenges such as desertification, degradation of land, drought, floods and water scarcity. In addition, agriculture is the main source of income for the majority of the working population in India. Ogungbenro and Morakinyo (2014) highlighted the supposed redistribution of rainfall due to global warming imposes a change in zonal rainfall received which could alter agricultural practices. The climatic cataclysms that have affected our planet in recent decades, the drought that hit Chad in 1973 has been particularly felt in the field of agriculture, ecology and also has a preponderant place in the drying up of Lake Chad (Jun and Bantin, 2017). Ogungbenro and Morakinyo (2014) recommended the farmers to plant drought resistant crops or early maturing crop varieties instead of over dependence on rain-fed agriculture.

Drought is one of the most worrying manifestations of variability. The effects of drought are many. It threatens on human life, socio-economic activities, and agricultural productivities through shortage of water. It also affects natural resources and environments, so the climate change has become popular by drawing attention of scientists, researchers and policy makers all over the world. The exchange and sharing of knowledge, information and experiences of different adaptation practices at all levels of the global community, including the regional, national and local community levels, therefore is necessary for the timely implementation of effective adaptation actions (FAO, 2011). The IPCC (2007) further concluded that global temperature increases are likely to persist in the 21st century and will probably be accompanied by changes in precipitation and runoff amounts. Limantol et al (2016) stated from their findings as others that climate change will affect agricultural systems ranging from commercial to subsistence/smallholder farmers in all countries. For developing countries where agriculture serves as the main pillar of the national economy, adaptation in smallholder farming systems becomes crucial for food security and poverty reduction, as well as for other socio-economic and structural development (FAO, 2010).

Limantol et al (2016) examined the farmers' perceptions and adaptation practices due to climate change and climate variability in recent years and their results showed that 90% of farmers believed that temperature increased over past 30 years, 94% of the farmers believed that decreasing of total rainfall, duration, intensity and rainy days; and 96% of the farmers believed that their farms were extremely vulnerable due to the above situations. Jun and Bantin (2017) also warned that variability in hydro-climatic parameters has a negative impact on agricultural, pastoral and fisheries activities which results major threat to the population.

The large variations in water levels and landscapes are directly dependent on rainfall which indicate that agricultural production be seriously compromised by climate change resulting in wide variability

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in the availability of water resources. The analysis of trends, shifts, variability and extremes is helpful in understanding the effect of climate changes.

For the above situations all over the world, it is imperative to understand the rainfall time series and also the probable break event of the year and month. These will help to estimate the future water availability and amount of water requirement during the break period. In this study, analyzing the break test using the available rainfall data and detecting the past break event.

A "break" can be defined by a change in the law of probability of the random variables whose successive achievements define studied time series (Galstyan et al., 2016). The break tests enable to detect a change in a time series and can be evaluated through different methods such as Pettitt test (1979), the procedure of segmenting series hydro rainfall Hubert Segmentation and Carbonnel (1987), and the Bayesian of Lee and Heghinian procedure (1977). These methods were originally programmed in a Pascal environment. The results of this program have been confirmed using the Khronostat program developed at the Science House of water (ESM) of Montpellier (Lubès et al., 1998). Test of Pettitt (Pettitt, 1979): recognized by its robustness, the test is non-parametric and derived from the wording of the Mann-Whitney test. The absence of a break in the time series X is the hypotheses is H_0 void. Procedure for segmentation (Hubert Segmentation et al, 1989): the method is to cut the series m segments (m >1) so that the average calculated on any segment is significantly different from the average of the adjacent segment. This procedure is seen as a test of stationary. If the procedure does not acceptable segmentation of order greater than or equal to 2, then the hypothesis invalid (stationary series) is accepted. Bayesian from Lee Héghinian (1977) procedure is used for research of the average change in the sequence of independent random variables. It assumes that a change in average exists in a time series, and then considers the probable date.

In literature several studies are available on similar work (Villar et al., 2009; Dounia et al., 2013; Ogungberno and Morakinyo, 2014; Taxak et al., 2014; Traore, 2014; Doumoua et al., 2016; Galstyan et al., 2016; Grelle et al., 2016; Limantol et al., 2016; Barry et al., 2017; Jun and Bantin 2017; Tossou et al., 2017; Traore et al., 2017).

In the present study, 8 number of regions were chosen from South India (Figure 1). They were Tamil Nadu (TN), Telangana (TS), Coastal Andhra Pradesh (CAP), Rayalaseema (RAP), Kerala (KL), Coastal Karnataka (CKA), North Interior Karnataka (NKA) and South Interior Karnataka (SKA) for the time period from 1901 to 2015; 115 years of annual and seasonal data was used to analyze the break in the rainfall time series. The South India covers Tamil Nadu, Andhra Pradesh, Telangana, Kerala and Karnataka. The complete South India was chosen for this study to analyze the break in annual precipitation. Both Andhra Pradesh (undivided) and Karnataka having three number of regions each namely Rayalaseema, Coastal Andhra Pradesh, and Telangana, and Coastal Karnataka, North Interior Karnataka and South Interior Karnataka respectively. Hence, these six regions and other two states were considered in this study.

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Study Area

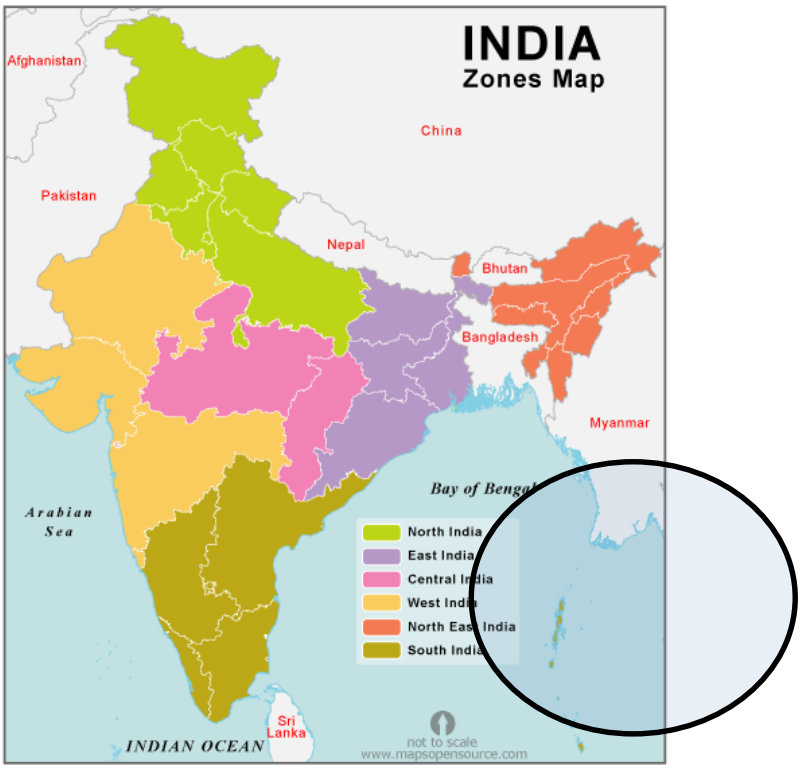

Figure 1: Study area from google image

Data Collection

Most of the studies used the rainfall data to evaluate the climate changes (Tossou et al., 2017; Traore et al., 2017; Jun and Bantin, 2017; Limantol et al., 2016; Doumouya et al., 2016; Ogungbenro and Morakinyo, 2014); Taxak et al., 2014; Dounia et al., 2013; and Espinoza Villar et al., 2009); Barry et al (2017) used both temperature and rainfall data; Galstyan, et al., (2016) used yearly air temperature data; Grelle et al (2016) used rainfall data from 1929-2006 and flow rates data from 1951 – 1997 and 1998-2006; Conway et al., (2009) used river flow data.

In this present study, monthly rainfall data for the time period 1901 to 2015 was collected through Open Government Data (OGD) Platform, India through the online link http://data.gov.in. The annual and seasonal (Autumn, Spring, Summer and Winter) precipitation were calculated by statistical test from the monthly data for all regions and for the same time period. No information is available about the instruments, methods, location of monitoring station used to record the preparation data in all eight regions.

Methodology

Statistical Test

The tests are to determine whether the characteristic of a sample data is identical to standard data or with another sample. Ordinarily, these tests are ranged in two categories: i) Parametric tests

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assume that the time series data and the deviations from the data trend follow a particular distribution ii) Non-parametric tests are generally distribution-free and the goal is to decide between two hypothesis, one called null or fundamental (denoted by H_0) and other called alternative (denoted by H_1).

The starting point of a statistical test is to define a null hypothesis (H_0) and an alternative hypothesis (H_1). To test for trend in the mean of a time series, H_0 would be that there is no change in the mean of the data, and H_1 would be that the mean is either increasing or decreasing with time.

Description of Khronostat Model

Khronostat is a statistical model developed by IRD (Research Institute for Development) at the House of Water Sciences (MSE) of Montpellier. It was developed as part of a study on climate variability in West and Central Africa and is oriented on the analysis of hydroclimatic series (Barry et al., 2017). It can evolve on an annual, monthly or daily scale depending on the needs expressed.

Khronostat includes various statistical tests: i) the first test category concerns the randomness (or independence test) series (correlation test on the rank and autocorrelogram test): they carry to the constancy of the average of the series throughout its observation period and ii) the second test category concerns the homogeneous character of the series (Pettitt test, Buishand test, Hubert Segmentation test, Bayesian methods or Lee & Heghinian test): they relate to the detection of breaks in a time series.

The application conditions of these tests are not very strict: i) allow to characterize as well as possible the evolution of climatic parameters; (ii) identify the pivotal years of climate change; (iii) complement climatic index calculations and iv) define the causes of series heterogeneity by sudden changes in climate series.

Khronostat is adapted to all variables (climatic, hydrological, meteorological,). However, it requires complete series with no gaps. It is also well advised by the meteorologists and climatologists over the world for surveillance or monitoring of drought.

Independence Test

The independence test is used to test the likelihood of no bond in a data. It provides information on the strength of the evidence and not on the strength of the association. These tests are powerful enough in general to distinguish between random and non-random nature of the series. The independent tests used in this article available in Khronostat are: the correlation test of rank and test the autocorrelogram.

Autocorrelogram Test

The estimation of the autocorrelogram is the first step in the statistical analysis of time series. The analysis of autocorrelogram is the finding of repeating pattern in a group of data.

Correlation Test of Rank

The rank correlation measures the relationship between rankings of different ordinal variables or different rankings of the same variable. A rank correlation coefficient measures the degree of similarity between two rankings.

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Homogeneity Test

A sample of data is said to be homogeneous if it presents no breaks. A break can be generally defined as an abrupt change in the probability law of the series at a given time, usually unknown. The break detection tests used in this article and available in Khronostat, are: Pettitt test, Buishand test, Hubert Segmentation test and Lee and Heghinian test. For all these tests, the null hypothesis that must be tested is H_0 « no break in the series ».

Pettitt Test

The Pettitt test is a non-parametric test that has been used in a number of hydroclimatological studies to detect abrupt changes in the mean of the distribution of the variable of interest. This test is often used to detect shifts in extremes because of the lack of distributional assumptions.

Hubert Segmentation Test

The Hubert Segmentation's segmentation procedure detects the multiple breaks in time series. The principle is to cut the series into m segments (m>1) such that the calculated means of the neighboring sub-series significantly differ.

Buishand Test

The Buishand method test the homogeneity of a time series, it is based on cumulative differences and is effective in the case of sharp changes of the mean.

Lee and Heghinian Test

The method determines a posteriori probability distribution function of the parameters μ and δ, considering their a priori distributions and supposing that the break time follows a uniform distribution. When the distribution is unimodal, the date of the break is estimated by the mode with more accuracy as the dispersion of the distribution is small.

Results

Time series analysis

The model was run in Khronostat software version 1.01; it results for both Independence tests and Homogeneity tests in data and graphical format. The results of each test executes in .kso file format, then .kso file can be opened by excel file.

The graphical representation of Autocorrelogram, Pettit, Hubert Segmentation, Buishand, and Lee and Heghinian tests are presented in Figures 2 to 41. The results of Break analysis are presented in Tables 1 to 8.

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Tamil Nadu

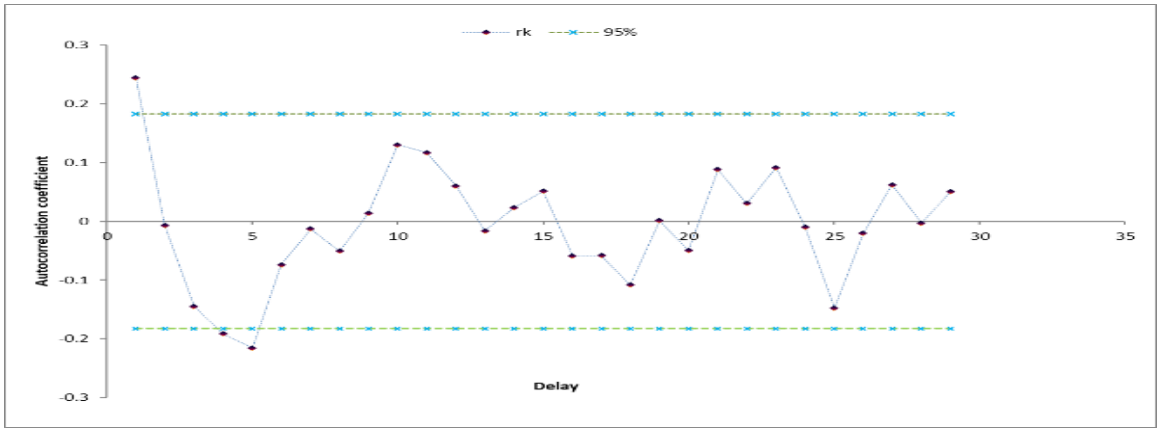

Figure 2: Tamil Nadu - Autocorrelogram

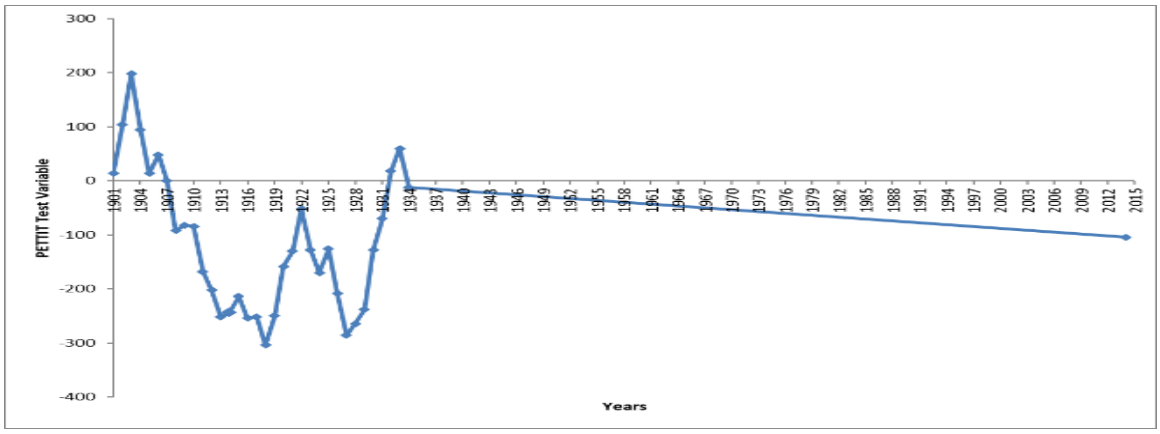

Figure 3: Tamil Nadu – Pettit test

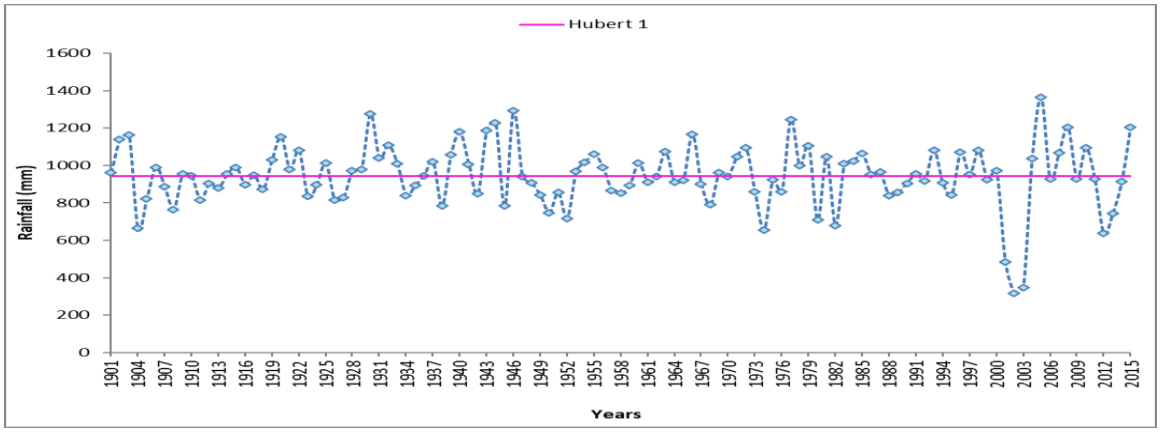

Figure 4: Tamil Nadu - Hubert Segmentation

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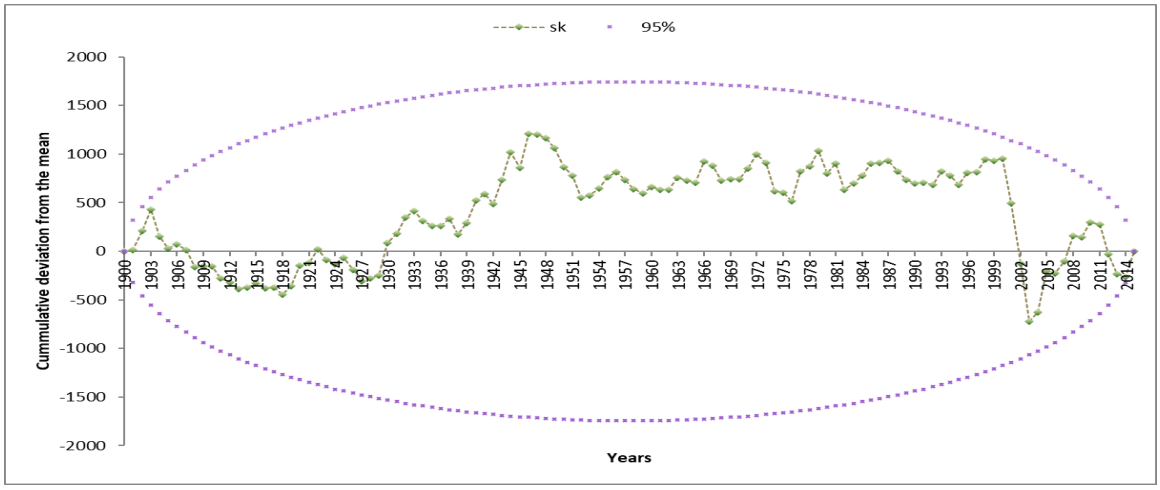

Figure 5: Tamil Nadu - Buisand Test

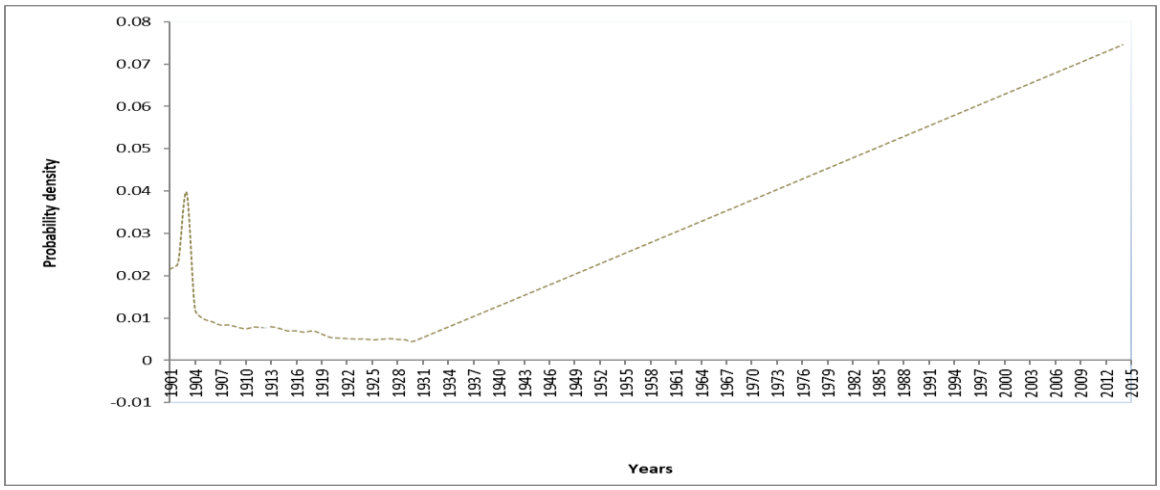

Figure 6: Tamil Nadu - Lee and Heghinian Test

Table 1: Tamil Nadu - Results of Break analysis

TEST	Null hypothesis	Break Date	Break Probability	Conclusion on series
Autocorrelogram	Accepted	-	-	Random series
Rank Correlation	Accepted	-	-	Random series
Pettitt	Rejected	1918	1.39e+00	Non-Homogeneous
Hubert Segmentation	Accepted	-	-	Homogeneous
Buishand	Accepted	-	-	Homogeneous
Lee and Heghinian	Rejected	2014	0.0745	Non-homogeneous

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For Tamil Nadu, the Independence test results that the data are in random series and the Homogeneity test results the break probability value of Pettitt test gives a very high probability of a break in the year of 1918 than the other tests. Hence, it is concluded that the break event happened in 115 years rainfall time series was in the year 1918.

Telangana

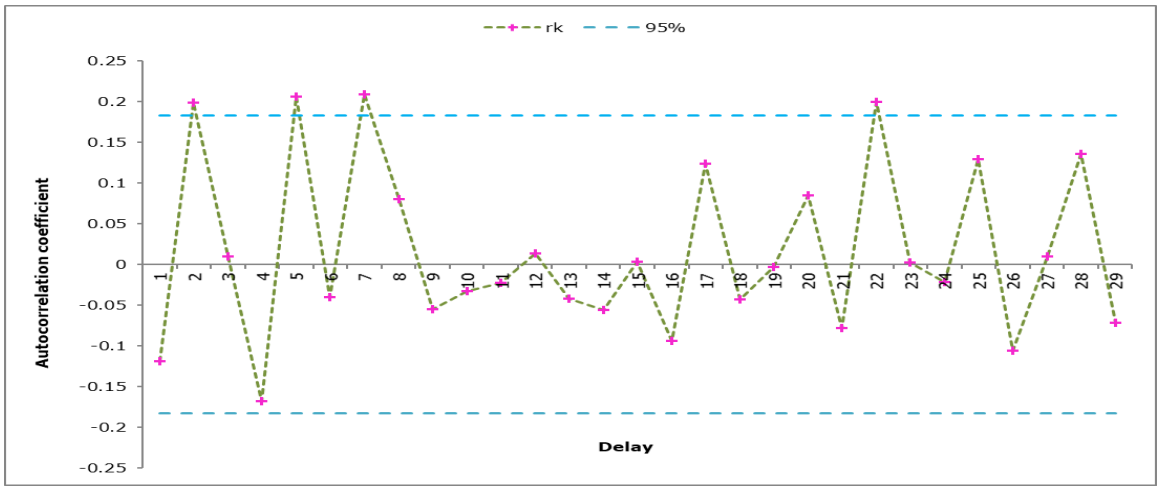

Figure 7: Telangana - Autocorrelogram Test

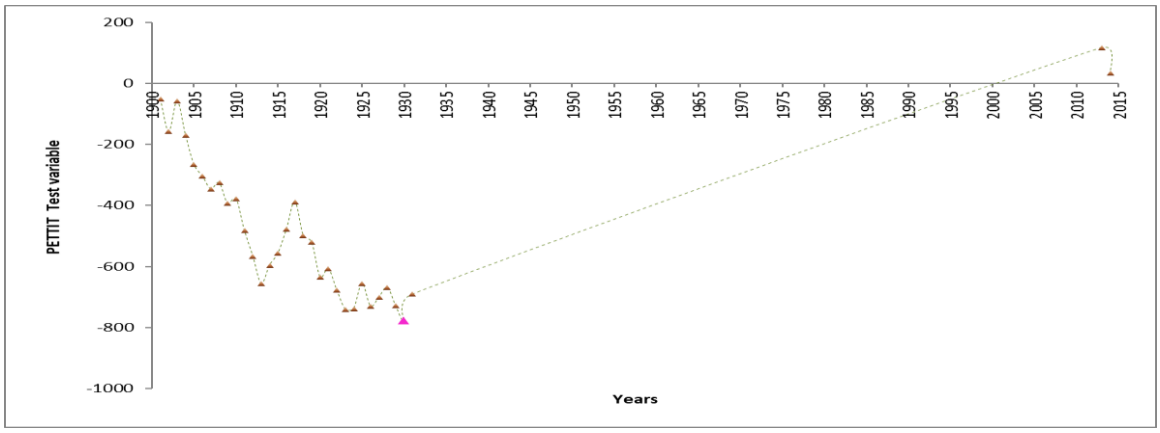

Figure 8: Telangana - Pettit Test

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Figure 9: Telangana - Hubert Segmentation test

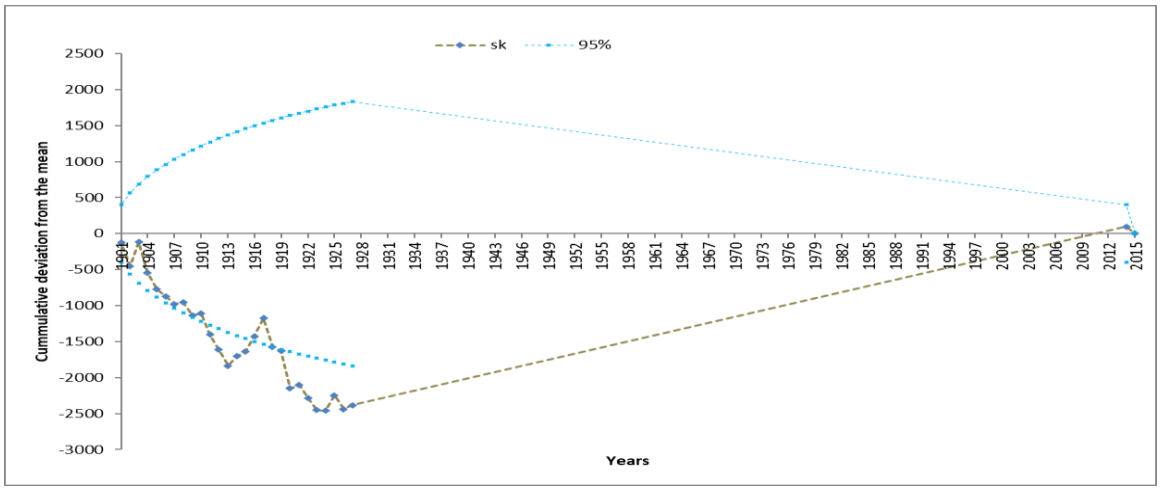

Figure 10: Telangana - Buishand Test

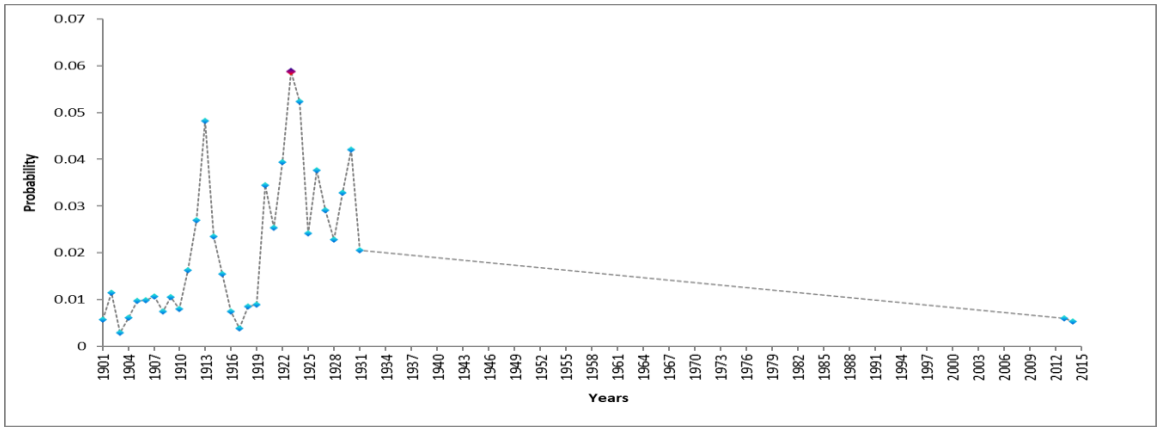

Figure 11: Telangana - Lee and Heghinian Test

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Table 2: Telangana - -Results of Break analysis

TEST	Null hypothesis	Break Date	Break Probability	Conclusion on series
Autocorrelogram	Accepted	-	-	Random series
Rank Correlation	Accepted	-	-	Random series
Pettitt	Rejected	1930	1.87e-01	Non-homogeneous
Hubert Segmentation	Rejected	1987,1990	-	Non-homogeneous
Buishand	Rejected	1952	-	Non-Homogeneous
Lee and Heghinian	Rejected	1923	0.0587	Non-homogeneous

For Telangana, the Independence test results that the data are in random series and the Homogeneity test results the break probability value of Pettitt test gives a very high probability of a break in the year of 1930 than the other tests. Hence, it is concluded that the break event happened in 115 years rainfall time series was in the year 1930.

Coastal Andhra Pradesh

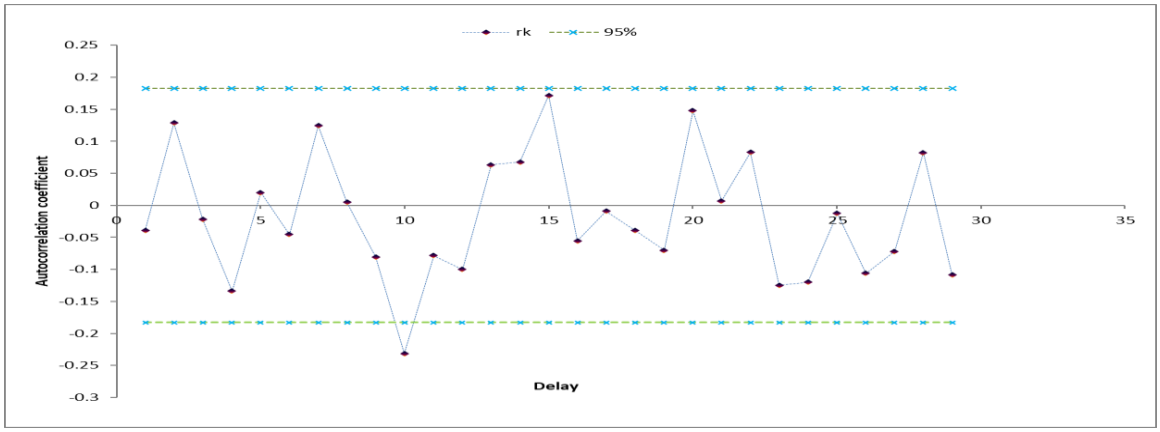

Figure 12: Coastal Andhra Pradesh – Autocorrelogram Test

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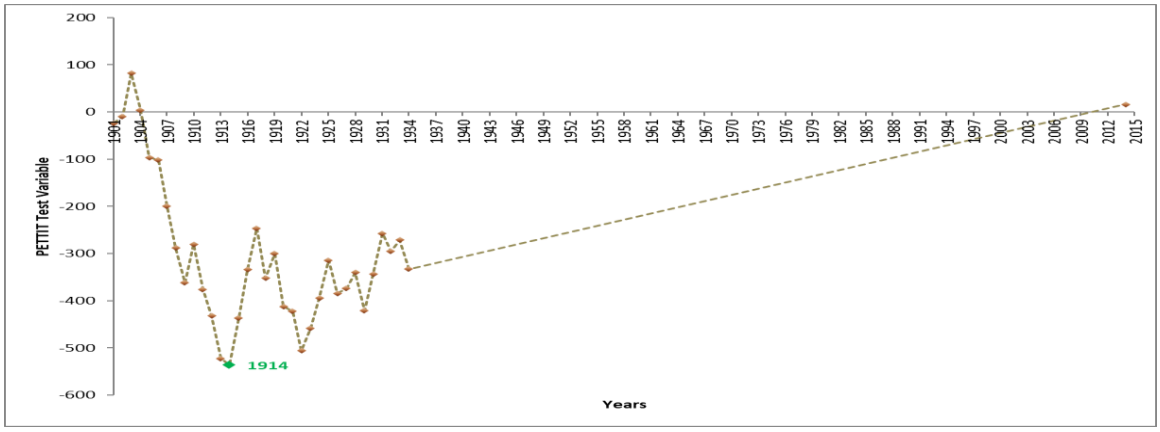

Figure 13: Coastal Andhra Pradesh - Pettit Test

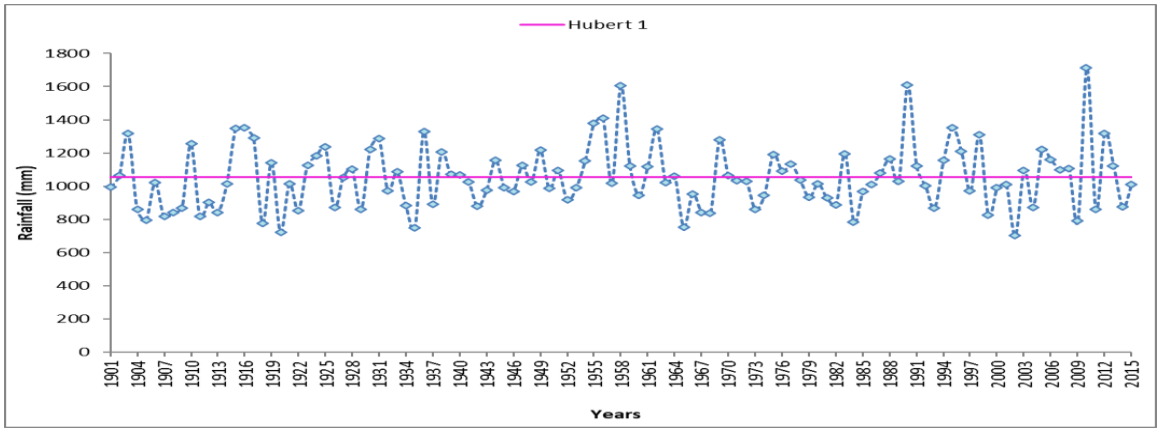

Figure 14: Coastal Andhra Pradesh - Hubert Segmentation Test

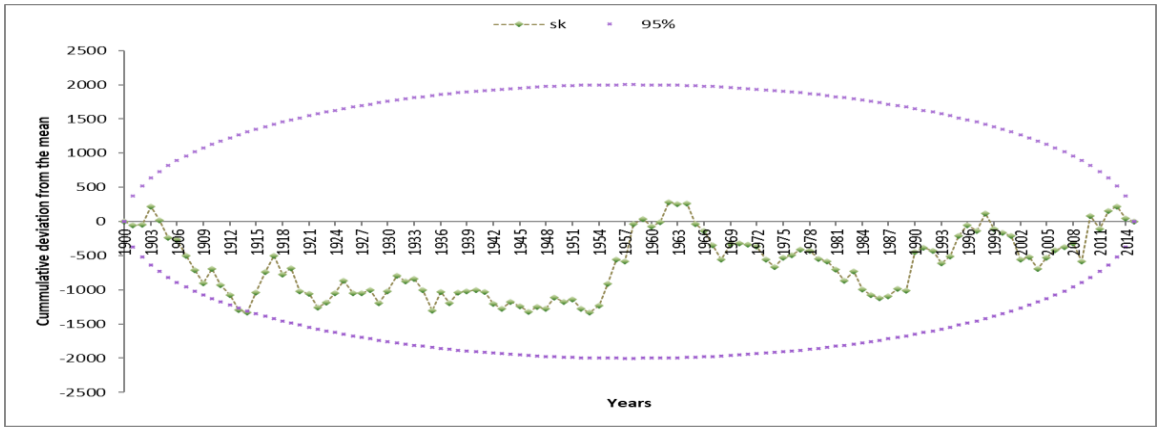

Figure 15: Coastal Andhra Pradesh - Buishand Test

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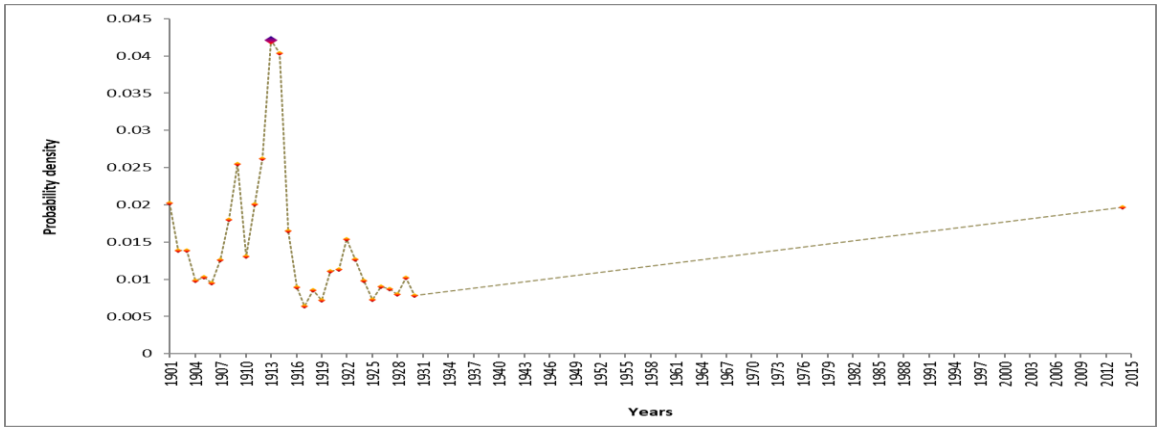

Figure 16: Coastal Andhra Pradesh - Lee and Heghinian Test

Table 3: : Coastal Andhra Pradesh – Results of Break Analysis

TEST	Null hypothesis	Break Date	Break Probability	Conclusion on series
Autocorrelogram	Accepted	-	-	Random series
Rank Correlation	Accepted	-	-	Random series
Pettitt	Rejected	1914	6.50e-01	Non-Homogeneous
Hubert Segmentation	Accepted	-	-	Homogeneous
Buishand	Accepted	-	-	Homogeneous
Lee and Heghinian	Rejected	1913	0.0421	Non-homogeneous

For Coastal Andhra Pradesh, the Independence test results that the data are in random series and the Homogeneity test results the break probability value of Pettitt test gives a very high probability of a break in the year of 1914 than the other tests. Hence, it is concluded that the break event happened in 115 years rainfall time series was in the year 1914.

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Rayalseema

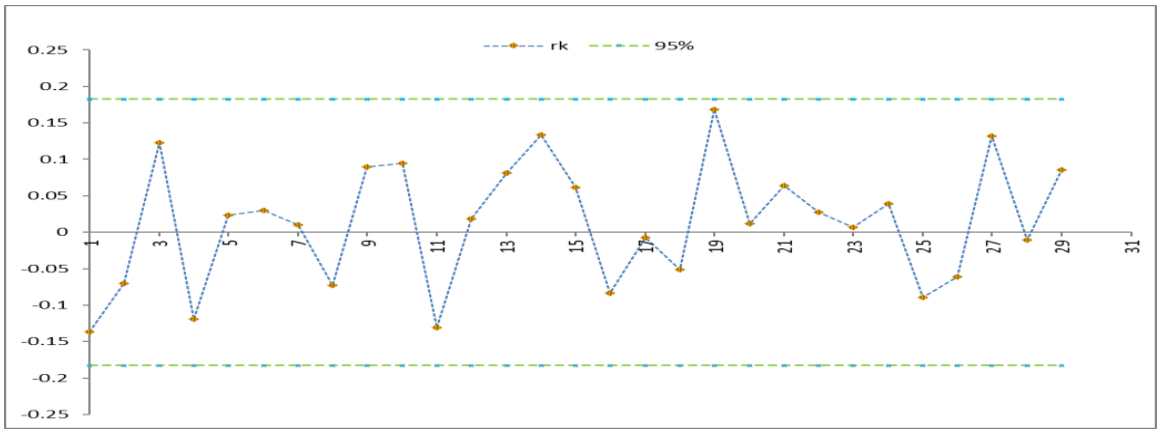

Figure 17: Rayalseema - Autocorrelogram Test

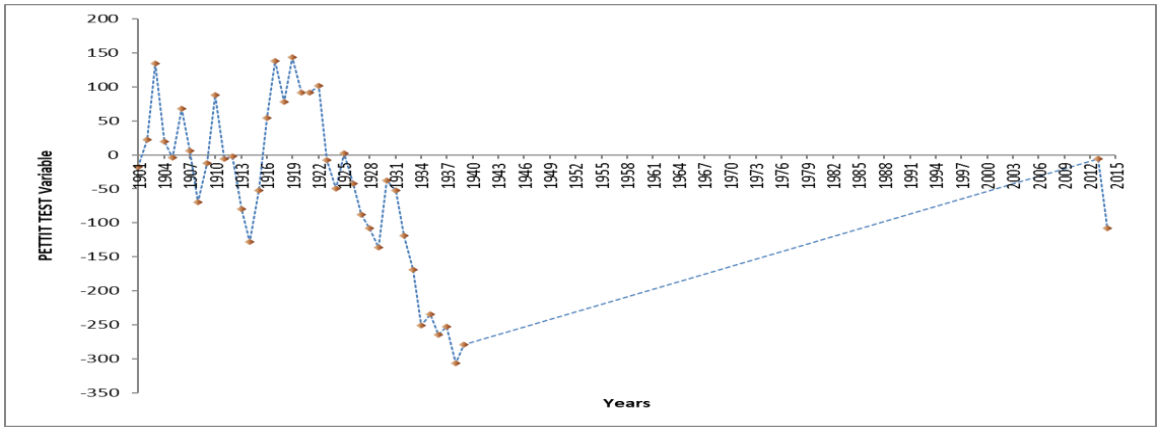

Figure 18: Rayalseema - Pettit Test

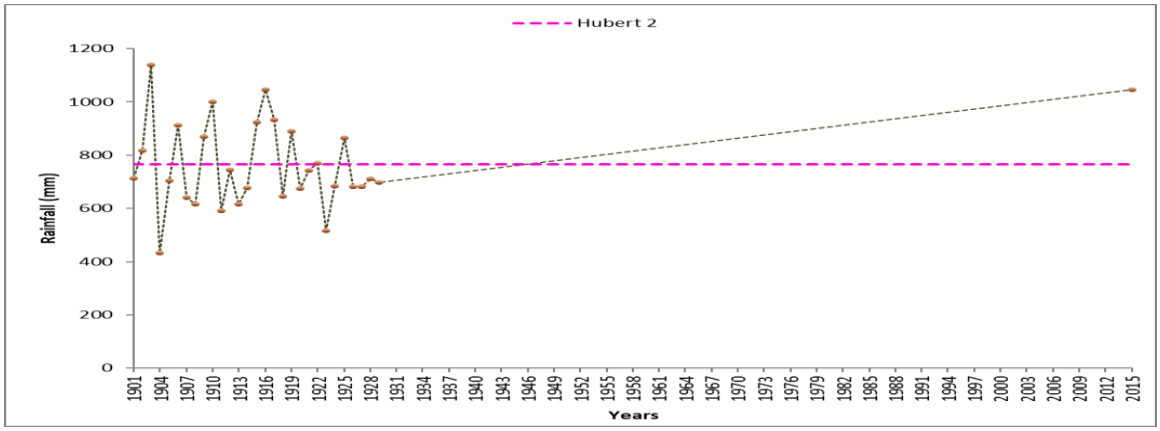

Figure 19: Rayalseema - Hubert Segmentation Test

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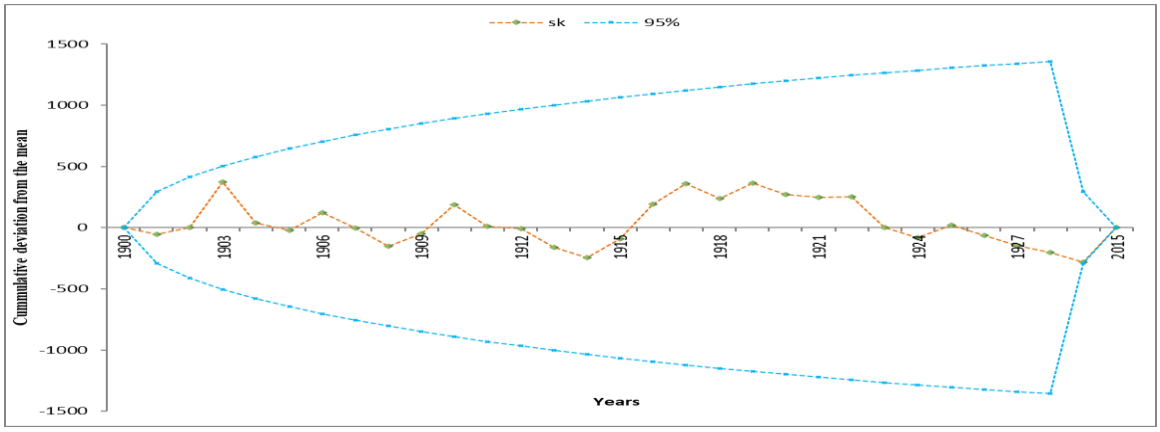

Figure 20: Rayalseema - Buishand Test

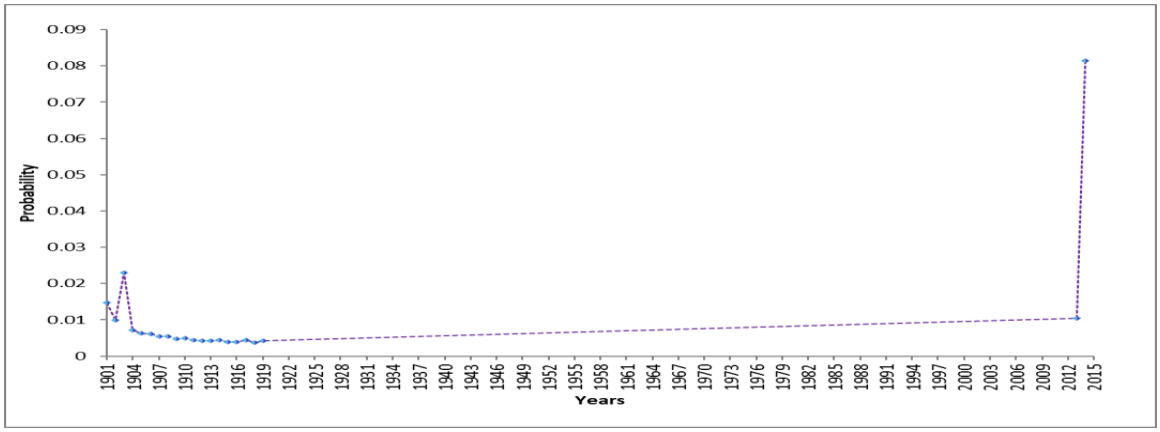

Figure 21: Rayalseema - Lee and Heghinian Test

Table 4: Rayalseema - Results of Break Analysis

TEST	Null hypothesis	Break Date	Break Probability	Conclusion on series
Autocorrelogram	Accepted	-	-	Random series
Rank Correlation	Accepted	-	-	Random series
Pettitt	Rejected	1974	3.70e-01	Non-Homogeneous
Hubert Segmentation	Accepted	-	-	Homogeneous
Buishand	Accepted	-	-	Homogeneous
Lee and Heghinian	Rejected	2014	0.0811	Non-homogeneous

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For Rayalseema, the Independence test results that the data are in random series and the Homogeneity test results the break probability value of Pettitt test gives a very high probability of a break in the year of 1974 than the other tests. Hence, it is concluded that the break event happened in 115 years rainfall time series was in the year 1974.

Kerala

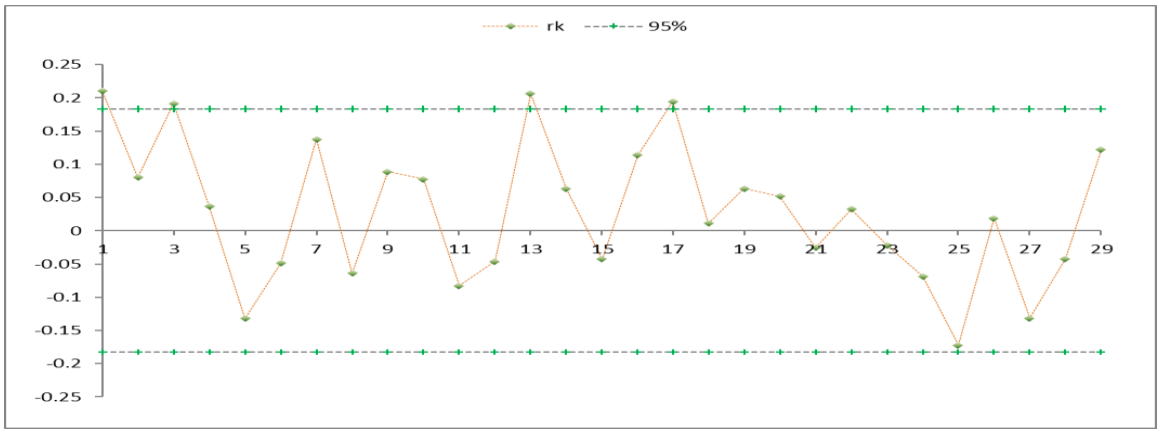

Figure 22: Kerala - Autocorrelogram Test

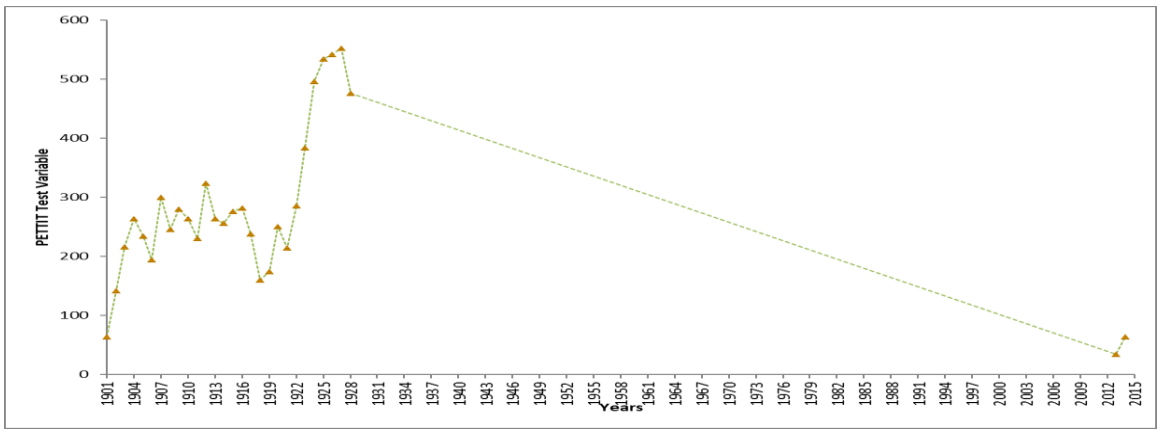

Figure 23: Kerala - Pettit Test

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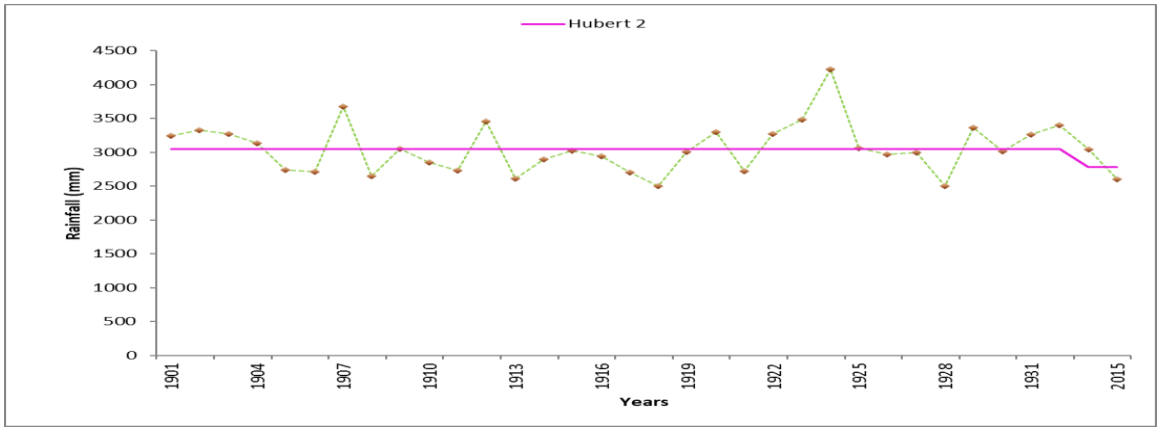

Figure 24: Kerala - Hubert Segmentation Test

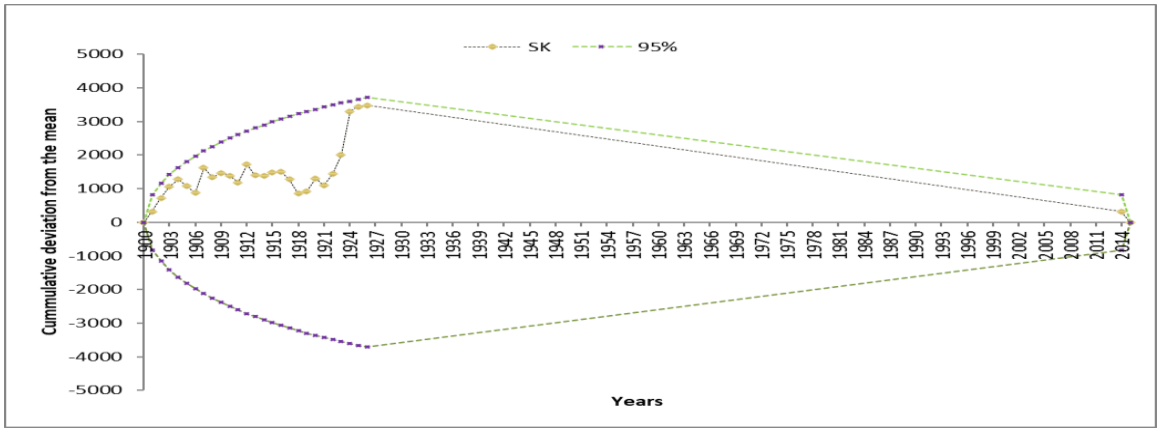

Figure 25: Kerala - Buishand Test

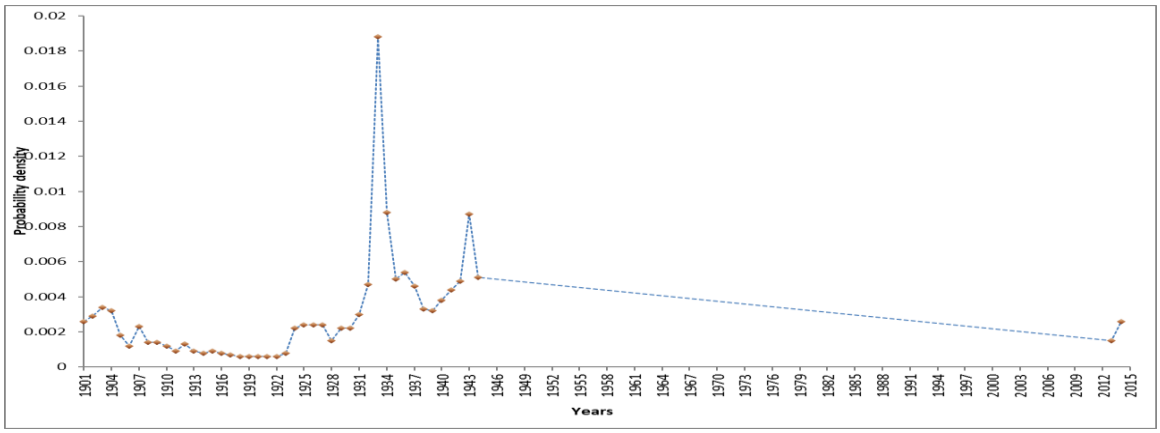

Figure 26: Kerala - Lee and Heghinian Test

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Table 5: Kerala - Results of Break Analysis

TEST	Null hypothesis	Break Date	Break Probability	Conclusion on series
Autocorrelogram	Accepted	-	-	Random series
Rank Correlation	Rejected	-	-	Non-Random series
Pettitt	Rejected	1962	1.42e-02	Non-homogeneous
Hubert Segmentation	Rejected	1958,1962	-	Non-homogeneous
Buishand	Rejected	-	-	Non-homogeneous
Lee and Heghinian	Rejected	1962	0.1652	Non-homogeneous

For Kerala, the Independence test of Autocorrelogram results that the data are in random series but Rank Correlation results Non-Random series, and the Homogeneity test of Pettit, and Lee and Heghinian results the break probability value in the year of 1962. Hence, it is concluded that the break event happened in 115 years rainfall time series was in the year 1962.

Coastal Karnataka

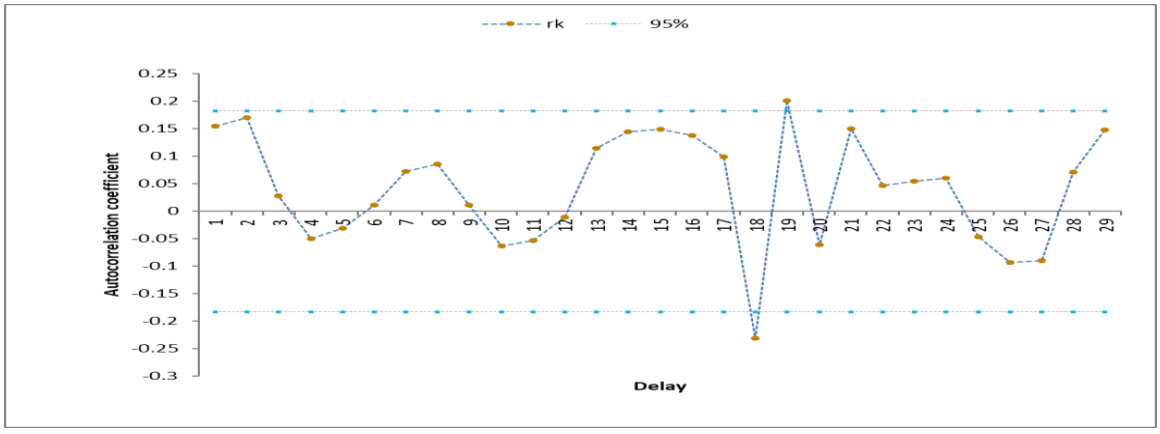

Figure 27: Coastal Karnataka - Autocorrelogram Test

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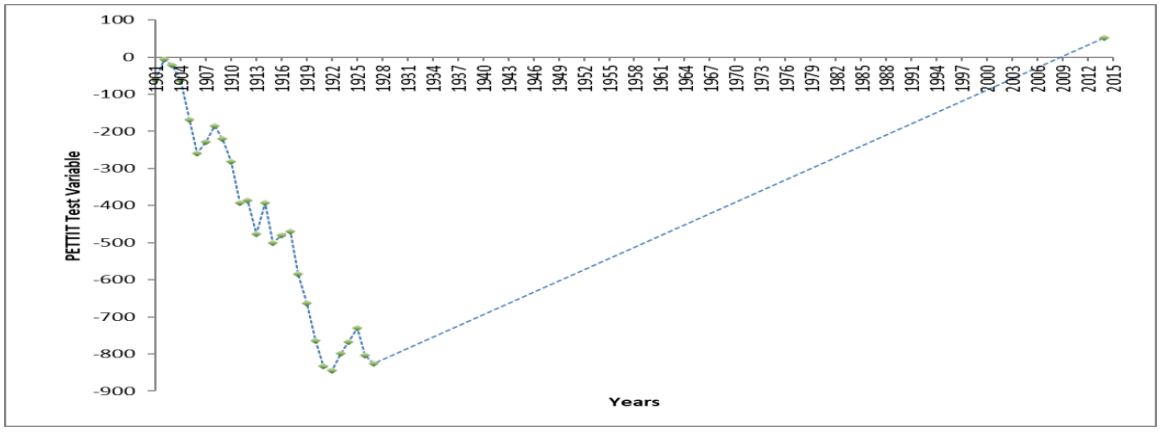

Figure 28: Coastal Karnataka - Pettit Test

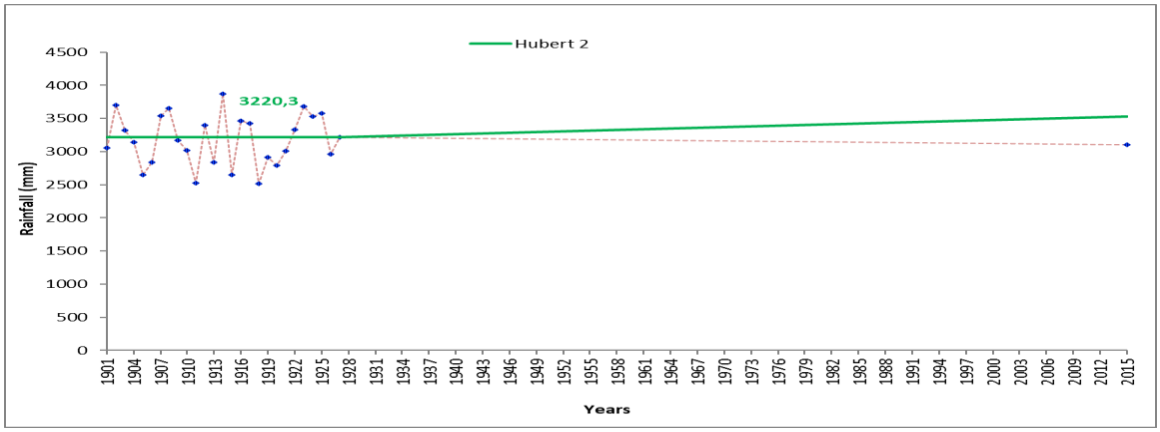

Figure 29: Coastal Karnataka - Hubert Segmentation Test

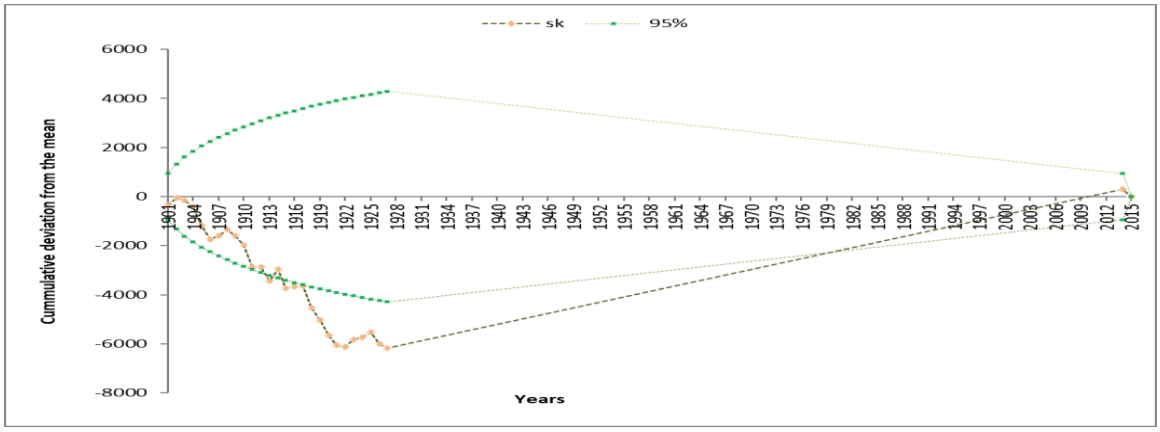

Figure 30: Coastal Karnataka - Buishand Test

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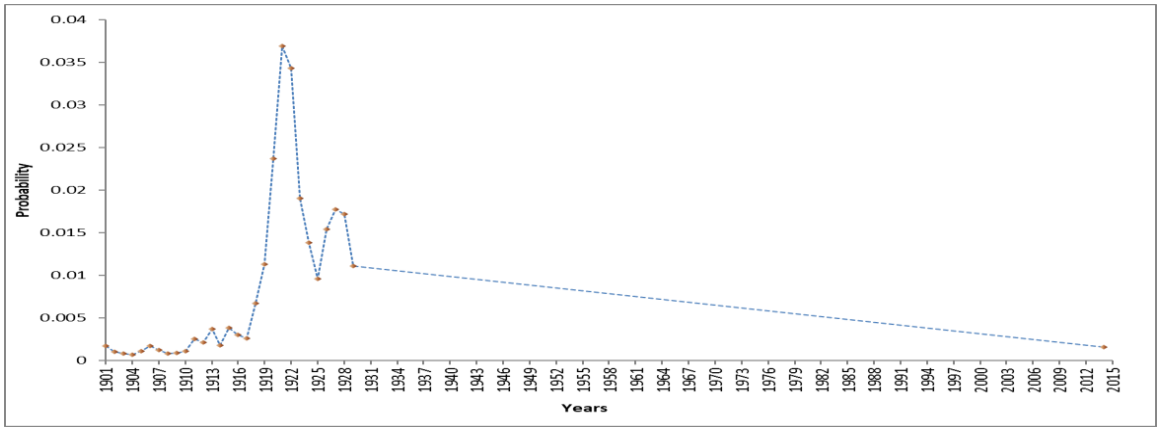

Figure 31: Coastal Karnataka - Lee and Heghinian Test

Table 6: Coastal Karnataka - Results of Break Analysis

TEST	**Null hypothesis**	**Break Date**	**Break Probability**	**Conclusion on series**
Autocorrelogram	Accepted	-	-	Random series
Rank Correlation	Rejected	-	-	Non-Random series
Pettitt	Rejected	1945	1.38e-02	Non-homogeneous
Hubert Segmentation	Rejected	1960,1961	-	Non-homogeneous
Buishand	Rejected	-	-	Non-homogeneous
Lee and Heghinian	Rejected	1945	0.0782	Non-homogeneous

For Coastal Karnataka, the Independence test of Autocorrelogram results that the data are in random series but Rank Correlation results Non-Random series, and the Homogeneity test of Pettit, and Lee and Heghinian results the break probability value in the year of 1945. Hence, it is concluded that the break event happened in 115 years rainfall time series was in the year 1945.

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North Interior Karnataka:

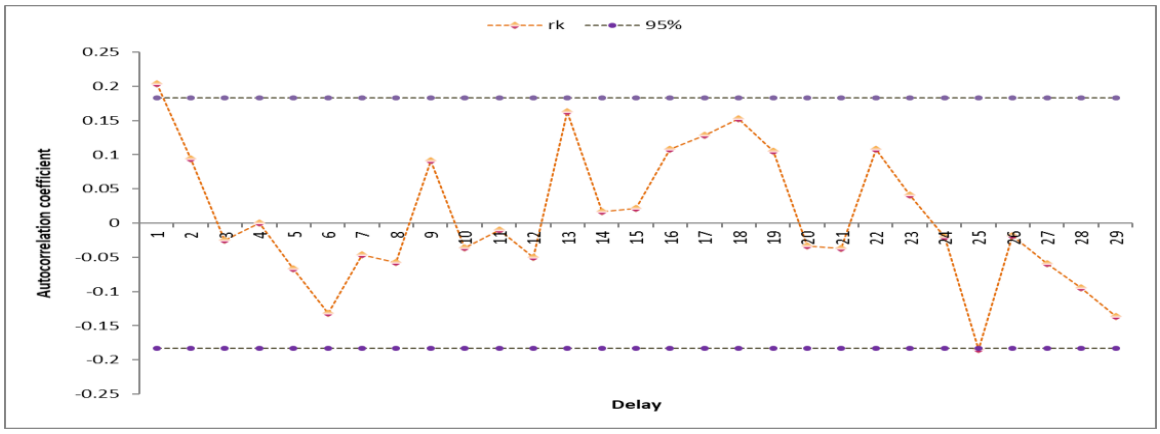

Figure 32: North Interior Karnataka - Autocorrelogram Test

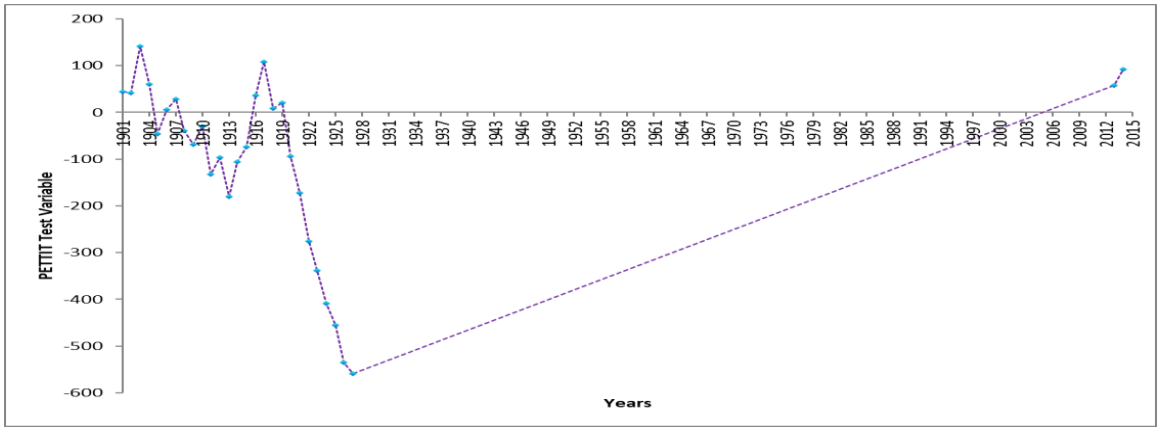

Figure 33: North Interior Karnataka - Pettit Test

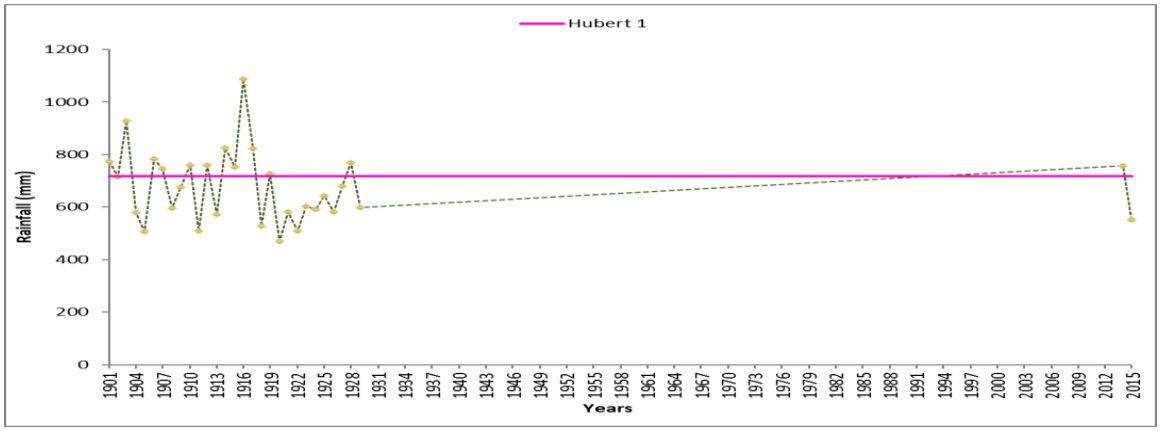

Figure 34: North Interior Karnataka - Hubert Segmentation Test

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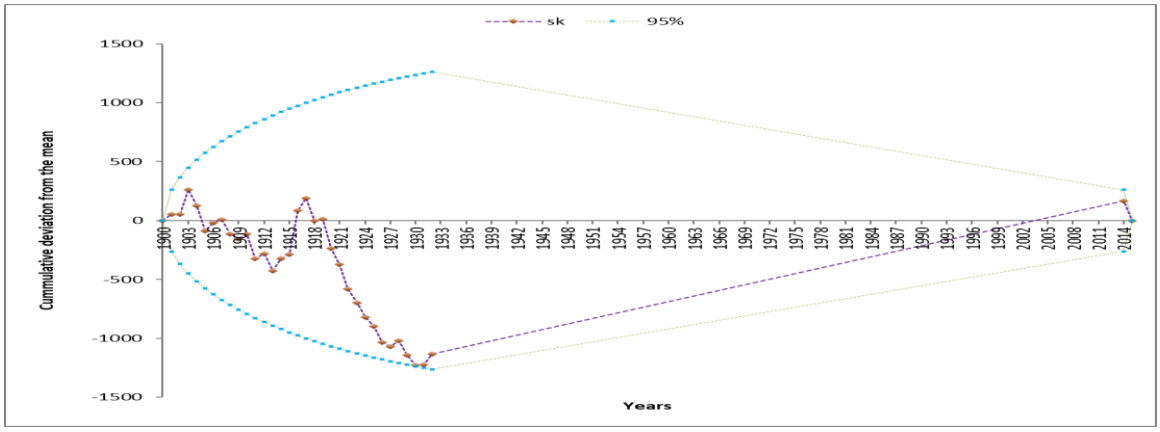

Figure 35: North Interior Karnataka - Buishand Test

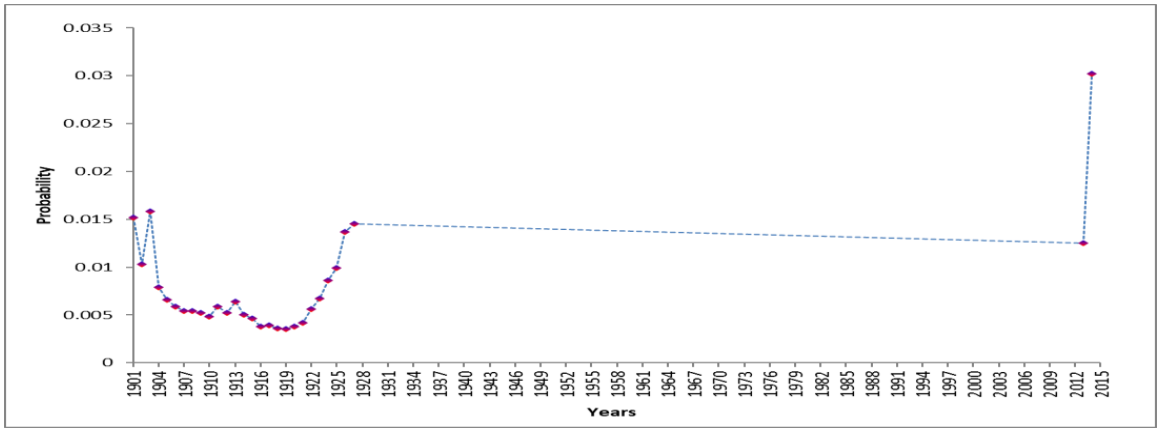

Figure 36: North Interior Karnataka - Lee and Heghinian Test

Table 7: : North Interior Karnataka – Results of Break Analysis

TEST	Null hypothesis	Break Date	Break Probability	Conclusion on series
Autocorrelogram	Accepted	-	-	Random series
Rank Correlation	Accepted	-	-	Random series
Pettitt	Rejected	1942	1.32e-01	Non-Homogeneous
Hubert Segmentation	Accepted	-	-	Homogeneous
Buishand	Accepted	-	-	Homogeneous
Lee and Heghinian	Rejected	1942	0.0424	Non-homogeneous

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For North Interior Karnataka, the Independence test results that the data are in random series and the Homogeneity test of Pettit, and Lee and Heghinian results the break probability value in the year of 1942. Hence, it is concluded that the break event happened in 115 years rainfall time series was in the year 1942.

South Interior Karnataka

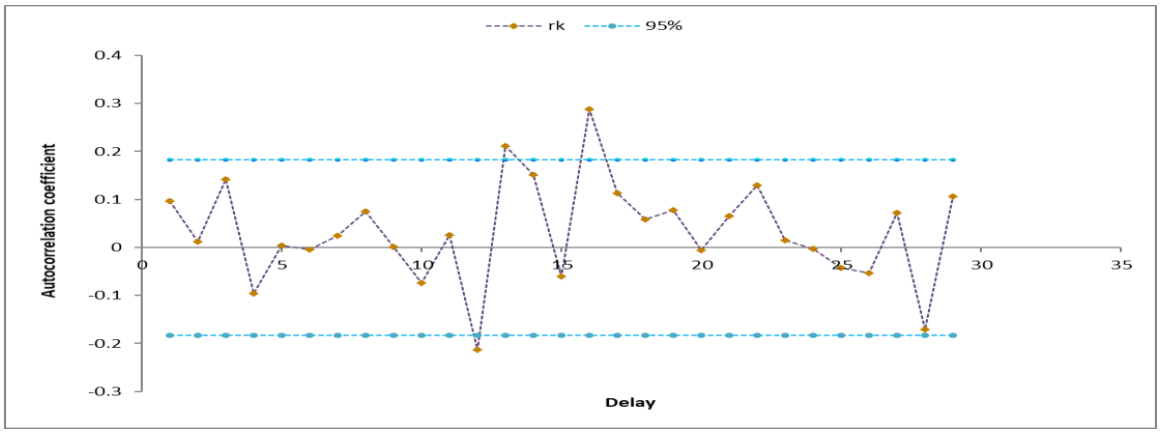

Figure 37: South Interior Karnataka - Autocorrelogram Test

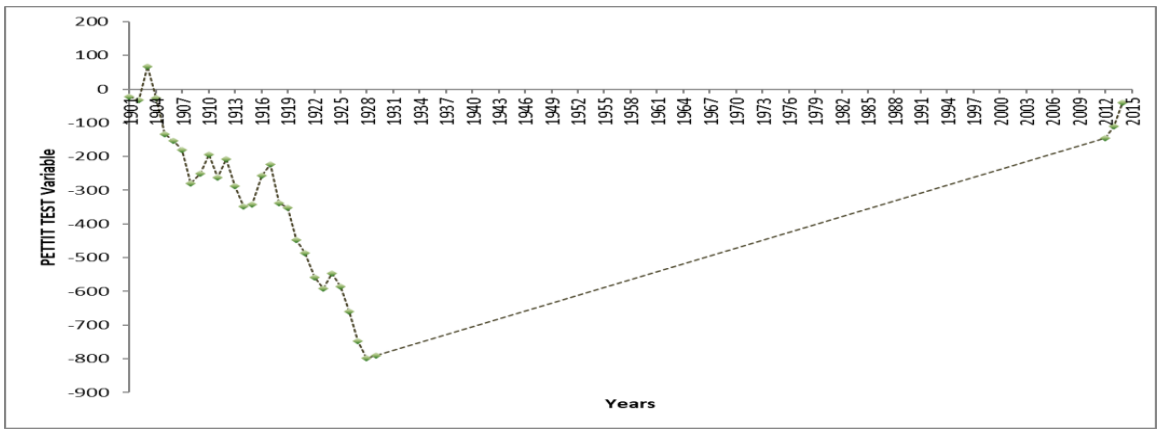

Figure 38: South Interior Karnataka - Pettit Test

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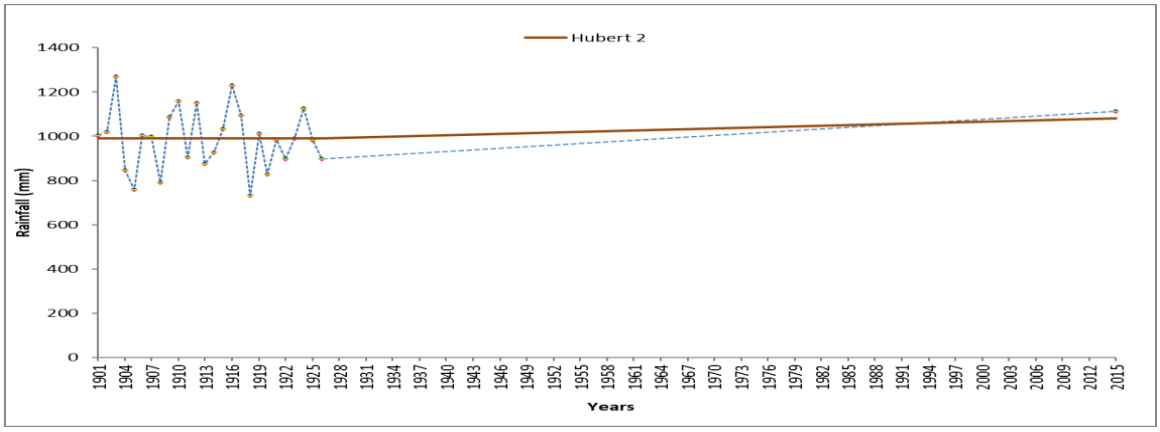

Figure 39: South Interior Karnataka - Hubert Segmentation Test

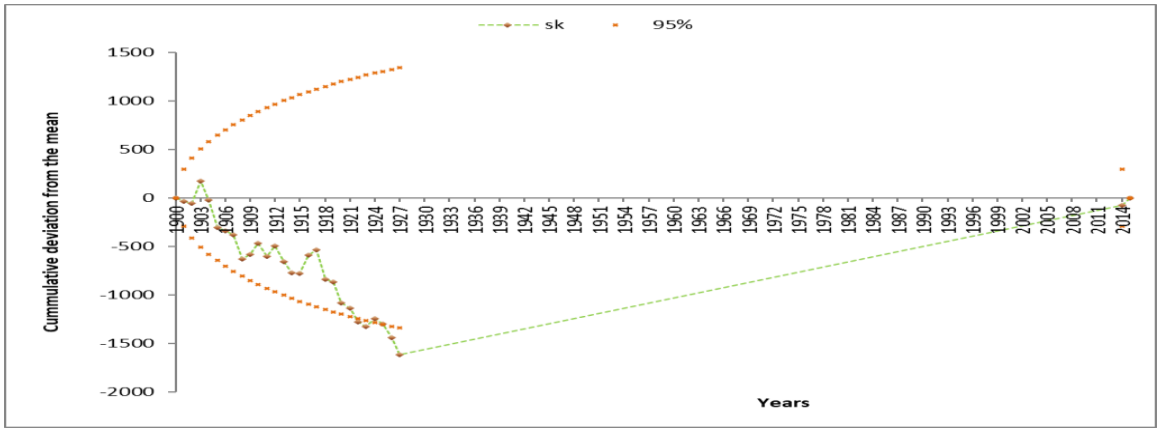

Figure 40: South Interior Karnataka - Buishand Test

Figure 41: South Interior Karnataka - Lee and Heghinian Test

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Table 8: South Interior Karnataka - Results of Break Analysis

TEST	Null hypothesis	Break Date	Break Probability	Conclusion on series
Autocorrelogram	Accepted	-	-	Random series
Rank Correlation	Rejected	-	-	Non-Random series
Pettitt	Rejected	1952	3.75e-03	Non-homogeneous
Hubert Segmentation	Rejected	1952	-	Non-homogeneous
Buishand	Rejected	-	-	Non-homogeneous
Lee and Heghinian	Rejected	1952	0.13	Non-homogeneous

For South Interior Karnataka, the Independence test of Autocorrelogram results that the data are in random series but Rank Correlation results Non-Random series, and the Homogeneity test of Pettit, and Lee and Heghinian results the break probability value in the year of 1952. Hence, it is concluded that the break event happened in 115 years rainfall time series was in the year 1952.

Conclusion

The eight regions of South India were considered in this study and found out that minimum of one break event and maximum of three break event happened in the last 115 years of rainfall time series. For a country, Agriculture and Tourism are playing a vital role in development, if both affects by shortage of water, then the income source of the country might result in poor condition. To overcome these types of problem, it is important to consider by climate experts and water stakeholders during the planning and construction of water resources project for sustainability nature.

Recommendation

This study was done with annual rainfall data; if more accurate data from the year of break even is needed, the same model could be run with monthly rainfall data. However, running with monthly data may be expensive.

References

1. Barry, N. Y., Traore, V. B., Ndiaye, M. L., Isimemen, O., Celestin, H., & Sambou, B. (2017). Assessment of Climate Trends and Land Cover/Use Dynamics within the Somone River Basin, Senegal. *American Journal of Climate Change*, *06*(03), 513–538. https://doi.org/10.4236/ajcc.2017.63026
2. Conway, D., Persechino, A., Ardoin-Bardin, S., Hamandawana, H., Dieulin, C., & Mahé, G. (2009). Rainfall and Water Resources Variability in Sub-Saharan Africa during the Twentieth Century. *Journal of Hydrometeorology*, *10*(1), 41–59. https://doi.org/10.1175/2008JHM1004.1

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3. Doumouya, F., Traore, V., Sadio, M., Sambou, H., Ali, A., Diaw, A., … Beye, A. (2016). Rainfall Variability in Sine Saloum River Basin in a Context of Climate Change and Variability. *Advances in Research*, *6*(6), 1–12. https://doi.org/10.9734/AIR/2016/25349
4. Dounia, M., Sabri, D., & Yassine, D. (2013). IMPACT OF CLIMATIC VARIABILITY ON THE WATER RESOURCES IN THE CATCHMENT SEYBOUSE, 6.
5. Espinoza Villar, J. C., Ronchail, J., Guyot, J. L., Cochonneau, G., Naziano, F., Lavado, W., … Vauchel, P. (2009). Spatio-temporal rainfall variability in the Amazon basin countries (Brazil, Peru, Bolivia, Colombia, and Ecuador). *International Journal of Climatology*, *29*(11), 1574–1594. https://doi.org/10.1002/joc.1791
6. FAO (Ed.). (2011). *Women in agriculture: closing the gender gap for development*. Rome: FAO.
7. Galstyan, H., Sfîcă, L., & Ichim, P. (2016). Long Term Variability of Temperature in Armenia in the Context of Climate Change, *10*(1), 7.
8. IPCC (Ed.). (2007). *Climate change 2007: the physical science basis: contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge ; New York: Cambridge University Press.
9. Jun, X., & Beyaitan Bantin, A. (2017). The Impact of Hydro Climatic Variability on Water Resources of Lake Chad Hydro Graphic Basin. *Hydrology: Current Research*, *08*(03). https://doi.org/10.4172/2157-7587.1000282
10. Limantol, A. M., Keith, B. E., Azabre, B. A., & Lennartz, B. (2016). Farmers' perception and adaptation practice to climate variability and change: a case study of the Vea catchment in Ghana. *SpringerPlus*, *5*(1). https://doi.org/10.1186/s40064-016-2433-9
11. Ogungbenro, S. B., & Morakinyo, T. E. (2014). Rainfall distribution and change detection across climatic zones in Nigeria. *Weather and Climate Extremes*, *5–6*, 1–6. https://doi.org/10.1016/j.wace.2014.10.002
12. Taxak, A. K., Murumkar, A. R., & Arya, D. S. (2014). Long term spatial and temporal rainfall trends and homogeneity analysis in Wainganga basin, Central India. *Weather and Climate Extremes*, *4*, 50–61. https://doi.org/10.1016/j.wace.2014.04.005
13. Tossou, E. M., Ndiaye, M. L., Traore, V. B., Sambou, H., Kelome, N. C., Aldiouma, B., & Diaw, A. T. (2017). Change point, Trend analysis, Rainfall variability, Climatic events, Water ressources, Khronostat software. *Resources and Environment*, 17.
14. Traore, V. B., Ndiaye, M. L., Mbow, C., Malomar, G., Sarr, J., Beye, A. C., & Diaw, A. T. (2017). Climatic events, Rainfed agriculture, Statistical approach, Climate monitoring, Sustainable water resources, Food safety. *World Environment*, 13.